Effect of Detector Dead Time on the Performance of Optical Direct-Detection Communication Links

C.-C. Chen
Communications Systems Research Section

Avalanche photodiodes (APDs) operating in the Geiger mode can provide a significantly improved single-photon detection sensitivity over conventional photodiodes. However, the quenching circuit required to remove the excess charge carriers after each photon event can introduce an undesirable dead time into the detection process. The effect of this detector dead time on the performance of a binary pulse-position-modulated (PPM) channel is studied by analyzing the error probability. It is shown that, when background noise is negligible, the performance of the detector with dead time is similar to that of a quantum-limited receiver. For systems with increasing background intensities, the error rate of the receiver starts to degrade rapidly with increasing dead time. The power penalty due to detector dead time is also evaluated and shown to depend critically on background intensity as well as dead time. Given the expected background strength in an optical channel, therefore, a constraint must be placed on the bandwidth of the receiver to limit the amount of power penalty due to detector dead time.

I. Introduction

The single-photon detection sensitivity of avalanche photodiodes (APDs) can be improved significantly by operating the APDs in the Geiger mode [1]. In contrast to conventional photodetectors that are susceptible to circuit noise, the extremely high gain of the Geiger-mode APD allows single-photon events to be detected effectively. For deep-space communications where the energy efficiency is critical, the use of the Geiger-mode APD will allow the implementation of near-quantum-limited optical communication links.

To operate the APD in the Geiger mode, the APD is first cooled to reduce the number of thermally excited charge carriers. The APD is then reverse-biased beyond its breakdown voltage. A single photon event will then initiate an avalanche of charge carriers and generate a detectable signal. Because of the strong reverse bias, however, the APD must be quenched after each photon event to remove the excess charge carriers and stop the avalanche. This quenching process introduces an undesirable dead time into the detection process. Photons that arrive after the initial photon event and before the end of the quenching process will therefore not be detected. The presence of detector dead time can also perturb the count statistics of the photodetector. Compared to a conventional detector for which the detection process can be modeled as a Poisson arrival process, the number of detectable photon events of a Geiger mode APD over a fixed interval is limited by its dead time. The distortion of count statistics can result in a higher bit error rate for optical communication channels. Consequently, a higher signal power is required to maintain the given error performance. In this article, the effect of detector dead time on the performance of an optical PPM system is analyzed. It is shown that in the presence of detector dead time, a higher signal power is required to maintain channel performance. This power penalty is then numerically evaluated for several different signal and background strengths.

II. Photocount Statistics

In the absence of the detector dead time, the detection of photon events can be modeled as a Poisson arrival process for which the probability of detecting k events over a time period $[0, T_s)$ is given by

$$p(k) = \frac{(\lambda T_s)^k}{k!} e^{-\lambda T_s}$$

where the constant λ is the average rate of arrival for photons. The photocount rate λ can be related to the intensity of the optical signal by

$$\lambda = \frac{\eta P_s}{h\nu}$$

where η is the quantum efficiency of the detector, P_s is the intensity of the incident optical signal, h is Planck's constant, and ν is the frequency of the optical signal.

When the detector dead time is considered, the actual count statistics of the receiver can be very different from the photon arrival statistics. This difference is illustrated in Fig. 1. The photons that arrive after the initial photon event and before the end of the quenching process will not be detected. In the presence of the detector dead time, the detection statistics are no longer Poisson distributed. In fact, for a detector with dead time Δ, the maximum observable count over a period of $[0, T_s)$ is $k_{max} = \lfloor T_s/\Delta \rfloor + 1$ where $\lfloor x \rfloor$ denotes the largest integer that is smaller than x. Furthermore, since the detector dead time can overlap two adjacent time slots, the count statistics over a given time slot will also depend on the received optical intensity over the previous time slot. For a receiver with dead time Δ, the probability of detecting k photon events where $k \leqslant k_{max} - 2$ is given by (Appendix)

$$p(k) = e^{-\delta K_0 - (1-k\delta)K_1} \times \left\{ \frac{K_1(1-k\delta)^k}{k!} \right.$$

$$+ \sum_{\ell=0}^{k-1} \frac{K_1^\ell}{\ell!} \left[(1-k\delta)^\ell - \left(1-(k-1)\delta\right)^\ell e^{-\delta K_1} \right]$$

