Design of an Optical PPM Communication Link in the Presence of Component Tolerances

C.-C. Chen
Communications Systems Research Section

A systematic approach is described for estimating the performance of an optical direct-detection pulse-position modulation (PPM) communication link in the presence of parameter tolerances. This approach was incorporated into the JPL optical link analysis program to provide a useful tool for optical link design. Given a set of system parameters and their tolerance specifications, the program will calculate the nominal performance margin and its standard deviation. Through use of these values, the optical link can be designed to perform adequately even under adverse operating conditions.

I. Introduction

Uncertainties in system parameters have a strong impact on the design of deep-space communication links. Traditionally, the design practice for communication systems is to reserve sufficient power margin to account for the parameter uncertainties. However, for deep-space systems in which the system power is at a premium, how to trim the design margin and still maintain a sufficiently high confidence range in system performance is an important problem that can be solved only by a systematic approach.

For radio frequency (RF) systems, a rigorous and well-established design procedure has been identified [1] based on extensive experience with RF system design. In this procedure, each parameter in the link control table will be specified by its *design value*, *favorable tolerance*, and *adverse tolerance*. The *design value* is the best estimate of the parameter under normal operating conditions. The *adverse* and *favorable tolerances* are derived based on past experience with the particular system component. These tolerance values are determined so that the actual parameter value generally falls within the specified tolerances. Probability distribution models of these parameters are also constructed based on experience. From these specifications, the mean and variance of each link control table entry can be calculated. These entries are then tabulated so that the final link margin can be calculated.

A similar problem occurs in the design of optical deep-space links. Despite the fact that optical systems generally consume less power than comparable RF systems, the scarcity of prime system power still implies that the communication system must be designed with a tight performance margin. Unfortunately, analysis of the optical link is much more complicated than that of the typical RF system. This is because RF systems' performance depends only on the receiver signal-to-noise ratio (SNR), whereas the performance of the optical link depends not only on the SNR but also on the actual signal and noise powers [2]. Furthermore, in contrast to RF systems, in which extensive design experience has been accumulated, comparatively little experience has been acquired for optical link design. As a result, larger uncertainties in parameter values can be expected for optical systems.

This article describes a systematic approach to estimate the performance of an optical direct-detection PPM communication link in the presence of uncertainties in component values. Section II outlines the standard procedure for calculating the performance of an optical communication link. Some shortcomings of this procedure are identified. The procedures and assumptions used to calculate the link control table in the presence of system parameter tolerances are then summarized and discussed in Section III.

II. Performance of an Optical Link

Given an optical system with source laser power P_T, the amount of signal power received by the detector is given by [3] as

$$P_S = P_T \eta_T G_T L_T \left(\frac{\lambda}{4\pi z}\right)^2 G_R \eta_R \eta_{\mathrm{atm}} \eta_F \qquad (1)$$

where η_T and η_R are the efficiencies of the transmitter and receiver optics, G_T and G_R are the transmitter and receiver antenna gains, λ is the optical wavelength, z is the link distance, L_T is the transmitter pointing loss factor, η_{atm} is the atmospheric transmission factor, and η_F is the narrowband filter transmission factor. The factor $(\lambda/4\pi z)^2$ is known as the space loss factor.

The transmitter antenna gain G_T is a function of the operating wavelength and the aperture diameter [4]. For a Gaussian input signal, G_T is given by [4] as

$$G_T = \left(\frac{\pi D_T}{\lambda}\right)^2 \frac{2}{\alpha_T^2} \left[e^{-\alpha_T^2} - e^{-\alpha_T^2 \gamma_T^2}\right]^2 \qquad (2)$$

where D_T is the aperture diameter, d_T is the obscuration diameter, $\gamma_T = d_T/D_T$ is the obscuration ratio of the transmitter, and $\alpha_T \approx 1.12 - 1.30\gamma_T^2 + 2.12\gamma_T^4$ is the optimal truncation ratio of the Gaussian beam. Similarly, the receiver antenna gain can be related to the receiver optics and obscuration diameters by

$$G_R = \left(\frac{\pi D_R}{\lambda}\right)^2 (1 - \lambda_R^2) \qquad (3)$$

where $\gamma_R = d_R/D_R$ is the receiver obscuration ratio.

