The Feasibility of the Disturbance Accommodating Controller for Precision Antenna Pointing

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The objective of this study is to investigate the feasibility of a new pointing (position loop) controller for the NASA–JPL Deep Space Network antennas using the Disturbance Accommodating Control (DAC) theory. A model that includes state dependent disturbances was developed, and an example demonstrating the noise estimator is presented as an initial phase in the controller design. The goal is to improve pointing accuracy by the removal of the systematic errors caused by the antenna misalignment as well as sensor noise and random wind and thermal disturbances. Preliminary simulation results show that the DAC technique is successful in both cancelling the imposed errors and maintaining an optimal control policy.

I. Introduction

Large, precision antennas for millimeter and submillimeter wave astronomical listening require precision pointing capability in the face of a host of nonlinear and random disturbances. Included in this category of noise sources are structural member deflections under wind, thermal and gravitational loading, bearing friction torques, and hysteresis as well as electrical, optical, and mechanical misalignments introduced by sensors, thermal deformations, and structure model imperfections. Traditional approaches for compensating systematic disturbances rely on laboratory measurements and field data and employ open-loop (or feed-forward) compensation using static look-up tables to refine predicted target positions. These techniques, although satisfactory in sub-X-band RF pointing, are marginal for the state-of-the-art telemetry requirements for the upcoming Voyager–Neptune flyby and beyond. The augmentation of the deep space telemetry channel to provide Ka-band (32-GHz) capability to increase mission performance will require 1-mdeg pointing accuracy for feasible reception at distances greater than 20 AU. The performance advantage between the current X-band and projected Ka-band is highly dependent on antenna pointing accuracy. Successful deep space telecommunications will require the NASA–JPL 34-m and 70-m antenna pointing systems (see for example Fig. 1) to exhibit pointing errors of 1 mdeg (rms) or better. With current accuracy on the order of 5 to 10 mdeg, the antenna pointing loss at 32 GHz [1] as compared to loss at X-band (8.4 GHz) is magnified by the frequency squared. The increased gain advantage of the Ka-band could easily be lost without comparable enhancement in the pointing accuracy, a performance requirement implicit to the higher gain antennas with narrower beamwidth.

Antenna pointing–tracking errors are typically functions of static and dynamic factors. Mechanical misalignment of sensors or inaccuracy in the predicts can be considered static error sources, whereas dynamic factors would include wind, thermal and gravitational loading, etc. The approach taken in this study is first to achieve a static error-free environment for precision pointing of the antenna. The philosophy employed to achieve this goal is to treat the systematic misalignment errors, as well as the servocommands, as disturbances to the controlled system. Hence, as an initial phase in the sequence of new controller design, this study addresses pointing–tracking in the presence of noise. The research goal is intended to produce the end product controller; hence, further investigation is necessary to augment the controller to include dynamic errors caused by random thermal and wind disturbances.

Antenna pointing improvements can be developed through a sequence of progressive controller modifications solely in the existing software routines. Current algorithms can be enhanced to simultaneously provide servotracking and correction of the systematic errors as well as beam stabilization in the presence of random disturbance torques. For a given antenna, servodrive, feed configuration, and surface distortion profile a computer software package could be developed to optimize the performance to achieve, adaptively, the maximum antenna gain for a prescribed direction vector with a "smart" controller.

Typical antenna controllers consist of an analog rate loop and a position loop closed through a digital computer. The control algorithms for the position control are either proportional–integral (PI) or state feedback control. The PI control is accomplished by applying gains to the position error and the integral of position error. The weighted sum of these signals is the commanded rate for the velocity loop. Zero steady-state error to a ramp input is realized with the PI controller in this Type II system.

The more sophisticated method utilizing state feedback allows specification of the eigenvalues of the closed position loop. The initial disadvantage of the state feedback is the requirement that all the states of the system be available for the feedback control. The technique of state estimation has circumvented this problem, providing the controller with an estimate value for each of the unmeasurable or uncertain state signals. The feedback gains are selected to yield the designer's selected eigenvalues to achieve desired performance of the system. This technique was incorporated in the upgrade of the 70-m antenna axis servos [2], [3] with the estimator gain vector selection based on system specifications, minimal estimator error, and insensitivity to encoder and digital-to-analog (D/A) quantizations.

The unique controller enhancement proposed in this study suggests that, simultaneous with state estimation, another vector be estimated to represent the disturbance state which could then be used in determining a more complete control strategy. The philosophy of the Disturbance Accommodating Controller (DAC) is based on the concept of state modeling of the disturbance vector as developed in [4]. In the following section the basics of the DAC theory are discussed as applied to the antenna pointing problem. A history of this DAC technique appears in [5].

II. Theoretical Background

The antenna pointing system can be modeled by a set of first-order linear differential equations:

$$\left.\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{B}u + \mathbf{F}w; \qquad x(0) = x_0 \\ y &= \mathbf{C}x \end{aligned}\right\} \tag{1}$$

where $x(t)$ is an $(n \times 1)$ state vector, $u(t)$ is an $(r \times 1)$ control effort vector, $y(t)$ is the $(m \times 1)$ output vector, and $w(t)$ is a $(p \times 1)$ collective disturbance vector representing the combined effects of all the uncontrolled forces/torques acting on the antenna. For instance, $w(t)$ can be projected to include the effects of lateral winds, misalignments, etc. It is assumed that $w(t)$ cannot be predicted or measured accurately. The matrices **A**, **B**, **C**, and **F** represent the continuous-time transmission matrices for the respective signals.

The state variable feedback is represented by the control law

$$u = -\mathbf{K}_x x \tag{2}$$

where $\mathbf{K}_x$ is a feedback gain matrix, not necessarily producing an optimum controller. Two control cases will be analyzed further; the first assumes no disturbance **F** and the second case assumes that **F** exists.

A. Assuming No Disturbances, F = 0

Substitution of Eq. (2) into Eq. (1) yields the closed-loop form with no disturbances,

$$\dot{x} = \mathbf{A}x - \mathbf{B}\mathbf{K}_x x = (\mathbf{A} - \mathbf{B}\mathbf{K}_x)x \tag{3}$$

The advantage of the state feedback is the ease by which the closed-loop eigenvalues of the system, obtained from Eq. (3), are arbitrarily specified through the selection of the gain matrix $\mathbf{K}_x$ (also called the pole placement technique). However, the pole-placement method does not guarantee that the design is optimal. On the other hand, if the optimal controller is designed, the quadratic performance technique, from the theory of optimal control, provides the optimal steady-state solution to the minimum control effort and minimum transient deviation of the state from the origin problem, i.e.,

$$u = -\mathbf{K}_g x \tag{4}$$

where the Kalman gain $\mathbf{K}_g = -\mathbf{R}^{-1}\mathbf{B}^T \mathbf{P}$. The term $\mathbf{P}$ is a symmetric positive semidefinite solution to the steady-state Riccati equation in the matrix form, that is,

$$\mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} + \mathbf{Q} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} = 0 \tag{5}$$

The matrices $\mathbf{R}(>0)$ and $\mathbf{Q}(>0)$ are symmetric weighting matrices in the associated quadratic performance index for the continuous linear regulator, that is

$$J(t_0) = x(T)^T\mathbf{P}(T)x(T) + \int_{t_0}^{T} [x^T\mathbf{Q}(t)x + u^T\mathbf{R}(t)u]\, dt \tag{6}$$

Note that the control is a time-varying state feedback; even if $\mathbf{A}$, $\mathbf{B}$, $\mathbf{Q}$, and $\mathbf{R}$ are time-invariant, $K_g(t)$ varies with time.

Implementation of state feedback requires knowledge of the entire state vector. In practice, however, not all state variables are available for direct measurement. Hence, a closed-loop *estimator* is utilized to predict the values of the unmeasurable state variables based on the measurements of the output and control variables. The state estimator vector x' is described by

$$\dot{x}' = \mathbf{A}x' + \mathbf{B}u + \mathbf{K}_{0_x}(y - \mathbf{C}x'); \qquad x'(0) = x'_0$$

or

$$\dot{x}' = (\mathbf{A} - \mathbf{K}_{0_x}\mathbf{C})x' + \mathbf{B}u + \mathbf{K}_{0_x}y \tag{7}$$

where $\mathbf{K}_{0_x}$ is the estimator error gain matrix $(n \times m)$, and the eigenvalues of $(\mathbf{A} - \mathbf{K}_{0_x}\mathbf{C})$ are commonly called the observer poles. Recall that the system must exhibit complete observability in order to determine the state vector using the output and control variables.