$$+ \sum_{\ell=0}^{k} \frac{K_0 K_1^k}{\ell!(K_0+K_1)^{k-\ell+1}}$$

$$\times \left[\left(1-(k+1)\delta\right)^\ell e^{\delta(K_0+K_1)} - (1-k\delta)^\ell \right]$$

$$+ \sum_{\ell=0}^{k-1} \sum_{m=0}^{\ell} \frac{K_0 K_1^\ell}{m!(K_0+K_1)^{\ell-m+1}}$$

$$\times \left[\left(1-(k+1)\delta\right)^m e^{\delta(K_0+K_1)} \right.$$

$$- (1-k\delta)^m (1+e^{\delta K_0})$$

$$\left. \left. + \left(1-(k-1)\delta\right)^m e^{-\delta K_1} \right] \right\}$$

where $\delta = \Delta/T_s$ is the normalized dead time and K_0, K_1 are the average number of incident photons over the previous and the present time slots, respectively. The probability of observing $k_{max} - 1$ and k_{max} events can also be calculated:

$$p(k|k = k_{max} - 1) = 1 - e^{(1-(k+1)\delta)K_0}$$

$$+ \frac{K_0 K_1^k}{(K_0+K_1)^{k+1}} e^{(1-(k+1)\delta)K_0}$$

$$+ e^{-\delta K_0 - (1-k\delta)K_1} \left\{ \frac{K_1^k (1-k\delta)^k}{k!} \right.$$

$$+ \sum_{\ell=0}^{k-1} \frac{K_1^\ell}{\ell!} \left[(1-k\delta)^\ell \right.$$

$$\left. - \left(1-(k-1)\delta\right)^\ell e^{-\delta K_1} \right]$$

$$+ \sum_{\ell=0}^{k-1} \frac{K_0 K_1^\ell}{(K_0+K_1)^{\ell+1}} e^{(1-k\delta)(K_0+K_1)}$$

$$+ \sum_{\ell=0}^{k-1} \sum_{m=0}^{\ell} \frac{K_0 K_1^\ell}{m!(K_0+K_1)^{\ell-m+1}}$$

$$\times \left[\left(1-(k-1)\delta\right)^m e^{-\delta K_1} \right.$$

$$- (1 - k\delta)^m \left(1 + e^{\delta K_0}\right)\Big]$$

$$\left. - \sum_{\ell=0}^{n} \frac{K_0 K_1^k}{\ell!(K_0 + K_1)^{k-\ell+1}} (1 - k\delta)^{\ell} \right\}$$

$$p(k|k = k_{\max}) = e^{(1-k\delta)K_0} - e^{-\delta K_0 - (1-(k-1)\delta)K_1}$$

$$\times \sum_{\ell=0}^{k-1} \frac{K_1^{\ell}}{\ell!} \left(1 - (k-1)\delta\right)^{\ell}$$

$$+ e^{-\delta K_0 - (1-(k-1)\delta)K_1}$$

$$\times \sum_{\ell=0}^{k-1} \sum_{m=0}^{\ell} \frac{K_0 K_1^{\ell}}{m!(K_0 + K_1)^{\ell-m+1}}$$

$$\times \left(1 - (k-1)\delta\right)^m$$

$$- e^{(1-k\delta)K_0} \sum_{\ell=0}^{k-1} \frac{K_0 K_1^{\ell}}{(K_0 + K_1)^{\ell+1}}$$

The probability distribution $p(k)$ is plotted in Fig. 2 for different values of δ. Note that for a receiver with no dead time, the photocount distribution is Poisson with mean K_1. In the presence of the detector dead time, the distribution is shifted towards the lower end. Furthermore, the probability density is zero for $k > k_{\max}$.

III. Performance Impact

The performance of optical channels under Poisson statistics has been investigated thoroughly in the literature [2], [3]. It was shown that the pulse position modulation (PPM) channel offers superior performance over other modulation schemes. In this scheme, each transmitted word is divided into M time slots, and the signal is encoded such that only one of these M slots contains the signal pulse. Because the detector can be implemented by simply comparing the received photocounts over different slot periods and choosing the slot with the highest count, the PPM decoder is less sensitive to the fluctuation in background and signal strengths. Furthermore, by compressing the signal power into a narrow time slot, a higher signal-to-noise ratio can be achieved and the overall system performance is improved.