The pointing loss factor L_T is a function of the transmitter antenna parameters and the transmitter pointing bias and jitter. Given an instantaneous pointing error, ϕ, the farfield intensity of the transmitted signal is reduced from its maximum by a factor [4] of

$$L_T(\phi) = \left[\frac{\int_{\gamma_T^2}^{1} e^{-\alpha_T^2 u^2} J_0(\pi d_T \phi u/\lambda)\, u\, du}{\int_{\gamma_T^2}^{1} e^{-\alpha_T^2 u^2}\, u\, du}\right] \qquad (4)$$

The pointing efficiency of the transmitter, given in terms of the transmitter pointing bias error and the rms jitter, is simply the average of $L_T(\phi)$ over the probability distribution of ϕ. For a two-dimensional spatial tracking system, transmitter pointing errors in the azimuth and altitude directions can be modeled as independently distributed Gaussian random variables so that the resulting radial pointing error is Rician distributed. By averaging $L_T(\phi)$ over the distribution of ϕ, the pointing loss factor L_T can be written as

$$L_T = \int_0^{\infty} L_T(\phi) \frac{\phi}{\sigma_T^2} \exp\left(-\frac{\phi^2 + \epsilon_T^2}{2\sigma_T^2}\right) I_0\left(\frac{\phi \epsilon_T}{\sigma_T^2}\right) d\phi \qquad (5)$$

where ϵ_T is the static pointing error and σ_T is the root-mean-square (rms) jitter in the transmitter line of sight.

In addition to the transmitted signal, the optical receiver also collects background radiation from other sources. Given the total irradiance of the noise source, W_N, the amount of noise power collected by the receiver can be written as

$$P_B = \eta_F \frac{\pi D_R^2}{4} (1 - \gamma_R^2) \frac{\pi \Theta^2}{4} W_N \Delta\lambda \qquad (6)$$

where Θ is the receiver diameter field of view and $\Delta\lambda$ is the narrowband filter bandwidth.

As was previously stated, the performance of the optical link depends on both signal and background powers. Given P_S and P_B, the quantities of signal and background photons detected by the receiver are Poisson-distributed random variables with means

$$K_S = \left(\frac{\eta_D \lambda}{hc}\right) P_S T_w \qquad (7)$$

$$K_B = \left(\frac{\eta_D \lambda}{hc}\right) P_B T_s \qquad (8)$$

where η_D is the detector quantum efficiency and T_w and T_s are the PPM word and slot widths, respectively. The word width is related to the PPM order, M, and the data rate, R_b, by

$$T_w = \frac{\log_2 M}{R_b} \tag{9}$$

and the slot width is related to T_w and the dead time T_d by

$$T_w = MT_s + T_d \tag{10}$$

Since $T_w > T_s$, Eq. (7) shows that the peak signal power is much higher than the average power. This is because in a PPM signaling scheme, the laser is turned on only during the signal time slot, while during the rest of the word period, no signal is transmitted.

Given the expected photocounts, K_S and K_B, the bit error rate (BER) of an M-ary optical PPM link can be written [2] as

$$BER = \frac{M}{2(M-1)} \Bigg[1 - \frac{1}{M} e^{-(K_S + MK_B)} - \sum_{k=1}^{\infty} \frac{(K_S + K_B)^k}{k!} e^{-(K_S+K_B)} \times \left(\sum_{j=0}^{k-1} \frac{K_B^j}{j!} e^{-K_B} \right)^{M-1} \left(\frac{(1+a)^M - 1}{Ma} \right) \Bigg] \tag{11}$$

where

$$a = \frac{K_B^k}{k! \sum_{j=0}^{k-1} \frac{K_B^j}{j!}}$$

The procedure described above can be used to calculate effectively the expected BER of an optical channel. Given the required system BER, the signal power can also be iterated to achieve a desired power margin.