The inaccuracy, e, in the state dynamics incurred in using the full-order $(n \times 1)$ estimate x' rather than the actual state x is given by subtracting Eq. (7) from Eq. (1) (with $F \equiv 0$), i.e.,

$$\dot{e} \equiv \dot{x} - \dot{x}' = (\mathbf{A} - \mathbf{K}_{0_x}\mathbf{C})e \tag{8}$$

where $e = x - x'$. From Eq. (8) it is apparent that the dynamic behavior of the error signal is determined by the observer poles. If the matrix $(\mathbf{A} - \mathbf{K}_{0_x}\mathbf{C})$ is a stable matrix, the error vector converges to zero for any initial error $e(0)$.

Since $\mathbf{A}$, $\mathbf{C}$ are fixed by the system, matrix $\mathbf{K}_{0_x}$ determines the estimator performance. Again the pole-placement technique can position the estimator eigenvalues from Eq. (8) for proper performance, that is, x' will converge to x regardless of the initial states $x(0)$ and $x'(0)$. Hence, the overall closed-loop noiseless system with full state estimator feedback can be expressed in the state variable notation as:

$$\begin{bmatrix} \dot{x} \\ \hline \dot{x}' \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{A} & -\mathbf{B}\mathbf{K}_x \\ \hline \mathbf{K}_{0_x}\mathbf{C} & \mathbf{A} - \mathbf{B}\mathbf{K}_x - \mathbf{K}_{0_x}\mathbf{C} \end{array}\right] \begin{bmatrix} x \\ \hline x' \end{bmatrix}$$

and

$$y = \left[\begin{array}{c|c} \mathbf{C} & 0 \end{array}\right] \begin{bmatrix} x \\ \hline x' \end{bmatrix} \tag{9}$$

Note that the dynamics of the closed-loop system depend on the eigenvalues of both the controller and the estimator. However, the separation principle allows the independent design of the controller and the estimator gain matrices assuming the observer poles are chosen correctly.

The optimal regulator described so far accommodates only initial conditions or impulse type disturbances and hence is incapable of tracking or handling typical noise inputs. In the case of finite input disturbances, the control law of Eqs. (2) through (9) cannot attain and maintain track, i.e., $y(t) \neq y_c(t)$ where y_c is the commanded output.

B. Case of F ≠ 0

Consider now the plant equations in the form of Eq. (1) with $\mathbf{F} \neq 0$,

$$\dot{x} = \mathbf{A}x + \mathbf{B}u + \mathbf{F}w \tag{10}$$

where $\mathbf{F}$ is an $(n \times p)$ matrix and $w(t)$ is a p-dimensional disturbance vector. Let us formulate the optimal regulator problem in such a way that at the terminal time T, the resulting control law always brings the state $x(t)$ and the velocity $\dot{x}(t)$ back to the commanded state and velocity, $x_c(t)$ and $\dot{x}_c(t)$, respectively, in the presence of any finite constant disturbance $w(t) \equiv k$. With neither the noise nor the servocommand known a priori, treating $w(t)$ as either a deterministic input or a nondeterministic input with a known probability is impractical since reliable information about the disturbance is not available. The primary objective of the control is to manipulate $u(t)$ in Eq. (10) so that the output $y(t)$ approaches and maintains the commanded value $y_c(t)$ promptly. Hence, the problem is reduced to finding a control which minimizes the functional $J(u)$,

$$J(u) = \lim_{t \to T} \int_0^T [\langle x\, Q\, x \rangle + f(u, \dot{u}, \ldots)]\, dt \qquad (11)$$

subject to the constraints imposed by Eq. (10) where the vector disturbance $w(0) = 0$, and $w(t)$ is assumed to satisfy the linear differential equation:

$$\alpha_\rho \frac{d^\rho w}{dt^\rho} + \alpha_{\rho-1} \frac{d^{\rho-1} w}{dt^{\rho-1}} + \ldots + \alpha_1 \frac{dw}{dt} + \alpha_0 w = 0 \qquad (12)$$

where the α_i are known, real scalar constants.

The class of admissible disturbances $w(t)$ defined in Eq. (12) can be characterized as the set of scalar functions

$$w(t) = \mathbf{H} z(t) \qquad (13)$$

with $\mathbf{H}$ a real p $\times$ ρ matrix and where

$$\dot{z}(t) = \mathbf{D} z + \sigma(t) \qquad (14)$$

where z is a real ρ vector and $\mathbf{D}$ is real matrix (p $\times$ ρ). The $\sigma(t)$ in Eq. (14) represents the uncertainty in the noise model. This representation of $w(t)$, illustrated in Fig. 2, shows that the optimal controller is designed by first building a duplicate model for the disturbance process typified by Eqs. (13) and (14). This noise estimator is driven by the vector $\mathbf{C}x(t)$, as is the plant state estimator. The noise estimate and the system state estimate are weighted and summed to yield the control law $u(t)$. As $t \to T$, $x(t)$ approaches the steady-state $x(t) = x_c(t)$ prior to any change in the state command.

Mathematically the noise is not precisely known. Hence, the $\sigma(t)$ represent completely unknown sequences of random-intensity, random-occurring, isolated delta functions. Antenna experimental data have shown that the alignment uncertainty exhibits the less than noisy properties of a stochastic process, and thus, in this particular case, Eq. (13) appears to be a reasonable model of the systematic disturbances.

The control law is effectively divided into two parts; that is,

$$u = u_s + u_d \qquad (15)$$

where u_s is assigned the task of servoregulation, and the component u_d is responsible for counteracting both disturbances, $w(t)$ and the reference signal $y_c(t)$ in servotracking. In the case of regulation, $y_c(t) \equiv 0$, the control u_d must be capable of satisfying the relation

$$\mathbf{B}(t) u_d(t) \equiv -\mathbf{F}(t) w(t) = -\mathbf{FH} z(t) \qquad (16)$$

for all admissible $w(t)$ in order to cancel out the disturbance $\mathbf{F}$. Johnson in [5] has shown that if the rank of $[\mathbf{B} \mid \mathbf{FH}]$ = rank of $[\mathbf{B}]$, then

$$-\mathbf{F}(t)\mathbf{H}(t) \equiv -\mathbf{B}(t)\mathbf{K}_z(t) \qquad (17)$$

for some gain matrix $\mathbf{K}_z(t)$. Failure of this condition means that some residual effect of $w(t)$ will always occur. Assuming Eq. (17) is satisfied, the counteraction torque, u_d, is

$$u_d(t) = -\mathbf{K}_z(t) z(t) \qquad (18)$$

and the regulation control, u_s, is chosen by the conventional means as

$$u_s(t) = -\mathbf{K}_x(t) x(t) \qquad (19)$$

The open-loop system augmented to include the noise state is described by

$$\left.\begin{aligned} \begin{bmatrix} \dot{x} \\ \hline \dot{z} \end{bmatrix} &= \left[\begin{array}{c|c} \mathbf{A} & \mathbf{FH} \\ \hline 0 & \mathbf{D} \end{array}\right] \begin{bmatrix} x \\ \hline z \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \hline 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ \hline \sigma \end{bmatrix} \\ y &= [\, \mathbf{C} \mid 0 \,] \begin{bmatrix} x \\ \hline z \end{bmatrix} \end{aligned}\right\} \qquad (20)$$

The exact closed-loop state and disturbance state vectors, assuming all states are available for measurement, are described by the differential equations

$$\begin{bmatrix} \dot{x} \\ \hline \dot{z} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{A} - \mathbf{BK}_x & \mathbf{FH} - \mathbf{BK}_z \\ \hline 0 & \mathbf{D} \end{array}\right] \begin{bmatrix} x \\ \hline z \end{bmatrix} + \begin{bmatrix} 0 \\ \hline \sigma(t) \end{bmatrix} \qquad (21)$$

Redefining the representation of Eq. (20) so that the new state vector includes both system and disturbance states gives

$$\underline{\dot{x}} = \begin{bmatrix} \dot{x} \\ \hline \dot{z} \end{bmatrix} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} + \underline{\delta} \tag{22}$$

$$\underline{y} = \underline{C}\,\underline{x}$$

Examination of Eq. (21) reaffirms the criterion set forth in Eq. (17) to eliminate the noise term in the system state equations. However, an obvious problem arises: how is it possible to access the actual disturbance state $z(t)$? Of course, $z(t)$ is not completely measurable, but it is possible to resort to estimator theory once again to observe and predict the noise state in an approach similar to that used in the plant state estimation. Hence, an appropriate control is obtained by replacing the actual noise state in Eq. (18) with the estimate of the noise state $z'(t)$, i.e., where estimates of $z(t)$ and $x(t)$ can be obtained from $y(t)$ by on-line, real-time state reconstruction. In general $\mathbf{K}_z(t)$ is shown to be not unique [4].

