Apparent Brightness of Stars and Lasers

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Foremost among issues affecting the potential use of astrometric techniques to locate and track laser-carrying spacecraft is the apparent brightness (detected energy or photon flux) of a laser relative to reference stars. Broadband detectors offer improved sensitivity to stars (broadband "blackbody" sources), but not to lasers. The important and fundamental step of calibrating laser and star brightnesses according to detector spectral sensitivity is carried out here for four representative kinds of broadband detectors, located above and below the atmosphere. Stars are identified according to parameters traditionally used by astronomers: temperature (spectral class, or color) and apparent brightness at visible wavelengths. These are translated into an energy and photon flux for each kind of broadband detector and are then compared with the corresponding flux from a laser. The comparisons are also given as "magnitude correction factors," i.e., in astronomers' traditional units of 4.0 dB. Astrometrists typically characterize the sensitivity of their instruments in terms of the precision with which they can make a relative measurement of position and the minimum brightness needed to achieve that precision. (In an ideal instrument, limited only by $1/\sqrt{N}$ photon statistical fluctuations, these two descriptions would coincide.) Given this information and the instrument's spectral sensitivity, one can use the calculations described in this report to infer the detectability of a laser and the precision with which it could be located and tracked. Results show, for example, that while a star and a laser may have comparable brightnesses at visible wavelengths, to a broadband detector the star might appear brighter than the laser by anywhere from 2 to 17 dB, depending on the star's spectral class and the detector spectral sensitivity. Since the limiting brightness quoted for a particular instrument is typically firm to within 4.0 dB (one magnitude), judging a laser's detectability by its visible brightness alone could lead to serious underestimation of the requirements on its effective radiated power. The laser brightness corrections given here solve this problem.

I. Introduction

The use of astrometric techniques to locate and track laser-carrying spacecraft is an intriguing and real possibility. Measurement precision is influenced strongly by the number of reference stars in the field of view, which in turn depends on the sensitivity of the detector/telescope combination and the size of the field of view. One way to improve sensitivity to stars and other broadband sources is to use broadband detectors. Indeed, some of the most sensitive astrometric measurements made to date have used a CCD detector with a sensitivity range of at least 250 nm [1]. On the other hand, use of a broadband detector will not increase the sensitivity to a laser or other narrowband source. It is important, therefore, to calibrate the relative brightness of lasers and potential reference stars according to the kinds of detectors used. This article shows how to make careful comparisons of the flux that would be measured by prospective broadband detectors from stars (blackbody sources) and from lasers (monochromatic sources).

Three elements make up the definition of a star's or other celestial object's apparent brightness: the quantity to be measured, the location of the measurement, and the wavelength range over which the measurement is made. Astronomers traditionally take the measure of brightness to be received energy flux, or irradiance, and the measurement location to be the top of Earth's atmosphere. The measurement wavelength range should, of course, be a particular detector's spectral sensitivity range. Historically, to standardize descriptions, astronomers have used for this last element several different narrow spectral response functions, ranging from the ultraviolet through the visible. Unfortunately, these narrowband characterizations are awkward and inappropriate indicators of the brightness that would be measured by broadband detectors from stars; as such they can lead to erroneous conclusions about the relative brightness of lasers (narrowband sources) and stars when both are viewed by broadband detectors.

To make accurate comparisons between the brightness of lasers and stars, one therefore must translate from at least one of these narrowband brightness calibrations into the actual flux that would be measured by particular broadband detectors. These translations depend on the spectral sensitivities of the detectors and on the spectral character of the source radiation—its effective temperature if it is blackbody, and its wavelength if it is monochromatic. These translations are made in this article for apparent visual brightness (defined below) and four representative kinds of broadband detectors. Next, the detected flux from a monochromatic laser (a function of laser power, transmit-aperture size, wavelength, and distance from receiver) is compared with that from stars of the same apparent visual brightness but of various temperatures, for the same four kinds of broadband detectors.

Two pieces of information are needed to characterize the brightness of a star: the star's temperature T or color (spectral class) and its apparent brightness (measured irradiance) in one of several standard narrow spectral regions, e.g., the visible. In this article, apparent visual brightness is chosen for the latter; it is defined as the irradiance measured at the top of Earth's atmosphere by a detector whose spectral sensitivity is described by the "visual response function" $V(\lambda)$. This narrow, sharply peaked function approximates the spectral response of the human eye. Traditionally, astronomers describe apparent visual brightness in terms of "apparent visual magnitude" m_V, which is the visual irradiance normalized to a standard irradiance, in units of 4.0 dB (see Eq. [5]). Hence the first part of this report translates from a star's temperature T and apparent visual magnitude m_V to the energy or photon flux that would be measured by a broadband detector with spectral sensitivity $D(\lambda)$ (see Eqs. [12]–[15] and Table 3). Four representative kinds of detectors are considered, both above and below the atmosphere: photomultiplier tubes (PMTs), charge-coupled devices (CCDs), avalanche photodiodes (APDs), and multi-anode microchannel arrays (MAMAs); detector sensitivities are described in the Appendix and in Fig. 3(a) and (b).

A laser looking back at a detector can also be characterized by two quantities: its wavelength λ_t and the irradiance it produces, $H_t \simeq P_t(\pi/4)(D_t/r\lambda_t)^2$, a function of the laser's power P_t, transmit-aperture diameter D_t, wavelength, and distance r from the detector. Alternatively, it can be characterized by its wavelength and its apparent visual magnitude $m_V \equiv -2.5 \log[10^{8.5} V(\lambda_t)H_t]$ (see discussion in Section III and Eq. [20]). The irradiance or photon flux measured by a detector with spectral sensitivity $D(\lambda)$ is just the product of $D(\lambda_t)$ and H_t or $(\lambda_t/hc)H_t$, respectively (h is Planck's constant and c is the speed of light). The flux measured by each of the four kinds of detectors described above is calculated in this report for a 1-W laser at Saturn with a 30-cm aperture, emitting at 0.532 micrometer (μ) (see Eqs. [21]–[24] and Table 4). The results are easily scaled to lasers with different parameters at arbitrary distances.

These quantitative comparisons of the measured flux from stars and lasers verify the predicted inadequacy of visual brightness (or any other narrowband measure of brightness) as an indicator of the relative brightness of lasers and stars viewed with broadband detectors. A laser and a star might appear equally bright when viewed by the eye or other narrowband detector, but widen the spectral response of the detector and many additional photons will be detected from the star, whereas no additional photons will be detected from the laser. Hence, to a broadband detector, the laser will appear dimmer than the star; how much dimmer depends on the temperature of the star and the spectral sensitivity of the detector. For example, the G0, B0, and M0 rows of Table 5 indicate that with the broadband detectors considered here, a laser will appear between 4 and 8 dB (one and two magnitudes) dimmer than a solar-type (G0) star of the same apparent visual brightness, and as much as 16 dB (four magnitudes) dimmer than a very hot or very cold star of the same visual brightness. (The Sun is a G2 star with an effective blackbody temperature of about 5770 K; see Table 1 [6].)

Just how bright will a typical spacecraft laser appear? Consider a 1-W laser at Saturn, transmitting through a 30-cm aperture at 0.532 μ. An observer above the atmosphere would receive a photon flux of about $10^{5.5} \simeq 300{,}000\ m^{-2}s^{-1}$, and the laser would have an apparent visual magnitude $m_V \simeq 11.2$ (see Eqs. [17] and [20]). On the ground the flux would be about 80 percent of this, equivalent to an effective $m'_V \simeq 11.5$. From Uranus, this laser would have $m_V \simeq 12.7$, and from Neptune $m_V \simeq 14.2$. A broadband detector such as a CCD or an APD might count as many as 50 percent of the incident laser photons, while a PMT or a MAMA would typically count only 10 percent. But from a solar-type star of this apparent visual magnitude, all these detectors would count many more photons, because the star is also emitting photons at wavelengths that lie outside the peak of the visual response function, but inside the response function of the detector. Table 5(b) compares the photon flux that would be measured by various detectors from a laser and from stars of 12 different temperatures, all of which have the same apparent visual magnitude m_V. It says, for example, that the UV-flooded CCD considered here would count approximately 7.3 dB (five times) more photons from a Sun-like (G0) star, and as much as 12.9 dB ($\simeq$20 times) more photons from a very hot (B0) star than it would from a laser, given that they all had the same apparent visual magnitude m_V. In astronomers' language (Table 5[d]), this says that a laser of $m_V \simeq 11$, say, would look to this CCD detector only as bright as a Sun-like star of $m_V \simeq 12.8$, or as bright as a very hot star of $m_V \simeq 14.2$.