In some instances, having to repeat the calculation for several different links can be a tedious and time-consuming task. In order to ease the design of optical communication links, a simple yet elegant optical link analysis program was developed by W. Marshall and B. Burk in 1986 [3], [5]. The objective of this program is to predict the performance of an optical communication link given a set of component and operational parameters as well as the noise source specification. A list of system parameters needed to specify the optical link is shown in Table 1. After all system parameters are entered, the program calculates and displays the link control table. A sample link control table for an Earth–Saturn link generated by this program is shown in Table 2.

III. Performance Estimate in the Presence of Parameter Uncertainties

The simple link analysis program is very useful in providing a preliminary estimate of the link performance. For systems in which all component and operational parameters can be precisely specified, the simple link analysis program is sufficient. In most systems, however, the parameters may not be specified precisely. For instance, the atmospheric transmission factor can vary from less than 2 dB on a clear day to over 200 dB in a thunderstorm. Components may degrade over time so that their performance specification cannot be given accurately. Accidents and interactions with interplanetary environments can also reduce the efficiency of the optical system. For these reasons, a systematic approach must be devised for the design of optical links in the presence of parameter uncertainties.

The optical link tolerance estimate program is designed to provide a simple analytic tool for estimating the performance of an optical link in the presence of uncertainties in component and operating parameters. Since most parameters are susceptible to time-dependent degradations, tolerance specifications must be given on these parameters. The parameters that must have their tolerances specified include the source power, the optics efficiencies, the detector quantum efficiency, the atmospheric transmission factor, and the narrowband filter transmission factor. The probability distribution of these parameters must also be specified based on sample distributions. From the tolerance specification and the method of distribution, an estimate of the parameter variance can be derived. The procedure for determining the variance is similar to the one used in RF system design.

Some critical link parameters, however, can best be represented as functions of the basic physical quantities. These parameters include the transmitter and receiver antenna gains, the transmitter pointing loss, and the quantity of background photons received. The functional dependencies are in general very complicated so that it is infeasible to derive the tolerance specifications on these parameters based on the tolerance specifications of basic component parameters. Furthermore, the actual performance of these parameters can also depend on factors not previously considered in the ideal link analysis. For instance, the transmitter antenna gain G_T depends not only on the aperture and obscuration diameters but also on the surface tolerance and the incoming beam quality. Deviations from the ideal surface and optical wavefront can result in a degraded antenna gain. Consequently, instead of specifying tolerances on the aperture and obscuration diameters, *adverse* and *favorable tolerances* will be specified directly for the transmitter and receiver antenna gains. The transmitter pointing efficiency is also a complex function of the component parameters. For simplicity, tolerance values will be specified directly on the transmitter pointing efficiency rather than on the static and rms pointing errors. Similarly, tolerance values will be specified for the noise photocount rather than for the receiver FOV and the narrowband filter bandwidth.

Finally, those parameters that can be specified exactly will be given no tolerance specifications. These parameters include the order of the PPM, the slot width and the modulation dead time, and the required bit error rate (BER). The link distance and the laser wavelength are also predetermined parameters. These parameters will be entered without tolerance specifications.

Once all the system parameters are properly specified, the amount of signal power needed to achieve the desired error performance (receiver sensitivity) can be calculated. In general, the receiver sensitivity is a function of the noise power and the modulation format. Since the noise power received by the detector varies for different values of the system parameters, the required signal power must vary accordingly. Unfortunately, the required signal power cannot be related to the noise power by a simple functional form. This can easily be seen from the complexity of the BER expression in Eq. (11). The lack of a simple functional dependence implies that the statistics of the required signal level cannot be deduced easily from the probability distribution of the noise count. Some simplifications must therefore be made before the tolerance on the required signal level can be calculated.