The actual closed-loop plant state $x(t)$, with the assumption that all states are available for measurement, and the disturbance state error variable $e_z(t)$ are described by the differential equations

$$\begin{bmatrix} \dot{x} \\ \hline \dot{e}_z \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK}_x & \mathbf{FH} - \mathbf{BK}_{0_z} \\ \hline 0 & \mathbf{D} + \mathbf{FH} \end{bmatrix} \begin{bmatrix} x \\ \hline e_z \end{bmatrix} + \begin{bmatrix} 0 \\ \hline \sigma(t) \end{bmatrix} \tag{23}$$

where $e = z - z'$, with $\mathbf{K}_{0_z}$ the appropriate observer gain for the disturbance state estimator.

Examination of Eq. (23) reveals that if e_z is zero, the behavior of $x(t)$ is totally independent of the disturbance $w(t)$. Of course, the ideal case is rarely realized; however, $\mathbf{K}_{0_z}(t)$ is chosen so that $e_z(t) \to 0$ rapidly for all initial values of x_0, z_0, e_0 so that the closed-loop plant state $x(t)$ is essentially insensitive to external disturbances that can be generated by Eq. (14).

Using the composite model of Eq. (22), the corresponding augmented state estimator vector $x'(t)$ is described through observer theory as

$$\underline{\dot{x}}' = \underline{A}\,\underline{x}' + \underline{B}\,\underline{u} + \underline{K}_0(\underline{y} - \underline{C}\,\underline{x}')$$

or

$$\underline{\dot{x}}' = (\underline{A} - \underline{K}_0\underline{C})\,\underline{x}' + \underline{B}\,\underline{u} + \underline{K}_0\underline{y} \tag{24}$$

where $\underline{\mathbf{K}}_0$, the composite estimator gain matrix, is chosen to force $\underline{x}'(t) \to \underline{x}(t)$. Using the same principle mentioned earlier in Eq. (8), the eigenvalues of $(\underline{A} - \underline{K}_0\underline{C})$ are chosen for the augmented system. The practical control law for servoregulation in the face of noise becomes

$$\underline{u}(t) = -\underline{\mathbf{K}}_x(t)\underline{x}'(t) = -\mathbf{K}_x(t)x'(t) - \mathbf{K}_z(t)z'(t) \tag{25}$$

where z', x' are the estimates of $z(t)$ and $x(t)$, respectively, obtained from $\underline{y}(t)$ by on-line, real-time state reconstruction, and $-\mathbf{K}_x(t)x'(t)$ is the control required to minimize a performance index if the disturbances were not present in Eq. (10). In general $\mathbf{K}_z(t)$ is not unique, as shown in [4].

The system is reorganized now to include servotracking rather than just servoregulation. Similarly the servocommand can be treated as a "disturbance" to the plant. Recall that the primary control objective in the antenna pointing system is that of servotracking $y_c(t)$, where in general the command is related to the system variables $(x_1, \ldots, x_n)$ by the equation

$$y_c(t) = \overline{C}(t)x(t) \tag{26}$$

In this case, the objective is to control the plant output $y(t)$ so that $\overline{C}$ is equal to $\mathbf{C}$ in Eq. (1). The behavior of y_c is assumed expressible by the servocommand model

$$\left.\begin{aligned} y_c(t) &= \mathbf{G}(t)c(t) \\ \dot{c} &= \mathbf{E}(t)c(t) + \mu(t) \end{aligned}\right\} \tag{27}$$

where $\mathbf{G}(t)$, $\mathbf{E}(t)$ are determined beforehand by appropriate modeling procedures, and c represents the servocommand state vector. The vector $\mu(t)$ represents the uncertain impulse sequences, similarly introduced in the disturbance model in the form of $\sigma(t)$. Note that in the case of set point regulation, y_c is essentially a constant, and hence $\mathbf{E}(t) \equiv 0$ and $\mathbf{G}(t)$ is the identity matrix with the assumption that the y_i are independent outputs. In servotracking, the $y_c(t)$ are allowed to vary continuously with time and $\mathbf{E}(t)$ is chosen accordingly. Exact servotracking cannot be realized unless the servocommand error $e = \mathbf{G}c - \mathbf{C}x$ is zero. Hence, u_s must be chosen so that $(y_c \to y)$ rapidly approaches the null space of $\mathbf{C}$ for all initial conditions.

Hence, the three individual plant, disturbance, and servocommand models can be combined into a single composite open-loop model

$$\underline{\dot{x}} = \begin{bmatrix} \dot{x} \\ \hline \dot{z} \\ \hline \dot{c} \end{bmatrix} = \underline{A}\,x + \underline{B}\,u + \underline{\delta} \tag{28}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{A} & \mathbf{FH} & 0 \\ \hline 0 & \mathbf{D} & 0 \\ \hline 0 & 0 & \mathbf{E} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{B} \\ \hline 0 \\ \hline 0 \end{bmatrix}, \quad \underline{\delta} = \begin{bmatrix} 0 \\ \hline \sigma(t) \\ \hline \mu(t) \end{bmatrix}$$

$\underline{y} = \underline{C}\,\underline{x}$ where $\underline{C} = [\, \mathbf{C} \mid 0 \mid 0 \,]$.

Figure 2 illustrates the continuous-time system with the DAC installed. The solution approach is as follows. An appropriate generic control law for servotracking in the face of noise follows as

$$u = -\mathbf{N}(t)z'(t) - \mathbf{K}_x(t)x'(t) - \mathbf{K}_c(t)c'(t) \tag{29}$$

Utilizing the servocommand state estimator, the closed-loop plant can be described as

$$\begin{aligned} \dot{x} &= (\mathbf{A} - \mathbf{BK}_x)x + (-\mathbf{BK}_z + \mathbf{FH})z' - \mathbf{BK}_z c' \\ &\quad + \mathbf{B}\,[\, -\mathbf{K}_z(z - z') - \mathbf{K}_x(x - x') - \mathbf{K}_c(c - c') \,] \end{aligned} \tag{30}$$

With the appropriate choices of the gains, $\mathbf{K}_x$, $\mathbf{K}_z$, $\mathbf{K}_c$, the estimate errors will approach zero quickly and the noise terms in z should have no effect on $y(t)$.

III. Mathematical Models for the Discrete-Time DAC

The continuous-time model of the disturbed dynamical system described in Eq. (1) can be transformed into a discrete-time prototype for investigation of an analog plant controlled by a digital computer. For simplicity, assume that the signal sampling time is coincident with the control effort application time. In feedback controller designs, control decisions are determined in light of real-time data provided to the controller through sensors. The term "sampled-data" denotes data provided to the controller which are updated only at specific isolated points in time. Between these updates, the data provided to the controller typically are held constant. Likewise, the control decisions are updated only at specific isolated times. In between the decision updates, the control action $u(t)$ either remains constant or follows a prescribed interpolation rule. The computer or digital controller is capable of processing only sampled-data and executing discrete-time control policies usually written as difference equations.

An appropriate discrete-time representation of the system, disturbances, and servocommands, analogous to the continuous model, is required for the investigation. Reference [6] derives the discrete composite state variable representation with the assumption that the noise is not state dependent, i.e.,

$$\left.\begin{aligned} \begin{bmatrix} x[(n+1)T] \\ \hline z[(n+1)T] \\ \hline c[(n+1)T] \end{bmatrix} &= \begin{bmatrix} A(nT) & FH(nT) & 0 \\ \hline 0 & D(nT) & 0 \\ \hline 0 & 0 & E(nT) \end{bmatrix} \begin{bmatrix} x(nT) \\ \hline z(nT) \\ \hline c(nT) \end{bmatrix} \\ &\quad + \begin{bmatrix} B(nT) \\ \hline 0 \\ \hline 0 \end{bmatrix} \underline{u}(nT) + \begin{bmatrix} \gamma(nT) \\ \hline \sigma(nT) \\ \hline \mu(nT) \end{bmatrix} \\ y(nT) &= [\, C(nT) \mid 0 \mid 0 \,] \begin{bmatrix} x(nT) \\ \hline z(nT) \\ \hline c(nT) \end{bmatrix} \end{aligned}\right\} \tag{31}$$

where the discrete-time plant matrices are

$$A(nT) = \Phi_A \left[[t_0 + (n+1)T, t_0 + nT] \right]$$

$\triangleq$ plant state transition matrix in discrete time

$$B(nT) = \int_{t_0+nT}^{t_0+(n+1)T} \Phi_A \; [t_0 + (n+1)T, \tau] \; \mathbf{B}(\tau)\, d\tau$$

$$\gamma[(n+1)T] = \int_{t_0+nT}^{t_0+(n+1)T} \left[\Phi_A \,[t_0 + (n+1)T, \tau]\; \mathbf{F}(\tau)\mathbf{H}(\tau) \times \int_{t_0+nT}^{\tau} \Phi_D(\tau,\xi)\sigma(\xi)d\xi \right] d\tau$$

$$FH(nT) = \int_{t_0+nT}^{t_0+(n+1)T} \Phi_A \left[[t_0 + (n+1)T, \tau] \right] \times \mathbf{F}(\tau)\mathbf{H}(\tau)\, \Phi_D(\tau, t_0 + nT)\, d\tau$$

and the discrete-time noise terms become

$$D(nT) = \Phi_D \left[[t_0 + (n+1)T, t_0 + nT]\right]$$

$\triangleq$ noise state transition matrix

$$\sigma(nT) = \int_{t_0+nT}^{t_0+(n+1)T} \Phi_D [t_0 + (n+1)T, \xi] \; \sigma(\xi)\, d\xi \tag{32}$$