Given an ideal photon counting detector, the probability of error for a binary optical PPM system can be written as [2]

$$PBE = \frac{1}{2}\Big[1 + Q\left(\sqrt{2K_B},\ \sqrt{2(K_S + K_B)}\right)$$

$$- Q\left(\sqrt{2(K_S + K_B)},\ \sqrt{2K_B}\right)\Big]$$

where $K_S = \lambda_S T_s$, $K_B = \lambda_B T_s$ are the average number of signal and background photons received over the time slot, and $Q(\alpha, \beta)$ is the Marcum's Q function which is given by

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} e^{-(\alpha^2 + x^2)/2} I_0(\alpha x)\, dx$$

where $I_0(x)$ is the modified Bessel function of order zero. The above equation was derived by assuming that the photocount statistics of the receiver can be modeled as a Poisson arrival process. In practice, the detectors used rarely achieve Poisson counting statistics. Conventional PIN diodes and APDs, because of their low signal gain ($G = 1$ for PIN diodes, and $G \approx 100 - 300$ for APDs), are susceptible to the thermal noise that is in general not Poisson distributed. When operating under low signal intensities, the low detection gain can result in a reduced single-photon detection efficiency and non-Poisson output statistics.

The detection sensitivity can be improved by operating the avalanche photodiode in the Geiger mode [1]. Because of the detector dead time associated with the Geiger mode of operation, however, the photocount statistics cannot be modeled as Poisson-distributed random variables. Furthermore, because of the finite duration of the detector dead time, the detection statistics for neighboring time slots become correlated. The photocount statistics of a given time slot depend not only on the signal intensity, but also on the intensity received during the previous time slot. In effect, the presence of the detector dead time introduces an intersymbol interference into the detection statistics.

The actual error probability of the optical PPM channel under this intersymbol interference is difficult to analyze. A simple estimate of the channel error rate, nevertheless, can be derived by assuming that the count statistics over different time slots are independent. In this case the error probability of a binary PPM channel can be written as

$$PBE(K_0, K_1) = \frac{1}{2} \sum_{k=0}^{\infty} P(k|K_0) P(k|K_1)$$

$$+ \sum_{k=0}^{\infty} P(k|K_1) \sum_{\ell=k+1}^{\infty} P(\ell|K_0)$$

In the above equation, $P(k|K_0)$ and $P(k|K_1)$ denote the probability of receiving k counts in a slot where the average incident photons are K_0 and K_1, respectively. The first term on the right-hand side of the PBE expression is the probability that both time slots receive equal counts, in which case a random choice was assigned at the decoder. The second term on the right-hand side is simply the probability that the background time slot receives more photons than the signal time slot. It should be remembered that the above PBE expression was derived by assuming that the receiver photocounts for adjacent time slots are uncorrelated. The marginal count statistics for each time slot, however, still depend on the optical intensities received in the present and the previous time slots. Using this approximation, the PBE of a binary PPM channel can be evaluated by averaging over different distributions of the transmitted sequences. Plotted in Fig. 3 is the probability of bit error for a binary PPM system assuming independent slot count statistics. The PBE is evaluated by assuming that channel symbols are transmitted with equal probability. It is seen from the figure that, at small values of δ, the effect of dead time on the channel PBE is almost negligible. As the dead time increases, however, the channel error rate starts to increase. For $\delta > 0.2$, the discrete quantization of the detector count statistics can result in an observable distortion of the PBE curves. This fact can be seen from Fig. 3 where, at $\delta \approx 1/3$ and $1/2$, the PBE curves display significant distortions due to quantization.

It should be noted that the performance of the PPM receiver depends strongly on the background count statistics. For channels with no background noise, the performance of the detector with dead time is identical to that of a quantum-limited receiver. When background noise is present in the receiver, however, the performance of the receiver becomes very sensitive to the detector dead time. Plotted in Fig. 4 is the probability of bit error for a binary PPM channel at different values of the background intensity. It is seen that, even for $K_B \approx 0.5$, the receiver performance is very sensitive to the detector dead time at $\delta > 0.1$.