One such simplification is the functional dependence of the required signal level on the background strength. Under the condition of weak background, the BER can be approximated by the Union-Chernoff bound [2]:

$$BER \approx (M-1)e^{(\sqrt{K_S+K_B}-\sqrt{K_B})^2} \qquad (12)$$

By fixing the BER and solving for the required signal level as a function of K_B, it is seen that

$$K_S \approx c + \sqrt{4cK_B} \qquad (13)$$

where $c = \ln[BER/(M-1)]$. For a small fluctuation of background, $K_B = \bar{K}_B + \Delta K_B$, K_S varies as

$$K_S \approx \bar{K}_S + \sqrt{\frac{c}{\bar{K}_B}}\,\Delta K_B \qquad (14)$$

Therefore, when the noise fluctuation is small compared to the average noise level, the required signal level can be assumed to have the same statistical dependence as the background level.

By using the above approximation, the required signal level can be calculated by iterating Eq. (11) given the expected noise photocount. The variance of the required signal power can then be calculated directly based on the variance of the noise power, or the favorable tolerance values of the noise power can be substituted into the BER expression to calculate the *favorable* and *adverse* required signal photocounts. From these values and the assumption that the required signal photocount has the same statistical dependence as the background photocounts, the variance on the required signal level can be calculated.

Once all the system parameters and the required signal power level have been calculated, the generation of the link control table is straightforward. Since the system power margin is generally expressed in terms of decibels, all parameters and their tolerances should be converted into decibels before they are entered into the link control table. This is done as follows: Given a parameter $x = x + \Delta x$, where $E[(\Delta x)^2] = \sigma_x^2$, the value in decibels is given by

$$y = 10 \times \log x \approx 10 \times \left[\log \bar{x} + \frac{\Delta x}{(\ln 10)\bar{x}}\right] \qquad (15)$$

The standard deviation of y can therefore be approximated as

$$\sigma_y \approx \frac{10\sigma_x}{(\ln 10)\bar{x}} \qquad (16)$$

When all parameters and their variance values are computed, the final power margin can be calculated by algebraically summing all the loss factors. Given the independent parameter assumption, the final link performance variance is simply the sum of all parameter variances. Table 3 shows a typical output of the tolerance link control program for the same Earth–Saturn link given by Table 2. The program also calculates and displays the 3σ value of the link performance. Some parameter values are different because the link control table now displays the average values of the parameters instead of the design values. Note that the standard link analysis results in a 5.2-dB margin. When component tolerances are considered, however, the 3σ margin is only 0.8 dB.

IV. Conclusions

The inclusion of tolerance calculations in the existing link analysis program gives the link designer a simple and effective tool for estimating the performance of a deep-space optical PPM communication link. The inclusion of tolerance calculations will permit the design of a deep-space link with sufficient power margin even under adverse operating conditions. At the same time, by minimizing the required system power while maintaining a confident operating margin, the cost of the system can be minimized without seriously affecting link performance. Finally, the use of rigorous design methodology allows critical link parameters to be identified. Improvements in these parameters can then be directly reflected in the reduction of performance uncertainty.

References

[1] J. Yuen, *Deep Space Telecommunications Systems Engineering*, New York: Plenum Press, 1983.

[2] R. Gagliardi and S. Karp, *Optical Communications*, New York: Wiley, 1975.

[3] W. Marshall and B. Burk, "Received Optical Power Calculations for Optical Communications Link Performance Analysis," *TDA Progress Report 42-87*, vol. July–September 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 32–40, November 15, 1986.

[4] B. Klein and J. Degnan, "Optical Antenna Gain: I. Transmitting Antennas," *Appl. Opt.*, vol. 13, no. 10, pp. 2397-2401, October 1974.

[5] J. Lesh, W. Marshall, and J. Katz, "A Simple Method for Designing or Analyzing an Optical Communication Link," *TDA Progress Report 42-85*, vol. January-March 1986, Jet Propulsion Laboratory, Pasadena, California, pp. 25-31, May 15, 1986.