In the time-invariant case, the matrices **A**, **B**, **C**, **F** are constant element matrices, and Eq. (32) is simplified to

$$\left.\begin{aligned}
A &= e^{\mathbf{A}T} \\
D &= e^{\mathbf{D}T} \\
B &= \int_0^T e^{\mathbf{A}(T-\tau)}\,\mathbf{B}\, d\tau \\
\gamma &= \int_0^T e^{\mathbf{A}(T-\tau)}\,\mathbf{FH} \\
&\quad \times \left[\int_0^\tau e^{\mathbf{D}(\tau-\xi)}\,\sigma(\xi + t_0 + nT)\, d\xi\right] d\tau \\
FH &= \int_0^T e^{\mathbf{A}(T-\tau)}\, FH\, e^{\mathbf{D}\tau}\, d\tau \\
\sigma &= \int_0^T e^{\mathbf{D}(T-\xi)}\,\sigma(\xi + t_0 + nT)\, d\xi
\end{aligned}\right\} \tag{33}$$

Similarly, the discrete-time servotracking state model of Eq. (27) can be represented in the form

$$\left.\begin{aligned}
y_c(nT) &= G(nT)\, c(nT) \\
c[(n+1)T] &= E(nT)\, c(nT) + \mu(nT)
\end{aligned}\right\} \tag{34}$$

where

$$\left.\begin{aligned}
E(nT) &= \Phi_E [(n+1)T, nT] \\
&\triangleq \text{discrete transition matrix for the servocommand} \\
\mu(nT) &= \int_{t_0+nT}^{t_0(n+1)T} \\
&\quad \times \Phi_E [t_0 + (n+1)T, \xi]\; \mu(\xi) d\xi
\end{aligned}\right\} \tag{35}$$

In the time-invariant case, Eq. (35) is reduced to

$$\left.\begin{aligned}
E(nT) &= e^{\mathbf{E}T} \\
\mu(nT) &= \int_0^T e^{\mathbf{E}(T-\xi)}\,\mu(\xi + t_0 + nT)\, d\xi
\end{aligned}\right\} \tag{36}$$

The model may be generalized further to include various exceptional case studies [5]. For example, the antenna systematic errors appear to be dependent on the particular azimuth/elevation position of the target; hence, the noise $w(t)$ can be made a function of the system orientation or of the state of the plant. In this case the disturbance model can be augmented to include the state dependency by adding extra terms as follows:

$$\left.\begin{aligned}
w(nT) &= H(nT) z(nT) + L(nT) x(nT) \\
z[(n+1)T] &= D(nT) z(nT) + M(nT) x(nT) \\
&\quad + \sigma(nT)
\end{aligned}\right\} \tag{37}$$

In this case it is necessary to derive the appropriate relationships from the continuous to discrete-time case in order to ascertain the mathematical meaning of the additional terms $L(nT)$ and $M(nT)$ in Eq. (37). These relations are given in Eq. (38).

$$
\left.
\begin{aligned}
\begin{bmatrix} x[(n+1)T] \\ \hline z[(n+1)T] \end{bmatrix} &= \left[\begin{array}{c|c} A(nT) & FH(nT) \\ \hline M(nT) & D(nT) \end{array}\right] \begin{bmatrix} x(nT) \\ \hline z(nT) \end{bmatrix} \\
&+ \begin{bmatrix} B(nT) \\ \hline 0 \end{bmatrix} u(nT) \\
&+ \begin{bmatrix} \gamma_1(nT) + \gamma_2(nT) \\ \hline \sigma(nT) \end{bmatrix} \\
y(nT) &= [\, C(nT) \mid 0 \,] \begin{bmatrix} x(nT) \\ \hline z(nT) \end{bmatrix}
\end{aligned}
\right\} \quad (38)
$$

where in the time-variant case,

$$
\gamma_2[(n+1)T] = \int_0^T e^{\mathbf{A}(T-\tau)}\,\mathbf{FH} \times \left[\int_0^\tau e^{\mathbf{D}(\tau-\xi)}\, x(\xi + t_0 + nT)\, d\xi \right] d\tau
$$

and

$$
M(nT) = \int_0^T e^{\mathbf{D}(T-t)}\, \mathbf{L} x(t + t_0 + nT)\, d\tau
$$

Construction of the on-line, real-time estimation of the three states--plant state $x(t)$, disturbance state $z(t)$, and the command state $c(t)$--requires the discrete-time state estimator given by

$$
\begin{aligned}
\begin{bmatrix} x'[(n+1)T] \\ z'[(n+1)T] \\ c'[(n+1)T] \end{bmatrix} &= \begin{bmatrix} A(nT) & FH(nT) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & E(nT) \end{bmatrix} \begin{bmatrix} x'(nT) \\ z'(nT) \\ c'(nT) \end{bmatrix} \\
&+ \begin{bmatrix} B(nT) \\ 0 \\ 0 \end{bmatrix} \underline{u}(nT) + \begin{bmatrix} K_{0x} \\ K_{0z} \\ K_{0c} \end{bmatrix} \\
&\times \left\{ \begin{bmatrix} C(nT) \mid 0 \mid 0 \\ C(nT) \mid 0 \mid 0 \\ 0 \mid 0 \mid 0 \end{bmatrix} \begin{bmatrix} x(nT) \\ z(nT) \\ c(nT) \end{bmatrix} - \begin{bmatrix} y(nT) \\ y(nT) \\ y_c(nT) \end{bmatrix} \right\}
\end{aligned} \quad (39)
$$

where $\underline{u}(nT)$, $y(nT)$ and $y_c(nT)$ denote the inputs to the observer. The matrices K_{0x}, K_{0z}, K_{0c} are arbitrary gain matrices that the DAC designer selects in accordance with the desired estimator response.

In order to establish the dynamics of the estimators, consider the state errors e_x, e_z and e_c defined as

$$
\left.
\begin{aligned}
e_x &= x(nT) - x'(nT) \\
e_z &= z(nT) - z'(nT) \\
e_c &= c(nT) - c'(nT)
\end{aligned}
\right\} \quad (40)
$$

These error dynamics are described by the discrete equations derived from Eq. (39), considering only first-order variations, as

$$
\begin{bmatrix} e_x[(n+1)T] \\ e_z[(n+1)T] \\ e_c[(n+1)T] \end{bmatrix} = \begin{bmatrix} A + K_{0x}C & FH & 0 \\ K_{0z}C & D & 0 \\ 0 & D & E + K_{0c}G \end{bmatrix} \times \begin{bmatrix} e_x(nT) \\ e_z(nT) \\ e_c(nT) \end{bmatrix} + \begin{bmatrix} \gamma(nT) \\ \sigma(nT) \\ \mu(nT) \end{bmatrix} \quad (41)
$$

In order to produce reliable estimates, the observer gain matrices (K_{0x}, K_{0z}, K_{0c}) are chosen so that the errors in Eq. (40) decay toward zero rapidly between control updates. Hence the homogenous solution of Eq. (41) is made asymptotically stable to the errors equal to zero. In general, Eq. (41) is a time-varying set of difference equations. The gains K_{0x}, K_{0z}, K_{0c} can be solved using the discrete Riccati equation from optimal control theory. For the case of constant element matrices A, C, FH, D, the design of the estimator gain matrices can be accomplished by the conventional eigenvalue placement method. Defining the error dynamics as a system with a characteristic matrix A_e

$$A_e = \begin{bmatrix} (A + K_{0x}) & FH & 0 \\ K_{0z}C & D & 0 \\ 0 & 0 & (E + K_{0c}G) \end{bmatrix} \tag{42}$$

the eigenvalues are positioned suitably (say, at zero) within the unit circle. A block diagram of the composite observer is shown in Fig. 4.

IV. Determination of the Discrete-Time Control Function *u*

The determination of the control function u in the discrete-time case involves several steps.