These laser brightness "correction factors" are significant but not necessarily discouraging, for they still leave the laser sources envisioned for use on planetary spacecraft within the actual or expected capabilities of ground-based detectors and telescopes. (The situation is, of course, better for space-based detectors and telescopes.) For example, astrometrists at USNO in Flagstaff, using a 4-m ([1.6-arcminute]^2 field-of-view) reflector telescope with a CCD at its prime focus, claim a nightly precision of 10 milliarcseconds (mas), or 50 nrad, for relative position measurements on stars as dim as $m_V = 19$ [1]. Their CCD was less sensitive (typically only 15 percent effective quantum efficiency) and narrower spectrally ($\simeq$250 nm wide, centered at $\simeq$575 nm) than the one considered here, so the corresponding laser brightness corrections would be less severe than those given here.

Gatewood et al. at the Allegheny Observatory in Pittsburgh, using a 30-in. ([10-arcminute]^2 field-of-view) refractor, Ronchi ruling, and four PMTs, have demonstrated nightly precisions of 3 mas or better for solar-type stars of $m_V = 8$ or brighter [2], [3]. They used narrow-bandpass filters with their PMTs (50 nm wide, centered at 642.5 nm) in order to isolate the wavelengths nearest the minimum focus of the refractor's objective lens. Hence the laser brightness corrections (for a laser emitting near 642 nm) would again be smaller than those given here. Gatewood estimates that a 1-m or larger system like theirs, located where both seeing and visibility are better, could provide these precisions for stars down to $m_V = 14$, or improve them to tenths of milliarcseconds for $m_V = 8$ and brighter stars. He also estimates that a 1-m system in space could provide precisions of 0.1 microarcsecond.

Buffington at JPL's Table Mountain Observatory, using a 12-in. (1.2-degree by 0.5-degree field of view) meridian-mount reflector with Ronchi grating and 12 PMTs, claims to be capable of nightly precisions of 4 mas for solar-type stars as faint as $m_V = 12$, and 15 mas for $m_V = 14$ stars [4]. Finally, for comparison, the European Space Agency's HIPPARCOS is expected to provide positions, parallaxes, and proper motions for 100,000 stars with $m_V = 10$ and brighter, with a precision of several milliarcseconds [4].

These examples of current astrometric capabilities give good reason to believe that lasers could be located and tracked successfully with existing or modestly improved astrometric technology and techniques.

II. Organization

This report is organized as follows: Section III reviews the definitions of astronomy's standard magnitude systems for describing source brightness. Although it has just been argued that these narrowband brightness measures are inappropriate for comparing lasers and stars, familiarity with them is essential, since all existing star catalogs and other sources of information about star locations and brightnesses use them. Section IV compares the visual irradiance and photon flux from stars with that which would be measured by broadband detectors. Section V does the same for monochromatic laser sources, with quantitative calculations restricted to the visible wavelength 0.532 μ, that of a frequency-doubled Nd:YAG laser. Section VI puts these calculations together to compare irradiance and photon flux from a laser with that from stars. Section VII summarizes the results and some of their implications.

Numerical results and support information are contained in figures and tables. The Appendix describes the spectral sensitivities assumed for the four kinds of detectors considered here, and for the spectral transmission of the atmosphere and the visual response function. These functions are graphed together in Fig. 3(a)–(c).

III. Review of Visual and Other Magnitude Systems

By convention, apparent brightness refers to the irradiance measured by an observer located at the top of Earth's atmosphere. Since stars radiate over a range of wavelengths broader than the sensitivity ranges of most detectors, it is customary in astronomy to speak of apparent brightness in particular spectral regions, defined by certain standard spectral response functions. Four major spectral response functions are used by astronomers (see Fig. 1): the visual ("V") spans the wavelength range 500 nm to 600 nm and peaks at around 555 nm; the blue ("B") spans the range 400 nm to 500 nm and peaks at around 435 nm; the ultraviolet ("U") spans the range 300 nm to 400 nm and peaks at around 350 nm; and the photographic ("pg") closely resembles the blue response curve but peaks at 430 nm [5], [6]. A fifth measure of brightness used by astronomers is the bolometric brightness ("b"), or total irradiance at all wavelengths. This report compares apparent visual brightness, defined as the irradiance measured by a detector with spectral sensitivity $V(\lambda)$, with the brightness measured by various broadband detectors with spectral sensitivities $D(\lambda)$.

The irradiance H_S measured by a detector with spectral sensitivity $S(\lambda)$ from a source that produces a spectral irradiance $H(\lambda)$ is

$$H_S \equiv \int S(\lambda)H(\lambda)\,d\lambda \tag{1}$$

A photon detector measures a flux

$$n_S \equiv \int S(\lambda)\left(\frac{\lambda}{hc}\right)H(\lambda)\,d\lambda \tag{2}$$

where $(hc)^{-1} = 10^{18.7}\ \mathrm{J}^{-1}\ \mu^{-1}$. When $S(\lambda) = V(\lambda)$, these give the visual irradiance and photon flux; when $S(\lambda) = D(\lambda)$, they give the irradiance and photon flux measured by a detector with spectral sensitivity $D(\lambda)$. The total (bolometric) irradiance and photon flux correspond to $S(\lambda) = 1$.

Associated with each of these systems is a reference value for the spectral irradiance at a special wavelength, typically that at which the spectral response function peaks. For the visual response function, this wavelength is $\lambda_V \equiv 0.55\ \mu$ (Eq. [10]). The brightness of a star, defined with respect to some spectral response function $S(\lambda)$, is traditionally described by giving the ratio of measured to reference spectral irradiance at these special wavelengths, in units of 4.0 dB, called "magnitudes." It is sufficiently accurate for most purposes to use reference values for the irradiance H_S; these are given below. Thus, the irradiance from a source of apparent magnitude m_S is $10^{-0.4m_S} \simeq (0.4)^{m_S}$ times the irradiance from a source with $m_S = 0$:

$$\frac{H_S(m_S)}{H_S(m_S = 0)} \equiv 10^{-0.4m_S} \simeq (0.4)^{m_S} \tag{3}$$

The more positive the magnitude, the dimmer the source. The reference values $H_S(m_S = 0)$ for the standard spectral response functions described above are usually given in exponential form:

$$H_S(m_S = 0) \equiv 10^{C_S}\ \mathrm{W/m^2} \tag{4a}$$

The constants C_S are [6]:

$$C_V \simeq -8.5$$

$$C_B \simeq -8.19$$

$$C_U \simeq -8.55$$

$$C_{pg} \equiv C_B - 0.044 \simeq -8.23$$

$$C_b \equiv C_V + \log\left[\frac{H_b(\odot)}{H_V(\odot)}\right] \simeq -7.6 \tag{4b}$$

The constants for the U and B magnitude systems are derived from setting $m_B = m_V = m_U$ for the mean of six particular dwarf stars, all of spectral type A0 [7]. The value for C_b comes from the stipulation that the Sun's apparent visual and bolometric magnitudes be equal, together with measurements of the solar bolometric and visual irradiances (1.35 kW/m^2 and 160 W/m^2, respectively). The visual irradiance from a source of apparent visual magnitude m_V is therefore

$$H_V(m_V) \equiv \int V(\lambda)H(\lambda,m_V)\,d\lambda = 10^{-0.4m_V}\ 10^{-8.5}\ \mathrm{W/m^2} \tag{5}$$

where $H(\lambda, m_V)$ is the spectral irradiance ($H[\lambda]$ in the above discussion) produced by a source with apparent visual magnitude m_V. Note that the visual irradiance from a source with $m_V = 0$ corresponds approximately to a flux of 10^{10} photons/m^2-s at wavelengths between 520 nm and 560 nm (see Eq. [15a]).