Finally, the increasing probability of bit error implies that higher signal power is needed to maintain the system performance in the presence of detector dead time. The power penalty due to detector dead time can be defined as the ratio between the required signal power in the presence and in the absence of detector dead time. This power penalty can be evaluated numerically. Plotted in Fig. 5 is the receiver power penalty versus dead time for a binary PPM channel at several values of the probability of bit error. The background intensity is assumed to be $K_B = 1$. Note that the power penalty increases rapidly for $\delta > 0.1$. In particular, for PBE = 10^{-6}, the power penalty due to dead time is greater than 2 dB when $\delta > 0.1$. This implies that, in order to keep the power penalty due to detector dead time to within 2 dB, the system operating at PBE = 10^{-6} must have an effective detector bandwidth higher than $10/T_s$. Equivalently, the detector used in the receiver must be capable of counting photons at a rate higher than $10/T_s$. The required detector bandwidth is smaller for systems operating at lower bit error rates and weaker background intensities.

The power penalty also depends strongly on the background intensity. Shown in Fig. 6 are the power penalty curves for a system operating at PBE = 10^{-3} and at different background levels. Note that systems with higher background levels are more sensitive to the detector dead time. Given the detector dead time, the power penalty is smaller for systems with weaker backgrounds. In particular, for $\delta = 0.1$, the power penalty ranges from less than 0.1 dB for $K_B = 0.01$, to approximately 1.5 dB for $K_B = 2$.

IV. Conclusions

The effect of detector dead time on the performance of an optical communication link was analyzed. It was shown that, in the presence of a large detector dead time, the receiver photocount statistics are seriously distorted. The distortion of photocount statistics can result in a degradation of the receiver performance for systems with nonzero background intensities. As a result, higher signal power must be applied to maintain the performance of the optical channel. This power penalty was shown to be a function of the desired BER, the detector dead time, and the background intensity. When designing the optical channel using the Geiger mode APDs, therefore, sufficient power margin must be reserved to account for the losses due to detector dead time. Similarly, given the power budget, the desired BER, and the expected background level, care must be taken to ensure that the detector employed has a sufficient bandwidth.

References

[1] D. Robinson and B. Metscher, "Photon Detection With Cooled Avalanche Photodiodes," *Appl. Phys. Lett.*, vol. 51, pp. 1493–1494, September 1987.

[2] R. Gagliardi and S. Karp, "M-ary Poisson Detection and Optical Communications," *IEEE Trans. Commun. Tech.*, vol. CT-17, no. 2, pp. 208–216, April 1969.

[3] R. Gagliardi and S. Karp, *Optical Communications*, New York: Wiley, 1975.

[4] D. Snyder, *Random Point Processes*, New York: Wiley, 1975.

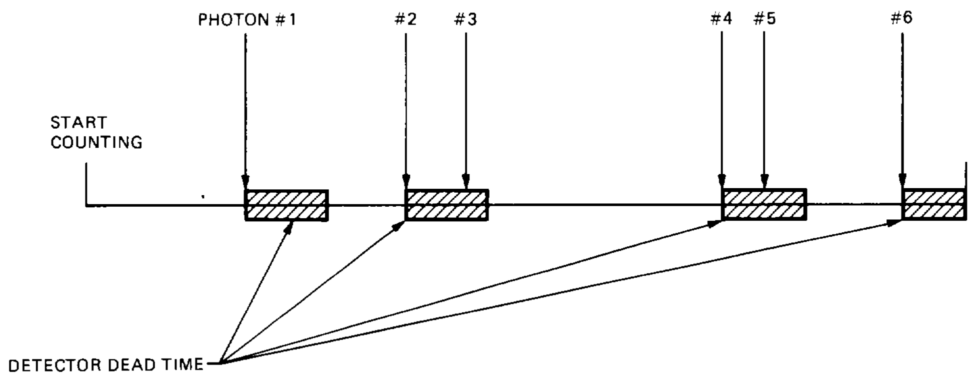

Fig. 1. The count statistics for a detector with dead time can be very different from the photon arrival statistics