(1) The state estimators are weighted and summed to determine the control law, i.e.,

$$u(nT) = f[x'(nT), c'(nT), z'(nT), nT] \tag{43}$$

(2) The control function is divided into two subtasks as mentioned previously,

$$u(nT) = u_s(nT) + u_d(nT) \tag{44}$$

where the component u_s is responsible for the servoregulation and the u_d effort is assigned the task of disturbance removal including servotracking. Substitution of Eq. (44) into Eq. (27) yields the plant state relation:

$$x((n+1)T) = A(nT)x(nT) + B(nT)u_s(nT) + B(nT)u_d(nT) + FH(nT)z(nT) + \gamma(nT) \tag{45}$$

Since the control effort in a discrete-time control problem is usually held constant between two consecutive sampling times, it is impossible generally to remove all the disturbance effects. Likewise, the presence of the uncertainty sequence $\gamma(nT)$ also limits the idea of complete time cancellation of the noise. Hence, the concept of "complete cancellation" means only that the noise effects $FHz(nT)$ are removed as they appear at isolated sample times, i.e.,

$$B(nT)\,u_d(nT) + FH(nT)\;\; z(nT) + E(nT)\,c(nT) = 0 \tag{46}$$

The condition for existence of $u_d(nT)$ to satisfy Eq. (46) is

$$[FH \mid A_e] = B[K_z \mid K_c]$$

Complete disturbance cancellation exists if, and only if,

$$FH(nT) = -B(nT)\,K_z(nT)$$

and

$$E(nT) = -B(nT)\,K_c(nT) \tag{47}$$

for some matrix $K_z(nT)$ and $K_c(nT)$. Assuming the conditions of Eq. (47) are satisfied, the control $u_d(nT)$ can be chosen in a practical sense as

$$u_d(nT) = -K_z(nT)\,z'(nT) - K_c(nT)\,c'(nT) \tag{48}$$

where $z'(nT)$ is the noise state determined by on-line, real-time estimation of $z(nT)$. The enclosed loop error dynamics using e_z, e_c may be incorporated into the model as

$$x[(n+1)T] = A(nT)\,x(nT) + B(nT)\,u_s(nT) - [B(nT)\,K_z(nT)]\;e_z(nT) - [B(nT)\,K_c(nT)]\;e_c(nT) + \gamma(nT) \tag{49}$$

Hence, the noise effects have been reduced to the ($BK_ze_z + BK_ce_c$) term, which should decay rapidly toward zero, and, of course, the isolated uncertainty sequence $\gamma(nT)$. The servoregulating control, $u_s(nT)$, can now be designed by conventional methods assuming the noise has been removed.

A complete block diagram of the original continuous-time plant model, and the proposed DAC with full state discrete-time composite observer, is shown in Fig. 4, with the control law

$$u_s = -K_x x'$$

and

$$u_d = -K_z z' - K_c c' \tag{50}$$

V. Summary of DAC Procedure

(1) The disturbance $w(t)$ is determined experimentally to ascertain distinguishing characteristics. Suppose that $w(t)$ is noted to consist of an uncertain bias at times and in other intervals $w(t)$ exhibits uncertain ramp features. Thus, the waveform of the disturbance has a general form

$$w(t) = k_1 + k_2 t \tag{51}$$

where k_1, k_2 are unknown constants which change value at unknown times.

(2) With the description of $w(t)$, the designer determines the simplest differential equation model for this class of disturbances, that is, the lowest order differential equation for which Eq. (51) is the general solution. The corresponding DAC matrices H, D are determined from the general form given in Eqs. (13) and (14). In observable canonical form the model becomes

$$w(t) = [1 \quad 0] \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \tag{52}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -b_1 & 1 \\ -b_2 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} \tag{53}$$

where (b_1, b_2) are constants. The characteristic polynomial of the disturbance model D is equated with the resulting characteristic polynomial attained from the Laplace transform of Eq. (51), that is,

$$det\,[\lambda I - D] = \lambda^2 + b_2\lambda + b_1 = \lambda^2 \tag{54}$$

and in this example, b_1 and b_2 equate to zero.

(3) The controller problem is separated into two subtasks,

$$u = u_s + u_d \tag{55}$$

such that u_s performs servoregulation and u_d is responsible for servotracking a command input and the disturbance removal. Hence, the control effort u_s is chosen for regulation assuming no noise; typically,

$$u_s = -K_x x \tag{56}$$

The control law u_d has the form

$$u_d = -K_z z - K_c c \tag{57}$$

where K_z and K_c are chosen to ensure disturbance removal and tracking, respectively.

(4) With available software tools, the equations can be incorporated into a single system simulation--see Eq. (28)--and tested for various noise inputs and servocommands.

VI. Test Model

A simplified test model for an antenna servomechanism is used to illustrate the DAC procedure (refer to Fig. 3). The objective is to control the elevation of an antenna designed to track an RF signal. The antenna and its drive mechanism have a moment of inertia J and damping B arising from bearing friction, aerodynamic friction, and the back emf of the dc-servodrive motor. The equations of motion are

$$J\ddot{\theta} + B\dot{\theta} = T_c + T_d \tag{58}$$

where T_c is the net torque developed by the drive motor, and T_d represents the disturbance torques possibly due to wind, static misalignments, etc. Substitution of the assumed coefficients in Eq. (58) yields:

$$\ddot{\theta} + 1\dot{\theta} + 6\theta = u + w \tag{59}$$

In this example, the coefficients for Eqs. (58) and (59) were arbitrarily selected and may be unrealistic. They were selected, however, to describe the effect of the new DAC controller. The general shape of the servocommand angle $\theta_c(t)$ is assumed to be composed of step and ramp functions. Hence, the servocommand $\theta_c(t)$ is modeled by y_c estimated using the commanded rate $\dot{\theta}_c(t)$, and the acceleration $\ddot{\theta}_c(t)$. For the purpose of maintaining a good tracking accuracy, it is reasonable to assume that the antenna drives are capable of following the peak velocity $\dot{\theta}_c(t)$ in the steady state with acceptable error. Since the objective is to permit acceptable communication signal reception, the dependence of the signal amplitude on pointing error is a major concern. The corresponding servocommand can be represented by the following state-space representation:

$$\begin{aligned} \dot{c}(t) &= \mathbf{E}(t)\,c(t) + \mu(t) \\ y_c(t) &= \mathbf{G}(t)\,c(t) \end{aligned} \tag{60}$$

Similarly the disturbance $w(t)$ (assumed to be step and ramp torques in this example) can be modeled as suggested in Eq. (54),

$$w(t) = \mathbf{H}z(t)$$
$$\dot{z}(t) = \mathbf{D}z(t) + \sigma(t) \quad (61)$$

Thus, the open-loop system can be represented in the form of Eq. (28) where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -1 \end{bmatrix}, \quad \mathbf{F} = \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{D} = \mathbf{E} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{G} = \mathbf{C} = [\ 1 \quad 0\ ]$$

$$\mathbf{H} = [\ 1 \quad 0\ ]$$

The state feedback gains have been chosen with $K_x = [94, 19]$ to ensure the undisturbed system closed-loop poles at $-10, -10$. The estimator poles for the plant, noise, and servocommand states are selected in the usual manner using pole-placement techniques for approximately 3 to 5 times faster response than the combined plant and feedback controller. A computer simulation of the closed-loop model in Eq. (28), shown in Fig. 5, is used to demonstrate the effectiveness of the DAC in disturbance rejection. Figure 6(a) illustrates the controlled output variable $\theta(t)$ servotracking the command input $\theta_c(t)$ without the DAC. The disturbance assumed in this example is plotted in Fig. 6(b) with the controlled variable $y(t)$. Without control other than state feedback, the output is unable to distinguish the control input from the disturbance and tends to follow the noise signal rather than the servocommand.

In Fig. 7(a) and (b), the same example with the inclusion of the DAC demonstrates the effective servotracking of the command in the presence of the noise input. Note that only a slight perturbation occurs in the controlled variable $y(t)$ at approximately 5.0 s just as the disturbance has occurred.

The analogous discrete-time system has been simulated to demonstrate the degradation expected in tracking when the position-loop of the controller is implemented via a digital computer. Figures 8 and 9 display the discrete-time system in a noisy environment both with and without the DAC incorporated in the loop, assuming a sampling time $T = 0.1$ second.

VII. Conclusions

The feasibility of implementing a disturbance accommodating controller has been investigated as applied to an analog servodrive for positioning an RF antenna. The DAC is designed for synthesizing and rejecting waveform-structured disturbances. The form of the systematic pointing errors inherent in antenna tracking systems appears viable to this characterization of the disturbance as structured waveforms rather than the noise generated through random processes with statistical descriptors. The waveform type of disturbances can be modeled according to a priori data by determination of the corresponding differential equation, and hence, the state representation of the waveform structured noise.