IV. Visual vs. Broadband-Detector Irradiance and Photon Flux From Stars

Although there is a one-to-one correspondence between a source's apparent visual magnitude m_V and the visual irradiance H_V it produces, there is no such correspondence between m_V and the general spectral irradiance $H(\lambda)$ produced by the source. But knowledge of the latter is required to calculate the irradiance or photon flux that would be measured by a detector with spectral sensitivity $D(\lambda)$. Since stars radiate as blackbodies, the spectral irradiances they produce are proportional to the Planck distribution for a blackbody of temperature T:

$$W(\lambda, T) = \frac{C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)}$$

$$C_1 \equiv 2\pi h c^2 = 10^{8.57}\ \mu^4\text{-W/m}^2 \tag{6}$$

$$C_2 \equiv \frac{hc}{k} = 10^{4.16}\ \mu\text{-K}$$

where $k = 10^{-22.86}\text{J-K}^{-1}$. Since the spectral irradiance is defined as that at the top of Earth's atmosphere, the proportionality factor for isotropically radiating blackbody sources is essentially independent of both wavelength and angle of incidence. Hence for a star of blackbody temperature T, the spectral irradiances at different wavelengths scale as the ratios of the corresponding blackbody functions $W(\lambda, T)$:

$$\frac{H(\lambda)}{H(\lambda')} = \frac{W(\lambda, T)}{W(\lambda', T)} \tag{7}$$

The function $W(\lambda, T)$ varies little over the narrow region where $V(\lambda)$ is significant. Hence the integral of $W(\lambda, T)$ weighted by $V(\lambda)$ can be approximated by the value of $W(\lambda, T)$ at λ_V:

$$\int V(\lambda) W(\lambda, T) d\lambda \simeq W(\lambda_V, T) \int V(\lambda)\, d\lambda$$

$$\lambda_V \equiv 0.55\,\mu \tag{8}$$

This equality holds for blackbody temperatures from 2,800 K to 28,000 K, or star spectral classifications B through M (see Table 1) [6]. The integral on the right is

$$\int V(\lambda) d\lambda \simeq 0.089\,\mu \simeq 10^{-1.05}\,\mu \tag{9}$$

Using the definition (Eq. [5]) of visual irradiance, one can now see that the spectral irradiance at the wavelength λ_V of a star of temperature T and apparent visual magnitude m_V is independent of T:

$$H^{bb}(\lambda_V; m_V) = \frac{H_V(m_V)}{\int V(\lambda)\, \hat{W}_V(\lambda, T) d\lambda}$$

$$= 10^{-0.4 m_V}\ 10^{-7.45}\ \text{W/m}^2\text{-}\mu$$

$$\hat{W}_V(\lambda, T) \equiv \frac{W(\lambda, T)}{W(\lambda_V, T)} \tag{10}$$

This is the relation used in practice to calibrate the visual magnitude system. The spectral irradiance at any wavelength from a star of temperature T and apparent visual magnitude m_V is therefore

$$H^{bb}(\lambda; T, m_V) = H^{bb}(\lambda_V; m_V)\, \hat{W}_V(\lambda, T)$$

$$= 10^{-0.4 m_V}\ 10^{-7.45}\ \hat{W}_V(\lambda, T)\ \text{W/m}^2\text{-}\mu \tag{11}$$

The normalized blackbody functions $\hat{W}_V(\lambda, T)$ for stars of spectral class B0 through M5, for wavelengths in the range 100 nm through 1200 nm, are plotted in Fig. 2 and listed in Table 2.

Given a star of temperature T and apparent visual magnitude m_V, and the spectral irradiance it produces as derived in Eq. (11), the irradiance H_D that would be measured by a detector with spectral sensitivity $D(\lambda)$ is derived from Eq. (1):

$$H_D^{bb}(T, m_V) \equiv \int D(\lambda) H^{bb}(\lambda; T, m_V) d\lambda$$

$$= 10^{-0.4 m_V}\ 10^{-7.45}$$

$$\times \int D(\lambda)\, \hat{W}_V(\lambda, T) d\lambda\ \text{W/m}^2 \tag{12}$$

Here and in all other equations in this report, the integration range for λ is to be specified in units of micrometers. This detected irradiance is now to be compared with the visual irradiance H_V (Eq. [5]) from the same source. Table 3(a) gives the logarithms of the ratios

$$\frac{H_D^{bb}(T, m_V)}{H_V(m_V)} = 10^{1.05} \int D(\lambda)\, \hat{W}_V(\lambda, T) d\lambda \tag{13}$$

for stars of 12 different spectral classes and the nine spectral response functions described in Appendix A. The detected irradiance $H_D(T, m_V)$ from a star of temperature T and apparent visual magnitude m_V is easily recovered from Table 3(a), with the help of Eq. (5).

It can be argued that in reality, most kinds of detectors that have reasonable sensitivities at visible wavelengths do not measure energy flux (irradiance), but rather measure photon flux. For a nearly monochromatic source of radiation the two are proportional. But for blackbody radiation viewed with a broadband detector, energy flux can be appreciably different from photon flux. For true photon detectors, the relevant quantity for describing apparent brightness is the photon flux n_D measured by a detector with spectral sensitivity $D(\lambda)$, defined in Eq. (2). From Eq. (11) for the spectral irradiance of a star, the detected photon flux n_D from a blackbody of temperature T and apparent visual magnitude m_V is

$$n_D^{bb}(T, m_V) = 10^{-0.4 m_V}\, 10^{10.99} \times \int D(\lambda)\left(\frac{\lambda}{\lambda_V}\right) \hat{W}_V(\lambda, T)\, d\lambda \quad \mathrm{m^{-2} s^{-1}} \tag{14}$$

($[\lambda_V/hc] = 10^{18.44}\,\mathrm{J^{-1}}$). The visual photon flux from a source of apparent visual magnitude m'_V is

$$n_V(m'_V) \simeq 10^{-0.4 m'_V}\, 10^{9.94} \quad \mathrm{m^{-2} s^{-1}} \tag{15a}$$

(The approximation sign refers to the assumption that all photons arrive with wavelength $\lambda_V \equiv 0.55\ \mu$, at which $V(\lambda)$ is maximum.) Hence the ratio of detected to visual photon flux from a star of apparent visual magnitude m_V is

$$\frac{n_D^{bb}(T, m_V)}{n_V(m_V)} = 10^{1.05} \int D(\lambda)\left(\frac{\lambda}{\lambda_V}\right) \hat{W}_V(\lambda, T)\, d\lambda \tag{15b}$$

The logarithms of these ratios are listed in Table 3(b). Note that the ratios of detected to visual irradiance or photon flux from a star (Eqs. [13] and [15]) need not be greater than one. That is, even though a broadband detector is sensitive to photons of many different wavelengths, its efficiency is typically much poorer than that of the eye at visible wavelengths.

The numbers in Table 3(a) and (b) need only be multiplied by -2.5 to give the brightness correction in magnitudes between visual and detected irradiance or photon flux. This has been done in Table 3(c) and (d). When the magnitude correction is negative, the detector measures a greater flux than does the eye from a given star. These magnitude corrections are similar in definition to astronomers' "bolometric correction" $BC(T)$. The latter is the ratio of total irradiance to visual irradiance, in units of magnitudes:

$$\begin{aligned} BC(T) \equiv m_b - m_V &= -2.5 \log\left[10^{-0.9}\, \frac{H_b^{bb}(T, m_V)}{H_V(m_V)}\right] \\ &= -2.5\left(0.15 + \log \int \hat{W}_V(\lambda, T)\, d\lambda\right) \end{aligned} \tag{16a}$$

Here the bolometric irradiance is defined by

$$\begin{aligned} H_b^{bb}(T, m_V) &\equiv \int H^{bb}(\lambda; T, m_V)\, d\lambda \\ &= 10^{-0.4 m_V}\, 10^{-7.45} \int \hat{W}_V(\lambda, T)\, d\lambda \quad \mathrm{W/m^2} \end{aligned} \tag{16b}$$

and the additive constant arises because of the difference in reference values for calibration of the visual and bolometric magnitude systems (Eq. [4]). Note that the bolometric correction is always negative, because a star's total irradiance is always greater than its irradiance at visible wavelengths. For reference, the bolometric corrections for stars of the 12 spectral types considered in this report are included in Table 1.