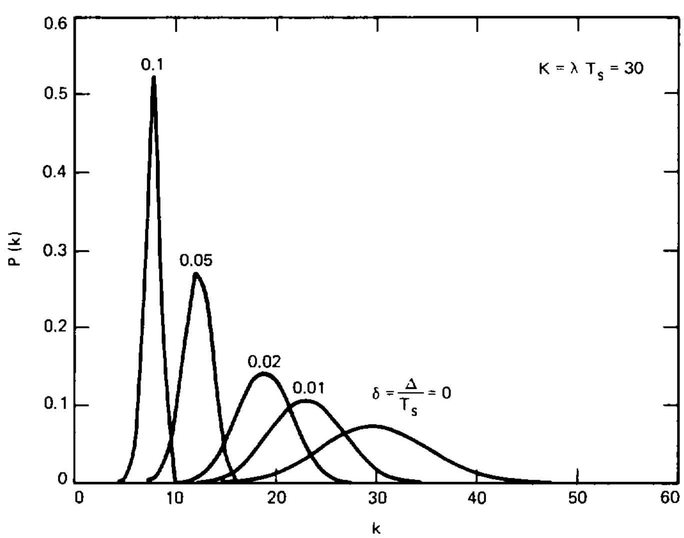

Fig. 2. Probability distribution of detector photocounts at different values of detector dead time

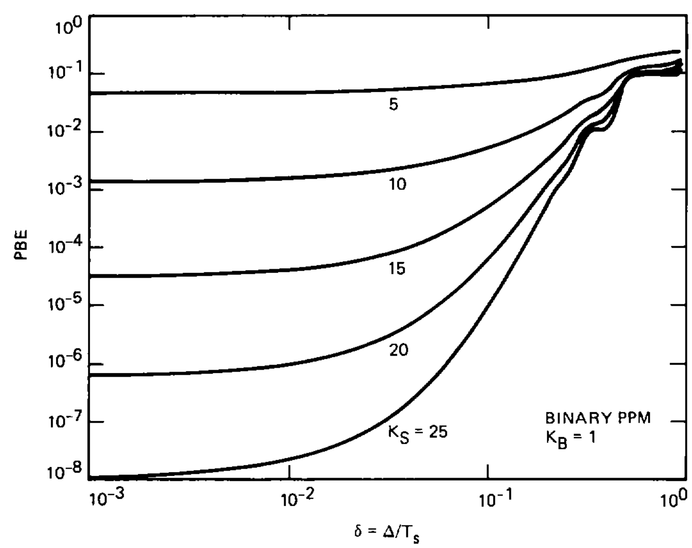

Fig. 3. Probability of bit error versus detector dead time for a binary PPM receiver with varying signal intensity

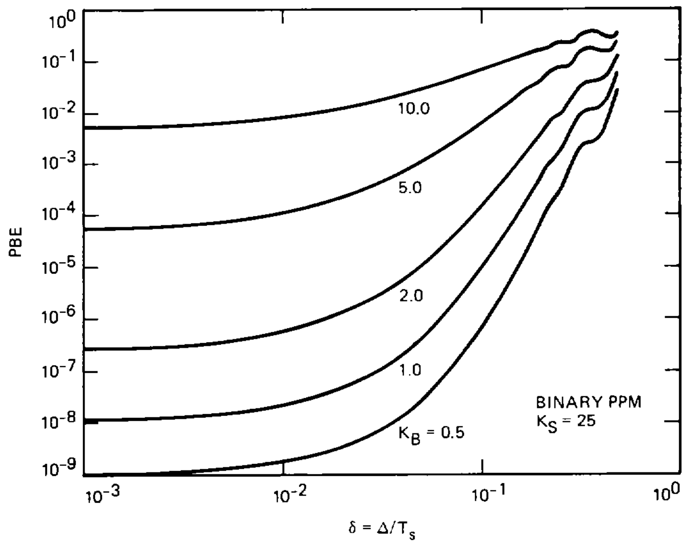

Fig. 4. Probability of bit error versus detector dead time for a binary PPM receiver with different background intensities