In this study, simulation results show that the DAC is an appropriate technique for cancellation of the systematic errors, while simultaneously allowing an optimal control policy to regulate the system. The ease with which the DAC is implemented along with the existing servo-control is another attribute of this technique. Practical implementation issues such as model order, computation time, and storage requirements offer no expected challenges for microprocessor-based controllers. Further study is necessary to incorporate the state-dependency issue in regard to systematic pointing errors expected in antenna position controllers.

References

[1] M. A. Koerner, "Relative Performance of 8.5 GHz and 32 GHz Telemetry Links on the Basis of Total Data Return per Pass," *TDA Progress Report 42-87*, vol. July–September 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 65–80, November 15, 1986.

[2] J. A. Nickerson, "A New Linear Quadratic Optimal Controller for the 34-Meter High Efficiency Antenna Position Loop," *TDA Progress Report 42-90*, vol. April–June 1987, Jet Propulsion Laboratory, Pasadena, California, pp. 136–141, August 15, 1987.

[3] R. E. Hill, "A Modern Control Theory Based Algorithm for Control of the NASA/JPL 70-Meter Antenna Axis Servos," *TDA Progress Report 42-91*, vol. July–September 1987, Jet Propulsion Laboratory, Pasadena, California, pp. 285–294, November 15, 1987.

[4] C. D. Johnson, "Theory of Disturbance Accommodating Controllers," *Control and Dynamic Systems: Advances in Theory and Application*, vol. 12, New York: Academic Press, chapter 7, pp. 387–489, 1976.

[5] C.D. Johnson, "Disturbance-Accommodating Control: A History of Its Development," *Proc. 15th Annual Meeting, Society of Engineering Science*, pp. 331–347, December 1978.

[6] C. D. Johnson, "A Discrete-Time Accommodating Control Theory," *Control and Dynamic Systems: Advances in Theory and Application*, vol. 18, New York: Academic Press, chapter 6, pp. 223–345, 1982.