V. Visual vs. Broadband-Detector Irradiance and Photon Flux From Lasers

Calculations similar to those just made for stars can be made for monochromatic sources. A laser at a distance r from the top of Earth's atmosphere, transmitting a power P_t at wavelength λ_t through a telescope of area $A_t = (\pi/4)(D_t^2)$, produces an irradiance

$$\begin{aligned} H_t &= \frac{P_t}{r^2 \Omega_t} = \frac{P_t A_t}{r^2 \lambda_t^2} = P_t \frac{\pi}{4}\left(\frac{D_t}{r\lambda_t}\right)^2 \\ &= 10^{-12.96} F_0 \quad \mathrm{W/m^2} \end{aligned} \tag{17a}$$

where the dimensionless scale factor F_0 is

$$F_0 \equiv \frac{\left(\frac{P_t}{1\,\mathrm{W}}\right)\left(\frac{D_t}{30\ \mathrm{cm}}\right)^2}{\left(\frac{r}{10\,\mathrm{AU}}\right)^2 \left(\frac{\lambda_t}{0.532\ \mu}\right)^2} \tag{17b}$$

Since $0.532\ \mu$ is a wavelength of special interest, it is denoted henceforth by λ_s:

$$\lambda_s \equiv 0.532\ \mu \tag{17c}$$

The spectral irradiance produced by the laser is

$$H_t(\lambda) = H_t\, \delta(\lambda - \lambda_t) \tag{18}$$

The visual irradiance $H_V^{mc}(\lambda_t, m_V)$ from a monochromatic (*mc*) laser emitting at wavelength λ_t follows from Eq. (5):

$$\begin{aligned} H_V^{mc}(\lambda_t, m_V) &= V(\lambda_t) H_t = 10^{-12.96} F_0 V(\lambda_t) \ \mathrm{W/m^2} \\ &= 10^{-12.98} F_0 \left[\frac{V(\lambda_t)}{0.95}\right] \mathrm{W/m^2} \\ &\equiv 10^{-0.4 m_V} 10^{-8.5} \ \mathrm{W/m^2} \end{aligned} \tag{19}$$

where $V(\lambda_s) \simeq 0.95$. The scale factor F_0 and apparent visual magnitude m_V of a laser are thus related by

$$\begin{aligned} m_V &= 11.15 - 2.5 \log [F_0 V(\lambda_t)] \\ &= 11.2 - 2.5 \log \left[\frac{F_0 V(\lambda_t)}{0.95}\right] \end{aligned} \tag{20}$$

The detected irradiance H_D from a laser of apparent visual magnitude m_V (or scale factor F_0) emitting at wavelength λ_t is

$$\begin{aligned} H_D^{mc}(\lambda_t, m_V) &= \int D(\lambda) H_t(\lambda) d\lambda = D(\lambda_t) H_t \\ &= 10^{-12.96} D(\lambda_t) F_0 \ \mathrm{W/m^2} \\ &= 10^{-0.4 m_V} 10^{-8.5} \left(\frac{D(\lambda_t)}{V(\lambda_t)}\right) \ \mathrm{W/m^2} \end{aligned} \tag{21}$$

The ratio of detected to visual irradiance is therefore

$$\frac{H_D^{mc}(\lambda_t, m_V)}{H_V^{mc}(\lambda_t, m_V)} = \frac{D(\lambda_t)}{V(\lambda_t)} \tag{22}$$

The logarithms of these ratios are given in Table 4(a) for $\lambda_t = \lambda_s$ and the usual nine spectral response functions. The detected irradiance $H_D^{mc}(\lambda_t, m_V)$ from a monochromatic source with apparent visual magnitude m_V (or scale factor F_0) emitting at wavelength λ_s is easily recovered from Table 4(a) and Eq. (21).

The irradiance (Eq. 17[a]) from a monochromatic source implies a photon flux

$$\begin{aligned} n_t &= \left(\frac{\lambda_t}{hc}\right) H_t = 10^{5.47} \left(\frac{\lambda_t}{\lambda_s}\right) F_0 \ \mathrm{m^{-2} s^{-1}} \\ &\simeq 300{,}000 \left(\frac{\lambda_t}{\lambda_s}\right) F_0 \ \mathrm{m^{-2} s^{-1}} \end{aligned} \tag{23}$$

The visual photon flux is

$$\begin{aligned} n_V^{mc}(\lambda_t, m_V) &= V(\lambda_t) n_t = 10^{5.45} \left(\frac{\lambda_t}{\lambda_s}\right) F_0 \left[\frac{V(\lambda_t)}{0.95}\right] \ \mathrm{m^{-2} s^{-1}} \\ &= 10^{-0.4 m_V} 10^{9.93} \left(\frac{\lambda_t}{\lambda_s}\right) \ \mathrm{m^{-2} s^{-1}} \end{aligned} \tag{24a}$$

(Eq. [20]). The flux measured by a photon detector with spectral sensitivity $D(\lambda)$ is

$$n_D^{mc}(\lambda_t, m_V) = D(\lambda_t) n_t = \left[\frac{D(\lambda_t)}{V(\lambda_t)}\right] n_V^{mc}(\lambda_t, m_V) \tag{24b}$$

For $\lambda_t = \lambda_s$, it is obtained simply by multiplying Eq. (24a) by the quantities whose logarithms are given in Table 4(a). Note that for monochromatic sources, the ratio of detected to visual irradiance or photon flux is typically less than one (the entries in Table 4[a] are negative), because most broadband detectors are less sensitive than the eye at visible wavelengths.

As was done in the previous section for stars, the numbers in Table 4(a) can be multiplied by −2.5 to give the visual-to-detector brightness correction in magnitudes; this is done in Table 4(b). These corrections are positive, reflecting the poor sensitivity of broadband detectors at visible wavelengths compared to that of the eye.

VI. Comparison of Star and Laser Brightnesses

It is now straightforward to compare the detected energy or photon flux from stars and lasers of the same (or different) apparent visual magnitudes. The ratio of the measured irradiance from a monochromatic laser of apparent visual magnitude m_V (or scale factor F_0) emitting at wavelength λ_t to that from a blackbody of temperature T and apparent visual magnitude m'_V is denoted here by $R(m_V - m'_V)$, a function of detector sensitivity $D(\lambda)$, laser wavelength λ_t, and star temperature T:

$$\frac{H_D^{mc}(\lambda_t, m_V)}{H_D^{bb}(T, m'_V)} \equiv R(m_V - m'_V)$$

$$= 10^{-0.4(m_V - m'_V)} R(0)$$

$$= 10^{0.4 m'_V}\, 10^{-4.48}\, F_0 \left[\frac{V(\lambda_t)}{0.95}\right] R(0) \tag{25a}$$

The ratio $R(0)$ is just the ratio of Eqs. (22) and (13) above:

$$R(0) = \frac{\left[\dfrac{D(\lambda_t)}{V(\lambda_t)}\right]}{10^{1.05} \int D(\lambda)\, \hat{W}_V(\lambda, T)\, d\lambda} \tag{25b}$$

Its logarithm for $\lambda_t = \lambda_s$, equal to the difference of the entries in Table 4(a) and Table 3(a), is tabulated in Table 5(a) for stars of 12 different temperatures and the nine different spectral response functions described in the Appendix.

A similar ratio, $\bar{R}(m_V - m'_V)$, can be defined for photon flux:

$$\frac{n_D^{mc}(\lambda_t, m_V)}{n_D^{bb}(T, m'_V)} \equiv \bar{R}(m_V - m'_V)$$

$$= 10^{-0.4(m_V - m'_V)} \bar{R}(0)$$

$$= 10^{0.4 m'_V}\, 10^{-4.48}\, F_0 \left[\frac{V(\lambda_t)}{0.95}\right] \bar{R}(0) \tag{26a}$$

The ratio $\bar{R}(0)$ is just the ratio of Eqs. (24) and (14):

$$\bar{R}(0) \equiv \frac{n_D^{mc}(\lambda_t, m_V)}{n_D^{bb}(T, m_V)} = \frac{\left(\dfrac{\lambda_t}{\lambda_V}\right)\left[\dfrac{D(\lambda_t)}{V(\lambda_t)}\right]}{10^{1.05} \int D(\lambda) \left(\dfrac{\lambda}{\lambda_V}\right) \hat{W}_V(\lambda, T)\, d\lambda} \tag{26b}$$

Its logarithm for $\lambda_t = \lambda_s$, equal to the difference of the entries in Table 4(a) and Table 3(b) minus 0.01 = log (λ_s/λ_V), is tabulated in Table 5(b). Note that, as expected, the ratios $R(0)$ and $\bar{R}(0)$ are always less than one, i.e., the entries in Table 5(a) and (b) are all negative.

The logarithms of the ratios $R(0)$ and $\bar{R}(0)$ listed in Table 5(a) and (b) need only be multiplied by –2.5 to give the brightness difference in magnitudes, as measured by a given broadband detector, between a laser (emitting at 0.532 μ) and a star of identical apparent visual magnitude. These positive "monochromatic magnitude corrections" are given in Table 5(c) and (d). For example, Table 5(d) shows that if a CCD looks at a laser and a very hot (B0) star of identical m_V, the laser will appear approximately 3.2 magnitudes (≃13 dB) dimmer than the star; for solar-type (G0) stars, the difference is only about 1.8 magnitudes (7 dB).