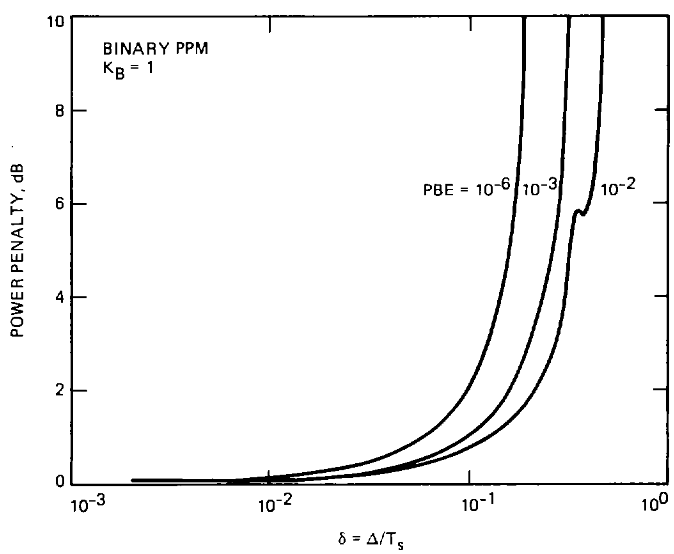

Fig. 5. Power penalty due to detector dead time for a binary PPM receiver at different values of the probability of bit error

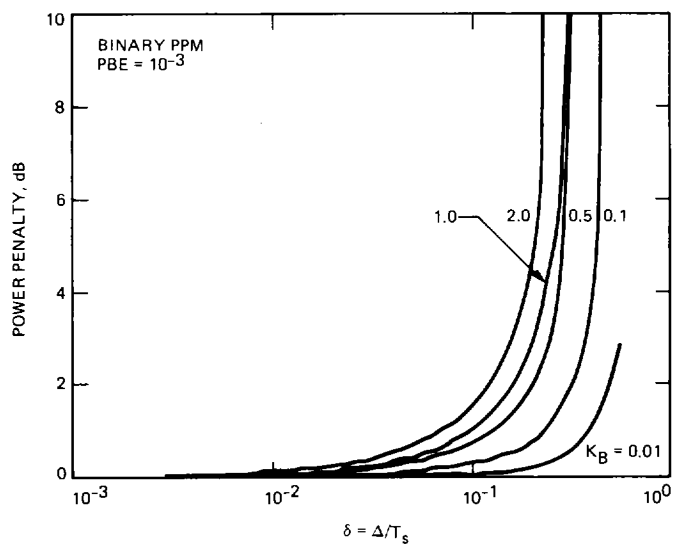

Fig. 6. Power penalty due to detector dead time for a binary PPM receiver at different background intensities

Appendix
Evaluation of the Probability Distribution

The probability distribution for the count statistics can be divided into 4 different cases, depending on the time of arrival t_0 of the last detected photon before the start of the current counting period, and the time of arrival of the kth photon. The probability density for each of the 4 conditions can be easily shown to be [4]

$$\text{Case 1}: t_0 < -\Delta, t_k > T-\Delta : p(t_1, t_2, \ldots, t_k) = \lambda_1^k e^{-\lambda_0 \Delta} e^{-\lambda_1 [t_k-(k-1)\Delta]} dt_1 \ldots dt_k$$

$$\text{Case 2}: t_0 < -\Delta, t_k < T-\Delta : p(t_0, t_1, \ldots, t_k) = \lambda_1^k e^{-\lambda_0 \Delta} e^{-\lambda_1 (T-k\Delta)} dt_1 \ldots dt_k$$

$$\text{Case 3}: t_0 > -\Delta, t_k > T-\Delta : p(t_0, t_1, \ldots, t_k) = \lambda_0 \lambda_1^k e^{\lambda_0 t_0} e^{-\lambda_1 (t_k - t_0 - k\Delta)} dt_0\, dt_1 \ldots dt_k$$

$$\text{Case 4}: t_0 > -\Delta, t_k < T-\Delta : p(t_0, t_1, \ldots, t_k) = \lambda_0 \lambda_1^k e^{\lambda_0 t_0} e^{-\lambda_1 [T - t_0 - (k+1)\Delta]} dt_0\, dt_1 \ldots dt_k$$

The probability of receiving k photons over the period $[0, T)$ is simply the integral of the probability density over the $k+1$ dimensional region spanned by $(t_0, t_1, \ldots, t_k)$. Depending on the number of photons k, the integral must be carried out over a different region.