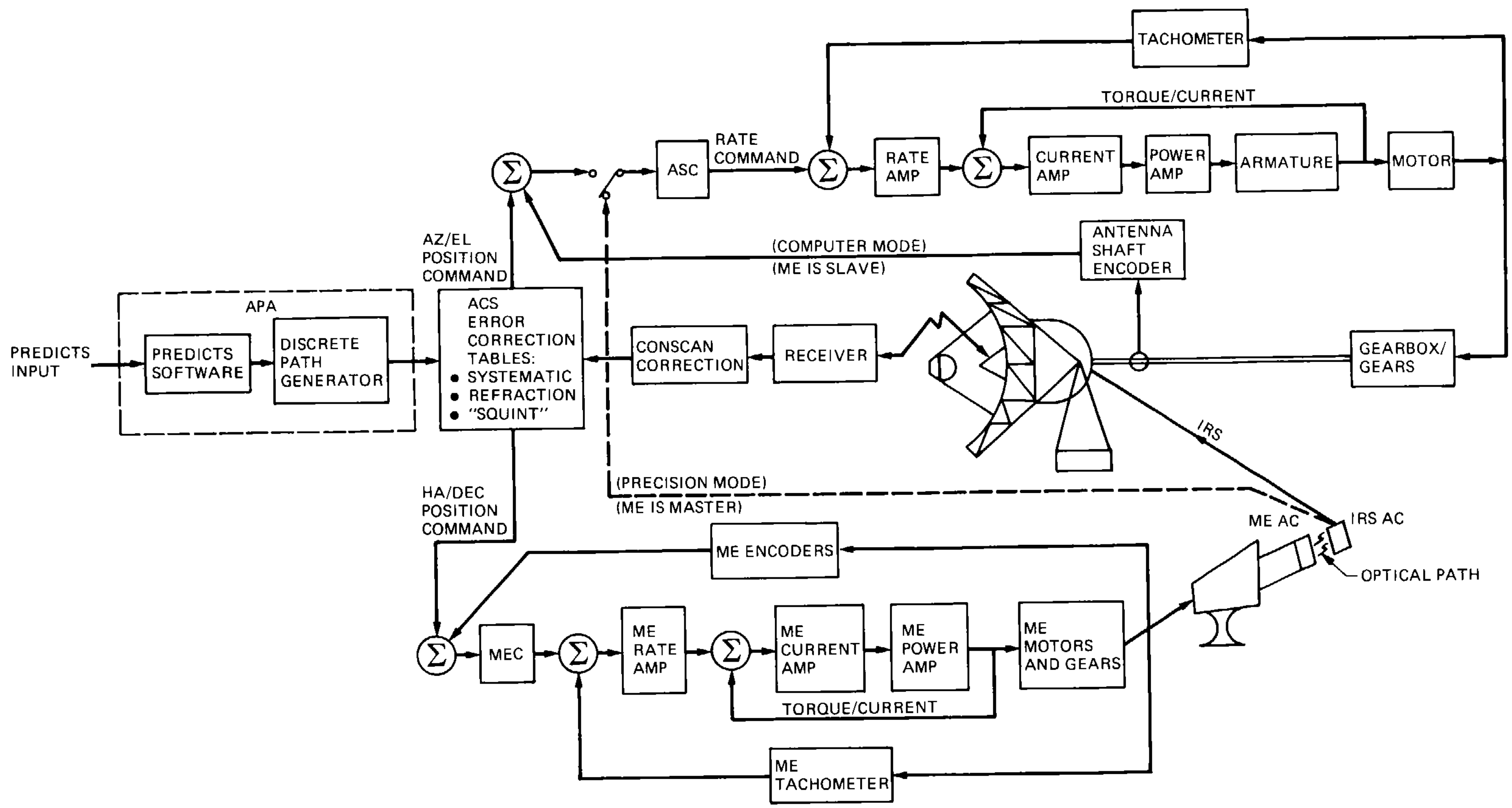

Fig. 1. Overview of the 70-m antenna pointing system at JPL

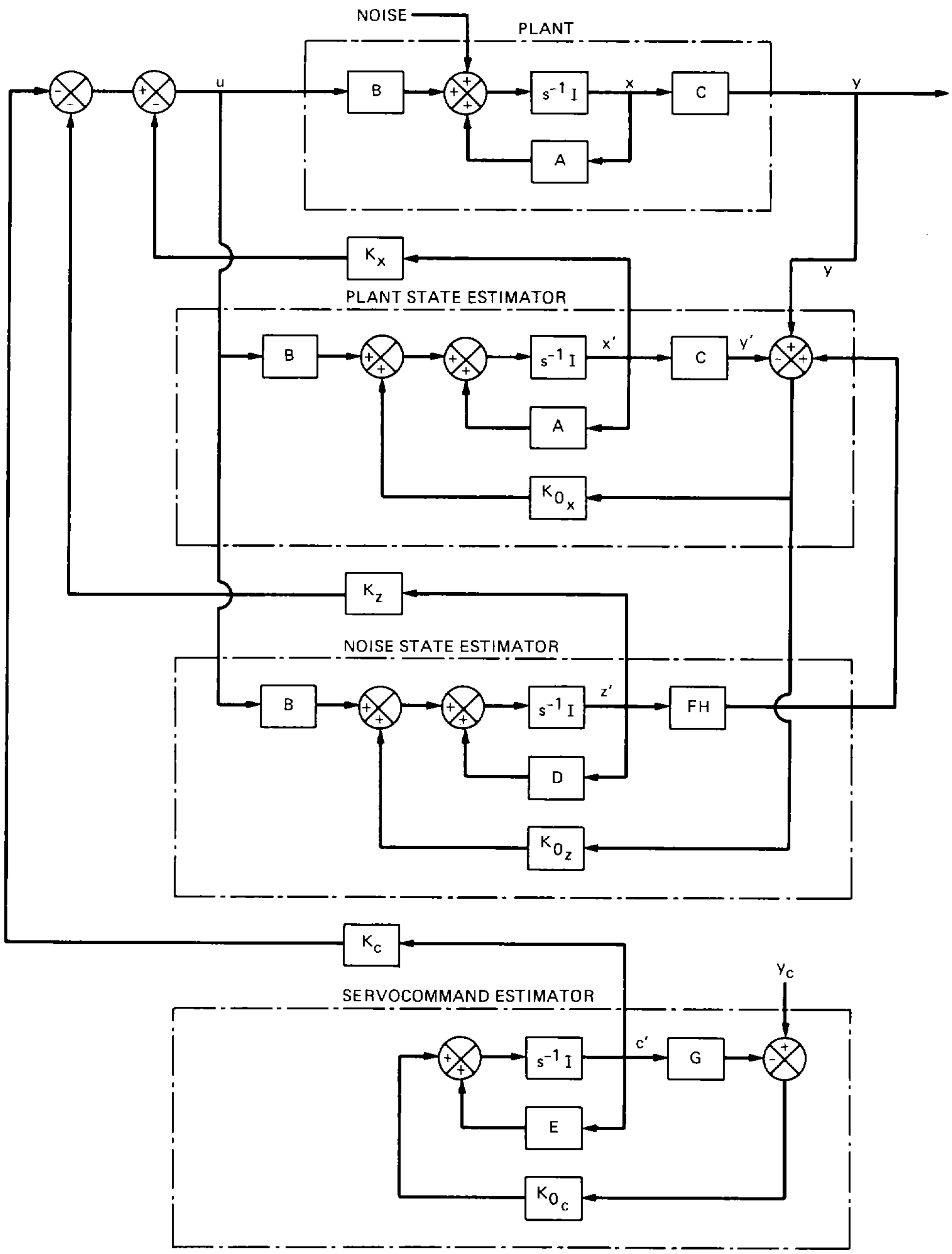

Fig. 2. Functional diagram of the servotracker with the DAC installed

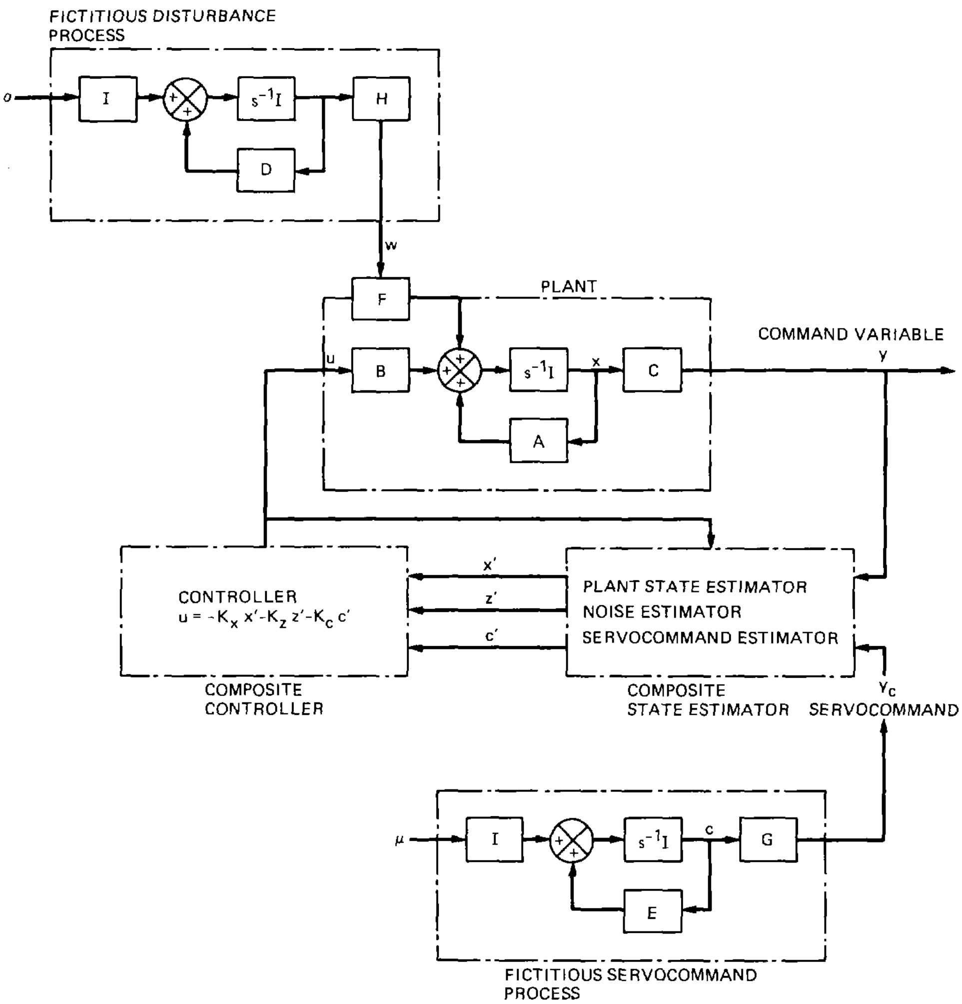

Fig. 3. Simplified block diagram of the plant and state feedback controller with composite estimator for servotracking

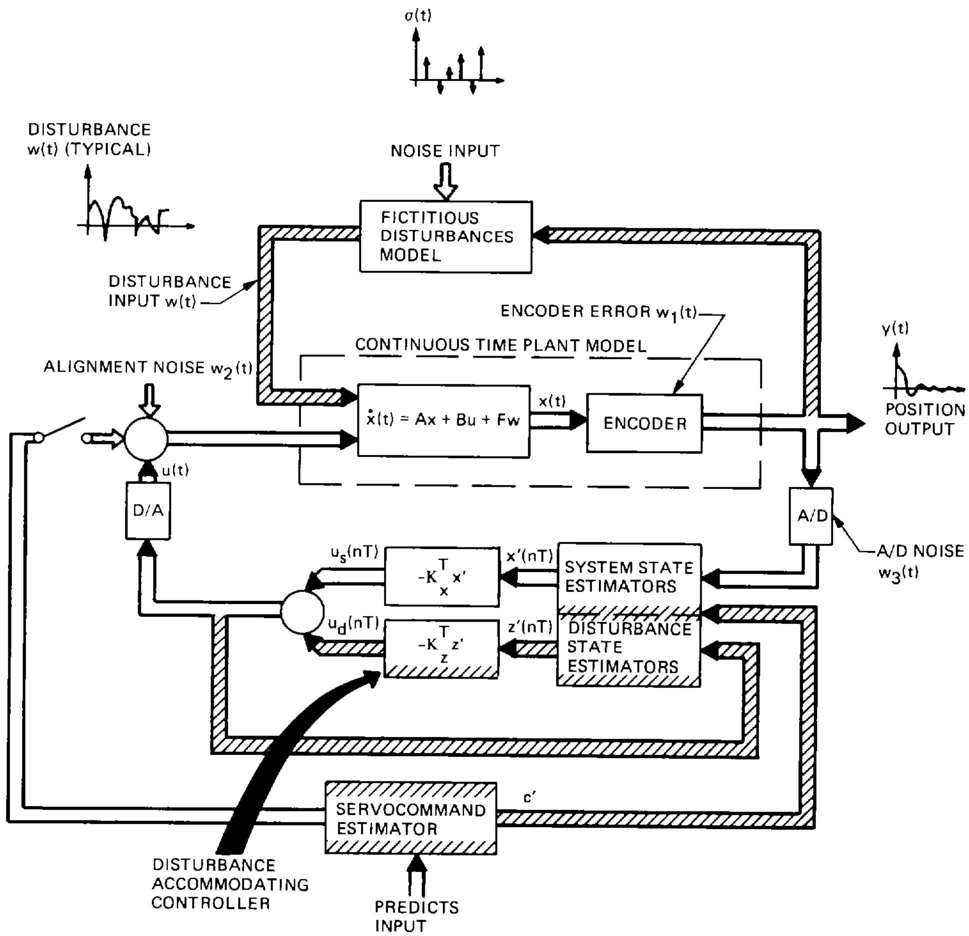

Fig. 4. Configuration for the discrete-time disturbance accommodating controller (DAC) included in a digital servotracking system; slashes represent changes required to include the DAC in a typical state estimate feedback controller scheme

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}'_1 \\ \dot{x}'_2 \\ \dot{z}'_1 \\ \dot{z}'_2 \\ \dot{c}'_1 \\ \dot{c}'_2 \\ \hline y_1 \end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-6 & -1 & -94 & -19 & -1 & 0 & 100 & 20 \\
19 & 0 & -19 & 1 & 0 & 0 & 0 & 0 \\
164 & 0 & -264 & -20 & 0 & 0 & 100 & 20 \\
500 & 0 & -500 & 0 & 0 & 1 & 0 & 0 \\
625 & 0 & -625 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -10 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & -25 & 0 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x'_1 \\ x'_2 \\ z'_1 \\ z'_2 \\ c'_1 \\ c'_2 \\ \\ \end{bmatrix}
+
\begin{bmatrix}
0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 25 & 0 \\ \hline 0 & 0
\end{bmatrix}
\begin{bmatrix} u & w \end{bmatrix}
$$