VII. Summary and Concluding Remarks

This article has shown how to make precise quantitative comparisons of the flux measured by certain broadband detectors from stars and lasers. Results show that for a given detector, the difference in measured flux from a star and a laser of similar apparent visual brightness depends strongly on the spectral class of the star. The difference is greatest for stars much hotter or colder than our Sun, since most of their radiation is outside the visible region of the spectrum. The difference is smallest, but by no means negligible, for stars of spectral classes A through K, or temperatures between 4000 K and 10,000 K. These comprise the majority of known stars (see Table 6[a] and [b]) [6]. In particular, results showed that a laser will appear dimmer than a Sun-like G0 star of equal apparent visual brightness by the following factors for the detectors considered: approximately 6 dB for the PMT, 7 dB for the CCD, 8 dB for the APD, and 6 dB for the MAMA. Ground-based observation did not change these numbers significantly for the CCD, and changed them by less than 1 dB for the others (see Table 5[a] - [d]).

For the astrometrist with a broadband detector trying to locate the position of a laser-carrying spacecraft, these results have the following implications: First, hot and cold stars that are as much as 1.5 magnitudes (6 dB) fainter in the visible than Sun-like stars may appear just as bright and be just as good reference stars as the visibly brighter Sun-like stars (refer to Table 3[a]-[d]). Second, the expected apparent brightness of the laser must be calculated for a particular detector in the manner described in this article; its visual brightness can give a seriously exaggerated estimate of its would-be measured brightness relative to stars of the same apparent visual brightness, by anywhere from 2 to 17 dB (Table 5[a]-[d]).

References

[1] D. G. Monet and C. C. Dahn, "CCD Astrometry I: Preliminary Results From the KPNO 4-m/CCD Parallax Program," *Astron. J.*, vol. 88, pp. 1489–1507, October 1983.

[2] G. Gatewood et al., "A Preliminary Look at Astrometric Accuracy as a Function of Photon Counts," *Astron. J.*, vol. 90, pp. 2397–2398, November 1985.

[3] G. Gatewood, "The Multichannel Astrometric Photometer and Atmospheric Limitations in the Measurement of Relative Positions," *Astron. J.*, vol. 94, pp. 213–224, July 1987.

[4] A. Buffington, "A Photometric Astrometric Telescope Using a Ronchi Ruling," submitted to *Review of Scientific Instruments*, 1987.

[5] H. L. Johnson, "Photometric Systems," in *Basic Astronomical Data: Stars and Stellar Systems III*, K. A. Strand (ed.), Chicago: University of Chicago Press, pp. 204–229, 1963.

[6] C. W. Allen, *Astrophysical Quantities*, London: Wm. Clowes and Sons, Ltd., pp. 197–207, 1976.

[7] K. R. Lang, *Astrophysical Formulae*, New York: Springer-Verlag, pp. 558–565, 1974.

[8] J. G. Timothy, "Multianode Microchannel Array Detector Systems: Performance Characteristics," *Optical Engineering*, vol. 24, pp. 1066–1071, 1985.

Table 1. Star spectral classes, temperatures, and bolometric corrections

Spectral type	Temperature, K	$BC(T)$
B0	28,000	−2.80
B5	15,500	−1.50
A0	9,900	−0.40
A5	8,500	−0.12
F0	7,400	−0.06
F5	6,600	0.0
G0	6,000	−0.03
G5	5,500	−0.07
K0	4,900	−0.20
K5	4,100	−0.60
M0	3,500	−1.20
M5	2,800	−2.30

Table 2. Normalized blackbody functions $\hat{W}_V(\lambda, T)$

(Eqs. [6] and [10] of text)

$$\hat{W}_V(\lambda, T) \equiv \frac{W(\lambda, T)}{W(\lambda_V, T)} = \frac{\left(\frac{\lambda_V}{\lambda}\right)^5 (e^{C_2/\lambda_V T} - 1)}{e^{C_2/\lambda T} - 1}$$

$$C_2 \equiv \frac{hc}{k} = 10^{4.16}\ \mu - K, \qquad \lambda_V \equiv 0.55\ \mu$$

λ	Star type											
	B0	B5	A0	A5	F0	F5	G0	G5	K0	K5	M0	M5
0.1	45.19	2.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.2	20.07	6.68	1.41	0.67	0.31	0.15	0.08	0.04	0.01	0.00	0.00	0.00
0.3	7.03	4.32	2.13	1.51	1.05	0.74	0.53	0.39	0.24	0.10	0.04	0.01
0.4	2.91	2.36	1.74	1.50	1.27	1.09	0.95	0.82	0.66	0.45	0.30	0.15
0.5	1.39	1.32	1.22	1.17	1.12	1.08	1.04	1.00	0.94	0.85	0.77	0.64
0.6	0.74	0.77	0.83	0.85	0.88	0.91	0.94	0.97	1.02	1.11	1.22	1.43
0.7	0.43	0.48	0.56	0.61	0.67	0.72	0.78	0.85	0.96	1.20	1.51	2.26
0.8	0.26	0.31	0.39	0.44	0.50	0.56	0.63	0.71	0.84	1.16	1.62	2.91
0.9	0.17	0.21	0.28	0.32	0.37	0.43	0.50	0.58	0.71	1.06	1.61	3.32
1.0	0.12	0.15	0.20	0.24	0.28	0.34	0.39	0.46	0.59	0.93	1.51	3.48
1.1	0.08	0.10	0.15	0.18	0.22	0.26	0.31	0.37	0.49	0.81	1.37	3.47
1.2	0.06	0.08	0.11	0.14	0.17	0.21	0.25	0.30	0.40	0.69	1.23	3.34

Table 3(a). Ratios of detected to visual irradiance for stars

(Eq. [13] in text)

$$\log\left[\frac{H_D^{bb}(T,m_V)}{H_V(m_V)}\right] \equiv \log\left(10^{1.05}\int D(\lambda)\,\hat{W}_V(\lambda,T)d\lambda\right)$$

Star type	Detector type: PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	0.21	1.48	0.32	0.48	−0.10	0.47	0.23	−0.14	−0.10
B5	0.10	0.99	0.32	0.29	−0.18	0.43	0.24	−0.23	−0.10
A0	−0.05	0.68	0.35	0.05	−0.29	0.38	0.27	−0.35	−0.10
A5	−0.11	0.60	0.36	−0.05	−0.34	0.37	0.29	−0.40	−0.10
F0	−0.17	0.54	0.38	−0.15	−0.38	0.36	0.31	−0.45	−0.10
F5	−0.23	0.51	0.40	−0.23	−0.42	0.36	0.34	−0.50	−0.10
G0	−0.27	0.49	0.42	−0.30	−0.46	0.36	0.36	−0.54	−0.10
G5	−0.32	0.48	0.45	−0.36	−0.49	0.37	0.39	−0.58	−0.10
K0	−0.37	0.48	0.49	−0.44	−0.53	0.39	0.43	−0.64	−0.10
K5	−0.44	0.52	0.58	−0.55	−0.58	0.44	0.53	−0.72	−0.09
M0	−0.50	0.59	0.68	−0.65	−0.61	0.52	0.63	−0.79	−0.09
M5	−0.53	0.76	0.89	−0.76	−0.62	0.71	0.85	−0.87	−0.07

Table 3(b). Ratios of detected to visual photon flux for stars

(Eq. [15b] in text)

$$\log\left[n_D^{bb}\,\frac{(T,m_V)}{n_V(m_V)}\right] \equiv \log\left(10^{1.05}\int D(\lambda)\left(\frac{\lambda}{\lambda_V}\right)\hat{W}_V(\lambda,T)d\lambda\right)$$

Star type	Detector type: PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	0.06	1.06	0.35	0.25	−0.22	0.42	0.27	−0.27	0.30
B5	−0.04	0.75	0.37	0.08	−0.29	0.40	0.30	−0.35	0.30
A0	−0.17	0.58	0.41	−0.13	−0.38	0.38	0.34	−0.45	0.30
A5	−0.22	0.54	0.43	−0.21	−0.42	0.38	0.37	−0.50	0.30
F0	−0.27	0.51	0.46	−0.29	−0.46	0.38	0.40	−0.55	0.30
F5	−0.32	0.51	0.49	−0.36	−0.49	0.40	0.43	−0.59	0.30
G0	−0.36	0.51	0.51	−0.42	−0.52	0.41	0.46	−0.63	0.30
G5	−0.39	0.51	0.55	−0.47	−0.54	0.43	0.49	−0.66	0.30
K0	−0.43	0.54	0.60	−0.53	−0.57	0.46	0.54	−0.71	0.31
K5	−0.48	0.60	0.70	−0.63	−0.60	0.54	0.65	−0.78	0.31
M0	−0.51	0.69	0.81	−0.70	−0.62	0.64	0.77	−0.83	0.32
M5	−0.52	0.90	1.04	−0.79	−0.60	0.86	1.00	−0.90	0.34