Case 1 : $t_0 < -\Delta,\ t_k > T - \Delta$:

Depending on the number of signal photons collected, the probability for this case is given by

$$k \leqslant k_{\max} - 1 : P(k) = \int_{t_1=0}^{t_2-\Delta} \int_{t_2=0}^{t_3-\Delta} \cdots \int_{t_{k-1}=(k-2)\Delta}^{t_k-\Delta} \times \int_{t_k=T-\Delta}^{T} p(t_1, \ldots, t_k)\, dt_1\, dt_2 \ldots dt_k$$

$$k = k_{\max} : P(k) = \int_{t_1=0}^{t_2-\Delta} \int_{t_2=0}^{t_3-\Delta} \cdots \int_{t_{k-1}=(k-2)\Delta}^{t_k-\Delta} \times \int_{t_k=(k-1)\Delta}^{T} p(t_1, \ldots, t_k)\, dt_1\, dt_2 \ldots dt_k$$

Case 2 : $t_0 < -\Delta,\ t_k < T - \Delta$:

$$k \leqslant k_{\max} - 1 : P(k) = \int_{t_1=0}^{t_2-\Delta} \int_{t_2=0}^{t_3-\Delta} \cdots \int_{t_{k-1}=(k-2)\Delta}^{t_k-\Delta} \times \int_{t_k=(k-1)\Delta}^{T-\Delta} p(t_1, \ldots, t_k)\, dt_1\, dt_2 \ldots dt_k$$

$$= \frac{(T-k\Delta)^k \lambda_1^k}{k!} e^{-\lambda_1 (T-k\Delta)} e^{-\lambda_0 \Delta}$$

$k = k_{\max}$: Not admissible

Case 3 : $t_0 > -\Delta,\ t_k > T - \Delta$:

In this case the dead time resulting from the last photon that arrived before the start of the time slot will overlap into the current time period, and the resulting photocount probability must be averaged over the probability of arrival for the 0th photon:

$$k \leqslant k_{\max} - 2 : P(k) = \int_{t_0=-\Delta}^{0} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots \int_{t_{k-1}=t_0+(k-1)\Delta}^{t_k-\Delta} \times \int_{t_k=T-\Delta}^{T} p(t_0, t_1, \ldots, t_k)\, dt_0\, dt_1 \ldots dt_k$$

$$k = k_{\max} - 1 : P(k) = \int_{t_0=-\Delta}^{T-(k+1)\Delta} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots \int_{t_{k-1}=t_0+(k-1)\Delta}^{t_k-\Delta}$$

$$\times \int_{t_k=T-\Delta}^{T} p(t_0, t_1, \ldots, t_k)\,dt_0\,dt_1 \ldots dt_k$$

$$+ \int_{t_0=T-(k+1)\Delta}^{0} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots$$

$$\times \int_{t_k=t_0+k\Delta}^{T} p(t_0, t_1, \ldots, t_k)\,dt_0\,dt_1 \ldots dt_k$$

$$k = k_{\max} \ : P(k) = \int_{t_0=-\Delta}^{T-k\Delta} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots$$

$$\times \int_{t_k=t_0+k\Delta}^{T} p(t_0, t_1, \ldots, t_k)\,dt_0\,dt_1 \ldots dt_k$$

Case 4 : $t_0 > -\Delta$, $t_k < T - \Delta$:

$$k \leqslant k_{\max} - 2 : P(k) = \int_{t_0=-\Delta}^{0} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots \int_{t_{k-1}=t_0+(k-1)\Delta}^{t_k-\Delta}$$

$$\times \int_{t_k=t_0+k\Delta}^{T-\Delta} p(t_0, t_1, \ldots, t_k)\,dt_0\,dt_1 \ldots dt_k$$

$$k = k_{\max} - 1 : P(k) = \int_{t_0=-\Delta}^{T-(k+1)\Delta} \int_{t_1=t_0+\Delta}^{t_2-\Delta} \cdots$$

$$\times \int_{t_k=t_0+k\Delta}^{T-\Delta} p(t_0, t_1, \ldots, t_k)\,dt_0\,dt_1 \ldots dt_k$$

$k = k_{\max}$: Not admissible

Since the 4 cases stated above are mutually exclusive, the total probability of detecting k photons is the sum of the probabilities evaluated in all 4 cases. By substituting the probability densities into the integrals, the probability for detecting k photons in the interval can be calculated.