Fig. 5. Simulation model for the DAC servocommand example

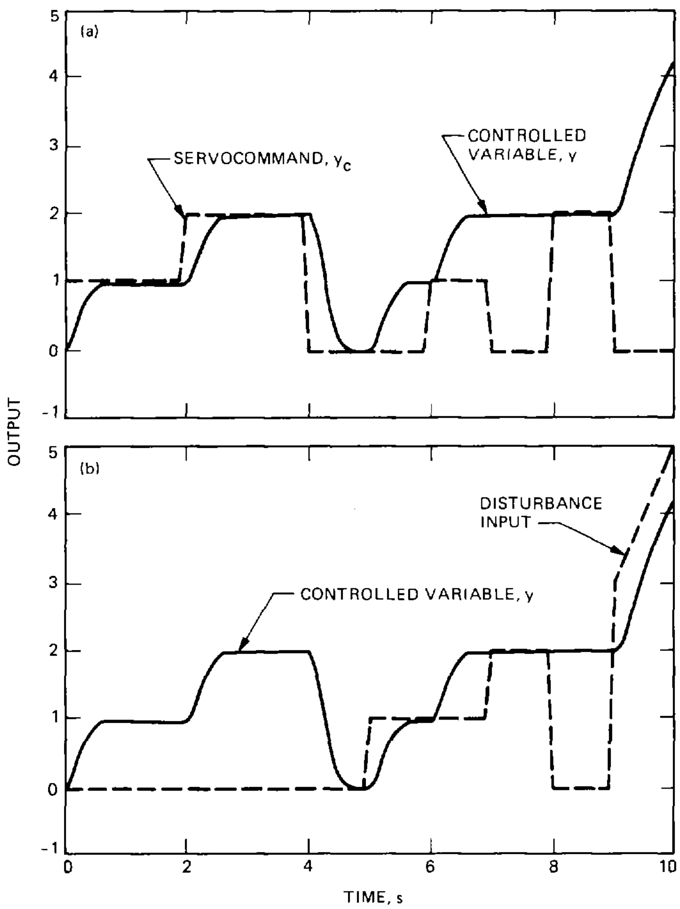

Fig. 6. Servocommand response for the continuous-time example without the DAC installed: (a) servocommand and controlled variable; (b) disturbance input and the controlled variable *y(t)*

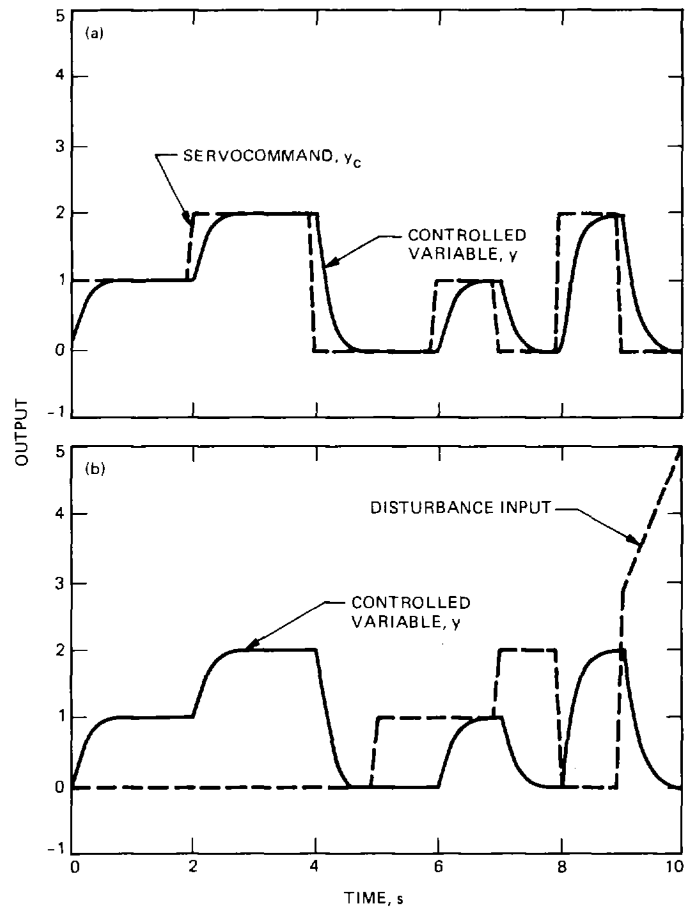

Fig. 7. Servocommand response for the continuous-time example with the inclusion of the DAC: (a) servocommand and controlled variable; (b) disturbance input and the controlled variable *y(t)*

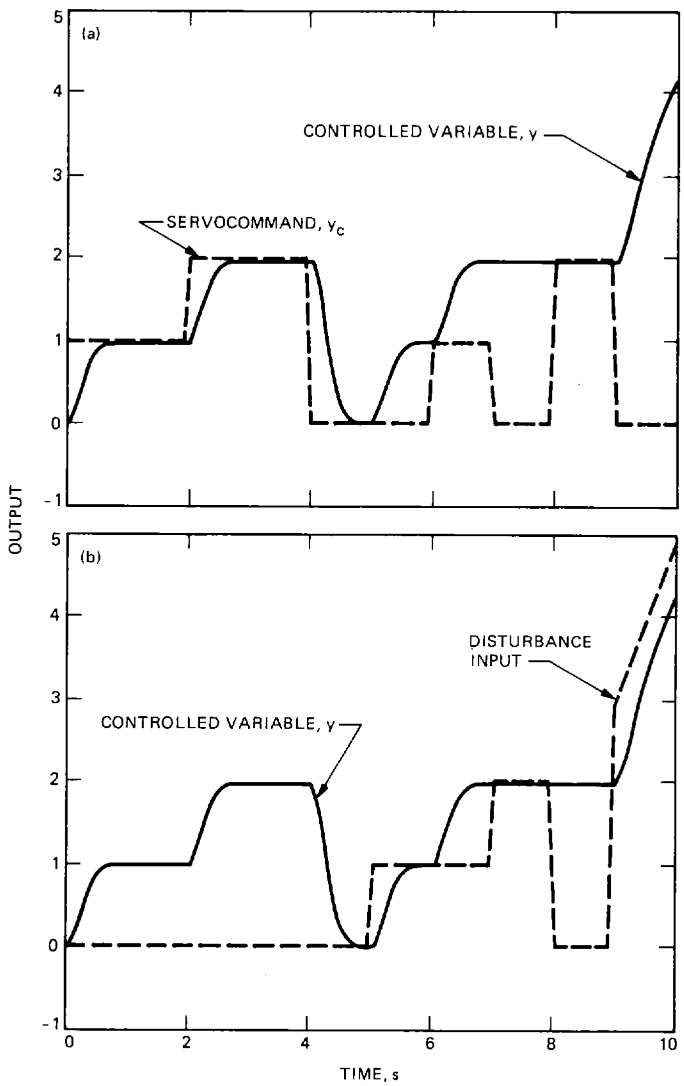

Fig. 8. Servocommand response for the discrete-time example without the DAC installed: (a) servocommand input and controlled variable; (b) disturbance input and the controlled variable *y*(*t*)

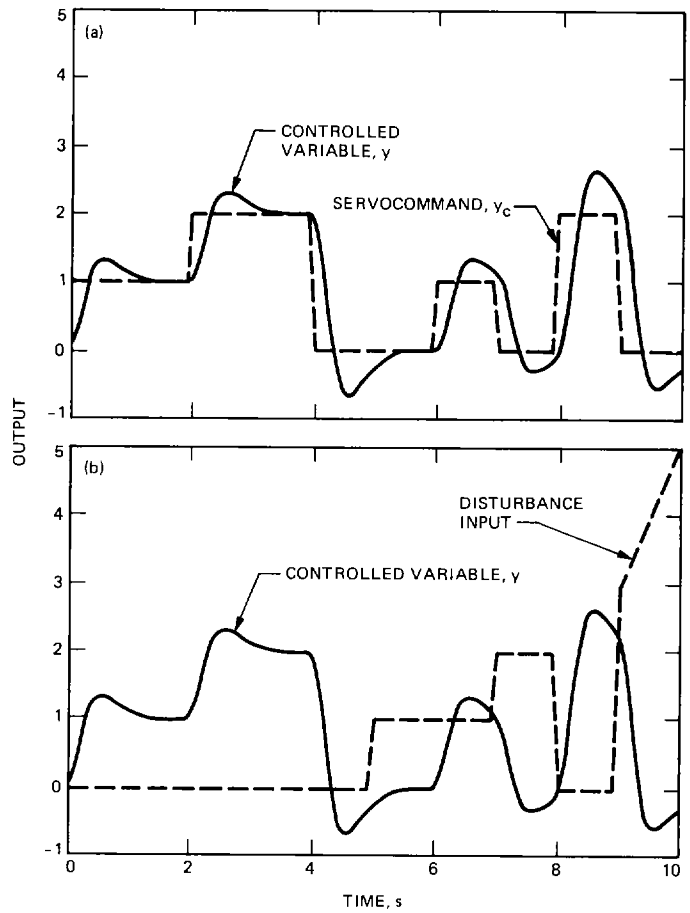

Fig. 9. Servocommand response for the discrete-time example with the DAC installed: (a) servocommand input and controlled variable; (b) disturbance input and the controlled variable *y*(*t*)