Table 3(c). Ratios of detected to visual irradiance for stars, in magnitudes

(Eq. [13] in text)

$$-2.5 \log \left[H_D^{bb} \frac{(T, m_V)}{H_V(m_V)} \right] \equiv -2.5 \log \left(10^{1.05} \int D(\lambda)\, \hat{W}_V(\lambda, T)\, d\lambda \right)$$

Star type	Detector type: PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	−0.53	−3.69	−0.79	−1.19	0.25	−1.18	−0.57	0.35	0.24
B5	−0.25	−2.49	−0.81	−0.71	0.46	−1.07	−0.60	0.57	0.24
A0	0.11	−1.69	−0.86	−0.12	0.73	−0.96	−0.68	0.87	0.25
A5	0.27	−1.50	−0.90	0.13	0.84	−0.92	−0.72	1.00	0.25
F0	0.43	−1.36	−0.95	0.36	0.96	−0.90	−0.78	1.13	0.25
F5	0.57	−1.28	−1.00	0.58	1.06	−0.90	−0.84	1.25	0.25
G0	0.68	−1.23	−1.06	0.74	1.14	−0.90	−0.90	1.36	0.25
G5	0.79	−1.21	−1.12	0.89	1.22	−0.92	−0.97	1.46	0.25
K0	0.92	−1.21	−1.23	1.09	1.31	−0.97	−1.08	1.59	0.24
K5	1.11	−1.29	−1.44	1.39	1.45	−1.11	−1.31	1.80	0.23
M0	1.24	−1.46	−1.71	1.62	1.53	−1.31	−1.59	1.97	0.22
M5	1.32	−1.89	−2.22	1.89	1.55	−1.77	−2.12	2.19	0.18

Table 3(d). Ratios of detected to visual photon flux for stars, in magnitudes

(Eq. [15b] in text)

$$-2.5 \log \left[n_D^{bb} \frac{(T, m_V)}{n_V(m_V)} \right] \equiv -2.5 \log \left(10^{1.05} \int D(\lambda) \left(\frac{\lambda}{\lambda_V} \right) \hat{W}_V(\lambda, T)\, d\lambda \right)$$

Star type	Detector type: PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	−0.15	−2.66	−0.88	−0.61	0.54	−1.05	−0.68	0.67	−0.75
B5	0.10	−1.88	−0.93	−0.19	0.73	−0.99	−0.74	0.87	−0.75
A0	0.42	−1.44	−1.02	0.32	0.95	−0.95	−0.85	1.14	−0.75
A5	0.55	−1.34	−1.08	0.53	1.05	−0.95	−0.92	1.25	−0.75
F0	0.68	−1.29	−1.14	0.73	1.14	−0.96	−0.99	1.37	−0.75
F5	0.79	−1.26	−1.21	0.90	1.22	−0.99	−1.07	1.48	−0.75
G0	0.89	−1.27	−1.29	1.04	1.29	−1.02	−1.15	1.57	−0.75
G5	0.97	−1.29	−1.36	1.16	1.35	−1.07	−1.23	1.65	−0.76
K0	1.07	−1.34	−1.49	1.33	1.42	−1.15	−1.36	1.77	−0.76
K5	1.20	−1.50	−1.74	1.57	1.50	−1.35	−1.62	1.94	−0.78
M0	1.28	−1.73	−2.03	1.76	1.54	−1.60	−1.92	2.08	−0.80
M5	1.29	−2.24	−2.59	1.98	1.51	−2.14	−2.49	2.25	−0.85

Table 4(a). Ratios of detected to visual irradiance and photon flux for lasers at $\lambda_s \equiv 0.532\ \mu$

(Eqs. [22] and [24] in text)

$$\log\left[\frac{H_D^{mc}(\lambda_s, m_V)}{H_V^{mc}(\lambda_s, m_V)}\right] = \log\left[\frac{n_D^{mc}(\lambda_s, m_V)}{n_V^{mv}(\lambda_s, m_V)}\right] = \log\left[\frac{D(\lambda_s)}{V(\lambda_s)}\right]$$

$$V(\lambda_s) \simeq 0.952$$

Detector type	Ratio
PMT	−0.92
CCD	−0.22
APD	−0.32
MAMA	−1.00
atm + PMT	−1.02
atm + CCD	−0.32
atm + APD	−0.42
atm + MAMA	−1.09
atm + vis	−0.09

Table 4(b). Ratios of detected to visual irradiance and photon flux for lasers at $\lambda_s \equiv 0.532\ \mu$, in magnitudes

(Eqs. [22] and [24] in text)

$$-2.5 \log\left[\frac{D(\lambda_s)}{V(\lambda_s)}\right]$$

$$V(\lambda_s) \simeq 0.952$$

Detector type	Ratio
PMT	2.30
CCD	0.56
APD	0.80
MAMA	2.50
atm + PMT	2.54
atm + CCD	0.79
atm + APD	1.04
atm + MAMA	2.74
atm + vis	0.23

Table 5(a). Ratios $R(0)$ of detected irradiance from laser and stars of identical m_V

(Eq. [25b] in text)

$$\log R(0) \equiv \log\left[\frac{H_D^{mc}(\lambda_s, m_V)}{H_D^{bb}(T, m_V)}\right]$$

$$= \log\left(\frac{\left[\dfrac{D(\lambda_s)}{V(\lambda_s)}\right]}{10^{1.05}\int D(\lambda)\,\widehat{W}_V(\lambda, T)\,d\lambda}\right)$$

$\lambda_s \equiv 0.532\,\mu$, $V(\lambda_s) = 0.952$

Star type	Detector type								
	PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	−1.13	−1.70	−0.64	−1.48	−0.92	−0.79	−0.64	−0.96	0.001
B5	−1.02	−1.22	−0.65	−1.29	−0.83	−0.74	−0.66	−0.87	0.003
A0	−0.88	−0.90	−0.67	−1.05	−0.73	−0.70	−0.69	−0.75	0.005
A5	−0.81	−0.82	−0.68	−0.95	−0.68	−0.69	−0.70	−0.69	0.005
F0	−0.75	−0.77	−0.70	−0.86	−0.63	−0.68	−0.73	−0.64	0.005
F5	−0.69	−0.73	−0.72	−0.77	−0.59	−0.68	−0.75	−0.59	0.005
G0	−0.65	−0.72	−0.75	−0.71	−0.56	−0.68	−0.78	−0.55	0.005
G5	−0.61	−0.71	−0.77	−0.65	−0.53	−0.69	−0.80	−0.51	0.004
K0	−0.55	−0.71	−0.81	−0.56	−0.49	−0.70	−0.85	−0.46	0.003
K5	−0.48	−0.74	−0.90	−0.45	−0.44	−0.76	−0.94	−0.37	−0.001
M0	−0.43	−0.81	−1.00	−0.35	−0.41	−0.84	−1.05	−0.30	−0.007
M5	−0.40	−0.98	−1.21	−0.25	−0.39	−1.02	−1.26	−0.22	−0.022

Table 5(b). Ratios $\bar{R}(0)$ of detected photon flux from laser and stars of identical m_V

(Eq. [26b] in text)

$$\log \bar{R}(0) \equiv \log \left[\frac{n_D^{mc}(\lambda_s, m_V)}{n_D^{bb}(T, m_V)} \right]$$

$$= \log \left(\frac{\left(\frac{\lambda_s}{\lambda_V}\right)\left[\frac{D(\lambda_s)}{V(\lambda_s)}\right]}{10^{1.05} \int D(\lambda) \left(\frac{\lambda}{\lambda_V}\right) \hat{W}_V(\lambda, T) d\lambda} \right)$$

$\lambda_s \equiv 0.532\,\mu$, $\lambda_V \equiv 0.55\,\mu$, $V(\lambda_s) = 0.952$

Star type	Detector type								
	PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	−0.98	−1.29	−0.67	−1.25	−0.80	−0.74	−0.69	−0.83	−0.395
B5	−0.88	−0.98	−0.69	−1.08	−0.73	−0.71	−0.71	−0.75	−0.394
A0	−0.75	−0.80	−0.73	−0.87	−0.63	−0.70	−0.76	−0.64	−0.393
A5	−0.70	−0.76	−0.75	−0.79	−0.60	−0.70	−0.78	−0.59	−0.393
F0	−0.65	−0.74	−0.78	−0.71	−0.56	−0.70	−0.81	−0.55	−0.394
F5	−0.60	−0.73	−0.81	−0.64	−0.53	−0.71	−0.84	−0.50	−0.395
G0	−0.57	−0.73	−0.84	−0.59	−0.50	−0.73	−0.87	−0.48	−0.396
G5	−0.53	−0.74	−0.87	−0.54	−0.48	−0.74	−0.91	−0.43	−0.397
K0	−0.49	−0.76	−0.92	−0.47	−0.45	−0.78	−0.96	−0.39	−0.399
K5	−0.44	−0.82	−1.02	−0.37	−0.41	−0.86	−1.06	−0.32	−0.405
M0	−0.41	−0.92	−1.13	−0.30	−0.40	−0.96	−1.18	−0.26	−0.413
M5	−0.41	−1.12	−1.36	−0.21	−0.41	−1.17	−1.41	−0.19	−0.432

Table 5(c). Ratios $R(0)$ of detected irradiance from laser and stars of identical m_V, in magnitudes

(Eq. [25b] in text)

$$-2.5 \log R(0) \equiv -2.5 \log \left[\frac{H_D^{mc}(\lambda_s, m_V)}{H_D^{bb}(T, m_V)} \right]$$

$$= -2.5 \log \left(\frac{\left[\frac{D(\lambda_s)}{V(\lambda_s)} \right]}{10^{1.05} \int D(\lambda)\, \widehat{W}_V(\lambda, T) d\lambda} \right)$$

$\lambda_s \equiv 0.532\ \mu$, $V(\lambda_s) = 0.952$

Star type	Detector type								
	PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	2.84	4.24	1.59	3.69	2.29	1.97	1.61	2.39	−0.003
B5	2.56	3.04	1.61	3.21	2.08	1.86	1.64	2.17	−0.008
A0	2.19	2.25	1.67	2.62	1.81	1.75	1.71	1.87	−0.013
A5	2.03	2.05	1.71	2.37	1.69	1.71	1.76	1.74	−0.013
F0	1.87	1.92	1.75	2.14	1.58	1.69	1.82	1.60	−0.013
F5	1.74	1.83	1.81	1.93	1.48	1.69	1.88	1.48	−0.013
G0	1.62	1.79	1.86	1.76	1.39	1.70	1.94	1.38	−0.013
G5	1.52	1.76	1.93	1.61	1.32	1.71	2.01	1.28	−0.011
K0	1.38	1.76	2.03	1.41	1.22	1.76	2.12	1.14	−0.008
K5	1.19	1.85	2.25	1.12	1.09	1.90	2.35	0.94	0.002
M0	1.07	2.02	2.51	0.88	1.01	2.10	2.62	0.76	0.017
M5	0.99	2.45	3.02	0.61	0.98	2.56	3.15	0.55	0.054

Table 5(d). Ratios $\bar{R}(0)$ of detected photon flux from laser and stars of identical m_V, in magnitudes

(Eq. [26b] in text)

$$-2.5 \log \bar{R}(0) \equiv -2.5 \log \left[\frac{n_D^{mc}(\lambda_s, m_V)}{n_D^{bb}(T, m_V)} \right]$$

$$= -2.5 \log \left(\frac{\left(\frac{\lambda_s}{\lambda_V}\right) \left[\frac{D(\lambda_s)}{V(\lambda_s)}\right]}{10^{1.05} \int D(\lambda) \left(\frac{\lambda}{\lambda_V}\right) \widehat{W}_V(\lambda, T) d\lambda} \right)$$

$\lambda_s \equiv 0.532\ \mu$, $\lambda_V \equiv 0.55\ \mu$, $V(\lambda_s) = 0.952$

Star type	Detector type								
	PMT	CCD	APD	MAMA	atm + PMT	atm + CCD	atm + APD	atm + MAMA	atm + vis
B0	2.45	3.21	1.68	3.11	1.99	1.85	1.72	2.07	0.99
B5	2.20	2.44	1.73	2.69	1.81	1.78	1.78	1.87	0.98
A0	1.89	1.99	1.82	2.18	1.58	1.74	1.89	1.60	0.98
A5	1.75	1.90	1.88	1.97	1.49	1.74	1.95	1.48	0.98
F0	1.62	1.84	1.95	1.77	1.39	1.75	2.03	1.37	0.98
F5	1.51	1.82	2.02	1.61	1.31	1.78	2.11	1.26	0.99
G0	1.42	1.82	2.09	1.46	1.25	1.81	2.18	1.17	0.99
G5	1.33	1.84	2.17	1.34	1.19	1.86	2.27	1.08	0.99
K0	1.23	1.90	2.29	1.17	1.12	1.94	2.40	0.97	1.00
K5	1.10	2.06	2.54	0.93	1.03	2.14	2.66	0.80	1.01
M0	1.03	2.29	2.84	0.74	1.00	2.39	2.96	0.65	1.03
M5	1.02	2.80	3.39	0.53	1.03	2.93	3.53	0.48	1.08

Table 6(a). Star numbers (Ref. [6], p. 243)

N_{m_V} = number of stars per square degree brighter than visual magnitude m_V

$\log N_{m_V}$

m_V	Galactic latitude, degrees									mean 0 – 90
	0	±5	±10	±20	±30	±40	±50	±60	±90	
0.0		−3.9			−4.2			−4.3		−4.1
1.0		−3.3			−3.6			−3.7		−5.56
2.0		−2.7			−3.0			−3.1		−3.00
3.0		−2.14			−2.5			−2.6		−2.43
4.0	−1.55	−1.63	−1.68	−1.81	−1.96	−2.05	−2.10	−2.12	−2.20	−1.90
5.0	−1.08	−1.16	−1.23	−1.36	−1.49	−1.56	−1.60	−1.63	−1.69	−1.41
6.0	−0.60	−0.68	−0.75	−0.88	−1.00	−1.07	−1.12	−1.15	−1.20	−0.93
7.0	−0.16	−0.23	−0.30	−0.43	−0.54	−0.61	−0.66	−0.69	−0.74	−0.46
8.0	+0.29	+0.23	+0.15	+0.02	−0.08	−0.16	−0.21	−0.24	−0.30	+0.00
9.0	+0.78	+0.69	+0.61	+0.48	+0.38	+0.30	+0.25	+0.20	+0.14	+0.45
10.0	+1.25	+1.16	+1.08	+0.94	+0.82	+0.74	+0.68	+0.63	+0.55	+0.91
11.0	1.73	1.63	1.53	1.38	1.26	1.17	1.10	1.05	0.96	+1.34
12.0	2.18	2.07	1.93	1.80	1.67	1.57	1.49	1.42	1.33	+1.76
13.0	2.60	2.49	2.37	2.20	2.08	1.94	1.84	1.77	1.69	+2.17
14.0	3.02	2.91	2.78	2.60	2.44	2.28	2.18	2.09	2.01	+2.56
15.0	+3.42	+3.30	+3.18	+2.95	+2.78	+2.61	+2.50	+2.40	+2.27	+2.94
16.0	3.78	3.71	3.54	3.30	3.09	2.91	2.78	2.68	2.54	+3.29
17.0	4.13	4.08	3.90	3.60	3.37	3.19	3.05	2.94	2.78	+3.64
18.0	4.50	4.40	4.23	3.93	3.65	3.44	3.29	3.17	3.02	+3.95
19.0	4.8	4.7	4.6	4.2	3.9	3.7	3.5	3.4	3.2	+4.20
20.0	+5.0	+5.0	+4.9	+4.5	+4.1	+3.9	+3.7	+3.6	+3.4	+4.5
21.0	5.3	5.2	5.1	4.8	4.3	4.1	3.9	3.7	3.5	+4.7

Table 6(b). Relative numbers of stars in each spectral class (up to m_V = 8.5)

Class	B	A	F	G	K	M
% stars	10	22	19	14	31	3

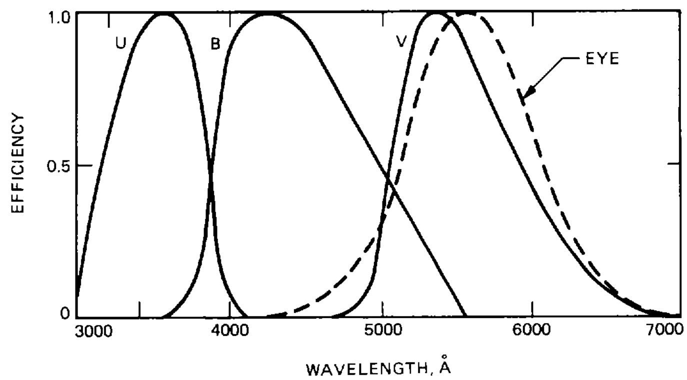

Fig. 1. Sensitivity curves of the eye and of the UBV photometric system

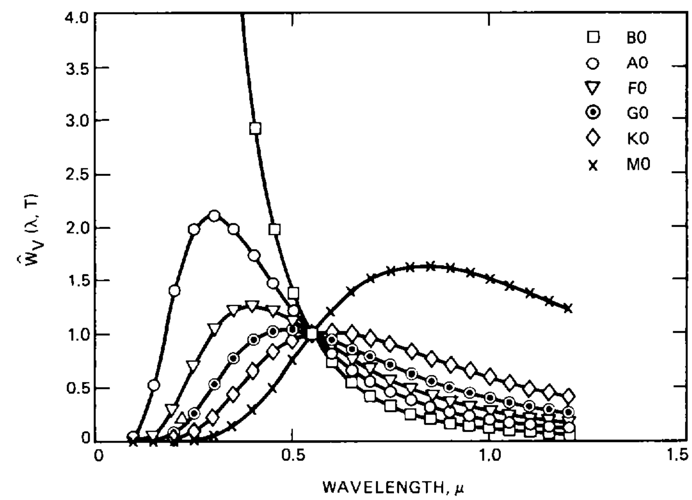

Fig. 2. Normalized blackbody distributions

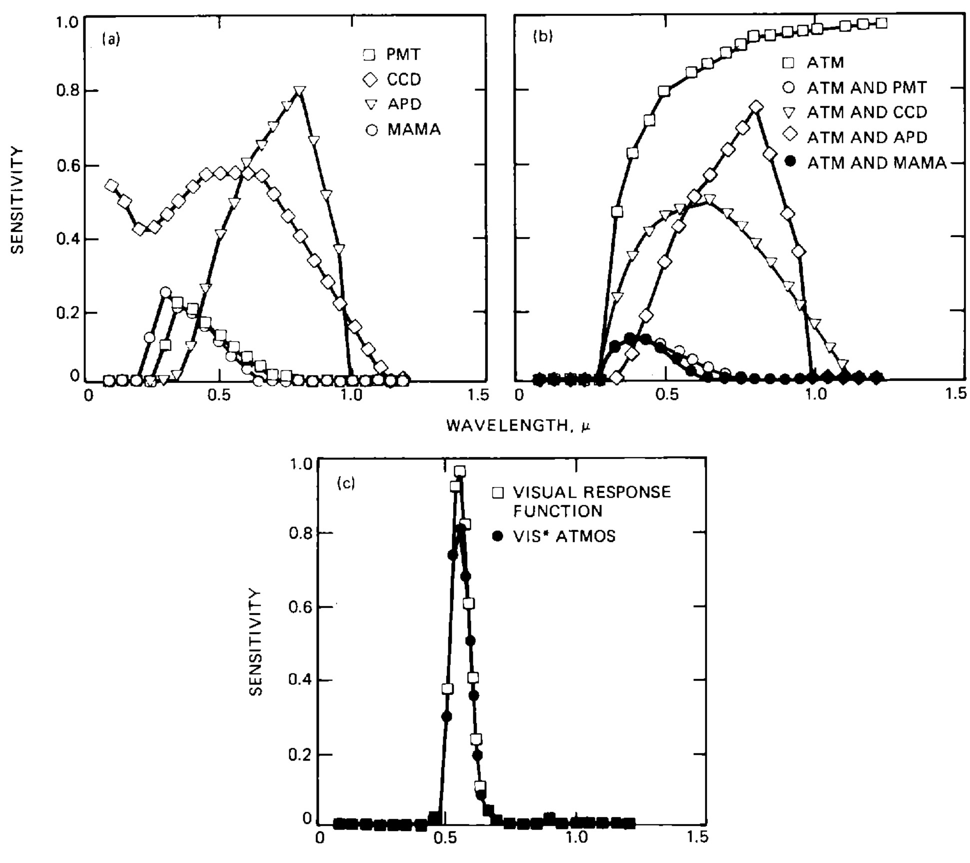

Fig. 3. Spectral sensitivities by detector type (a,b); visual response function (c)

Appendix

Detector Spectral Quantum Efficiencies

The detector spectral sensitivities $D(\lambda)$ referred to throughout this report do not in practice represent only the spectral quantum efficiencies of the detectors. They should also include the efficiency of the receiving optics as well as an atmospheric transmission function for detectors beneath the atmosphere. In these calculations the receiving-optics efficiency has been ignored, since it is highly system-dependent and typically is designed to be approximately constant over the range of the detector's spectral response. Atmospheric transmission is included, however. Therefore, nine different functions $D(\lambda)$ are considered. The first four are the spectral quantum efficiencies of four representative kinds of detectors currently used or likely to be used for spacecraft applications–photomultiplier tubes, charge-coupled devices, avalanche photodiodes, and multianode microchannel array detectors.[1] The next four are these efficiencies multiplied by the atmospheric transmission function defined in Allen [6]. The last is a product of the atmospheric transmission and visual response functions, which gives a visual-brightness reference for ground-based observations. The wavelength range chosen was 0.1 μ (a reasonable optics cutoff wavelength) through 1.2 μ (beyond which the sensitivity of most detectors is very poor). In all these functions, λ stands for wavelength in units of micrometers (μ). The functions are graphed together in Fig. 3(a)–(c).

I. Photomultiplier Tube (PMT)

The spectral quantum efficiency of a Hammamatsu (S-20) PMT (R64a) is modeled approximately by the following series of linear functions:

$2.13\lambda - 0.5325$	$(0.250 < \lambda < 0.350)$
0.213	$(0.350 < \lambda < 0.375)$
$-0.634\lambda + 0.4515$	$(0.375 < \lambda < 0.650)$
$-0.3065\lambda + 0.238$	$(0.650 < \lambda < 0.776)$

The quantum efficiency at $\lambda = 0.532\ \mu$ is 0.114.

II. Charge-Coupled Device (CCD)

The spectral quantum efficiency of a Texas Instruments three-phase (UV-flooded) CCD (J. Janesick, private communication) is modeled approximately by the following series of linear functions:

$-0.8\lambda + 0.62$	$(0.10 < \lambda < 0.20)$
0.42	$(0.20 < \lambda < 0.25)$
$0.75\lambda + 0.23$	$(0.25 < \lambda < 0.45)$
0.57	$(0.45 < \lambda < 0.65)$
$-1.2\lambda + 1.35$	$(0.65 < \lambda < 1.12)$

The quantum efficiency at $\lambda = 0.532\ \mu$ is 0.57.

III. Avalanche Photodiode (APD)

The spectral quantum efficiency of an RCA APD (30902S) is modeled approximately by the following series of linear functions:

$1.8\lambda - 0.62$	$(0.35 < \lambda < 0.4)$
$3.0\lambda - 1.1$	$(0.40 < \lambda < 0.5)$
$1.7\lambda - 0.45$	$(0.50 < \lambda < 0.6)$
λ	$(0.60 < \lambda < 0.8)$
$-3.0\lambda + 3.2$	$(0.80 < \lambda < 1.0)$

The quantum efficiency at $\lambda = 0.532\ \mu$ is 0.454.

IV. Multianode Microchannel Array (MAMA)

The spectral quantum efficiency of a MAMA device with a bi-alkali cathode [8] is modeled approximately by the following series of linear functions:

$2.5\lambda - 0.5$	$(0.20 < \lambda < 0.3)$
$-\lambda + 0.55$	$(0.30 < \lambda < 0.35)$
0.2	$(0.35 < \lambda < 0.4)$
$-0.78\lambda + 0.51$	$(0.40 < \lambda < 0.66)$

The quantum efficiency at $\lambda = 0.532\ \mu$ is 0.095.

V. Atmospheric Spectral Transmission Function

The spectral transmission function of Earth's atmosphere is modeled approximately by the following series of linear functions [6]:

$10.5\lambda - 3.14$	$(0.30 < \lambda < 0.34)$
$3.33\lambda - 0.7$	$(0.34 < \lambda < 0.4)$
$1.6\lambda - 0.01$	$(0.40 < \lambda < 0.5)$
$0.5\lambda + 0.54$	$(0.50 < \lambda < 0.8)$
$0.08\lambda + 0.87$	$(0.80 < \lambda < 1.6)$
1.0	$(1.60 < \lambda)$

The transmission at $\lambda = 0.532\ \mu$ is 0.806.

[1]Data on detector sensitivities were gathered by Jim Annis, a graduate student at the Institute of Astronomy in Hawaii, during his summer employment at JPL in 1987.