Table 1. Component and operational parameters of a typical optical link

Component parameters:

- Operating wavelength of the transmitter laser, λ
- Average laser output power, P_T
- Diameter of the transmitter aperture, D_T
- Diameter of the transmitter center obscuration, d_T
- Transmitter optics efficiency, η_T
- Transmitter pointing bias error, ϵ_T
- Transmitter rms pointing jitter, σ_T
- Diameter of the receiver aperture, D_R
- Diameter of the receiver center obscuration, d_R
- Receiver optics efficiency, η_T
- Narrowband filter transmittance, η_F
- Narrowband filter spectral bandwidth, $\Delta\lambda$
- Detector quantum efficiency, η_D
- Detector field of view, Θ

Operational parameters:

- Alphabet size, M
- System data rate, R_b
- Modulation slot width, T_s
- Link distance, z
- Atmospheric transmission factor, η_{atm}
- Desired/required link bit error rate, BER

Table 2. Link control table of an Earth–Saturn link generated by the optical link analysis program

		Factor	Decibels
Link specifications			
Component parameters:			
Wavelength, μm	0.532		
Average laser output power, W	2.000		
Modulation extinction ratio	0.100E+06		
Diameter of transmitter aperture, m	0.300		
Obscuration diameter of transmitter, m	0.600E–01		
Transmitter optics efficiency	0.650		
Transmitter pointing bias error, μrad	0.100		
Transmitter rms pointing jitter, μrad	0.100		
Diameter of receiver aperture, m	10.000		
Obscuration diameter of receiver, m	4.280		
Receiver optics efficiency	0.380		
Narrowband filter transmission factor	0.500		
Filter spectral bandwidth, Å	10.000		
Detector quantum efficiency	0.350		
Detector diameter field of view, μrad	100.000		
Receiver type (ideal = 0, APD-based = 1)	0.000		
Operational parameters:			
Alphabet size (M = ?)	256.000		
Data rate, kbps	114.350		
Dead time, μsec	67.401		
Slot width, nsec	10.000		
Distance between transmitter and receiver, AU	9.000		
Atmospheric transmission factor	0.500		
Required link bit error rate	0.200E–01		
Noise sources:			
Saturn receiver to source distance, AU	9.000		
Link control tables			
Laser output power, W		2.000	33.0 dBm
Minimum required peak power, W	0.130E+05		
Transmitter antenna gain		0.222E+13	123.5
Antenna diameter, m	0.300		
Obscuration diameter, m	0.060		
Beamwidth, μrad	3.068		
Transmitter optics efficiency		0.650	–1.9
Transmitter pointing efficiency		0.980	–0.1
Bias error, μrad	0.100		
RMS jitter, μrad	0.100		
Space loss (9.00 AU)		0.989E–39	–390.0
Atmospheric transmission factor		0.500	–3.0
Receiver antenna gain		0.285E+16	154.5
Antenna diameter, m	10.000		
Obscuration diameter, m	4.280		
Field of view, μrad	100.000		
Receiver optics efficiency		0.380	–4.2
Narrowband filter transmission		0.500	–3.0
Bandwidth, Å	10.000		

Table 2. (contd)

		Factor	Decibels
Link control tables (continued)			
Received signal power, W		0.758E–12	–91.2 dBm
Received background power, W	0.400E–09		
Detector quantum efficiency		0.350	–4.6
Photons/joule		0.268E+19	154.3 dB/mJ
Detected signal PE/second		0.711E+06	58.5 dB/Hz
Symbol time, sec		0.700E–04	–41.6 dB/Hz
Detected signal PE/symbol		46.500	16.7
Required signal PE/symbol (ideal)		13.900	11.4
Detected background PE/slot	3.750		
Margin		3.340	5.2

Table 3. Link control table of an Earth–Saturn link when component value tolerances are included in the calculation

	Design value	Favorable tolerance	Adverse tolerance
Link specifications			
Component parameters:			
Wavelength, μm	0.5320		
Transmitter power, W	2.0000	0.5E–01	0.2
Transmitter aperture, m	0.3000		
Transmitter obstruction, m	0.6000E–01		
Transmitter antenna gain, dB	123.4700	0.0	1.0
Transmitter optics efficiency	0.6500	0.2E–01	0.5E–01
Transmitter pointing bias, μrad	0.1000	0.0	0.0
Transmitter pointing jitter, μrad	0.1000		
Transmitter pointing loss, dB	–0.8821E–01	0.3E–01	0.3E–01
Receiver aperture, m	10.0000		
Receiver obstruction, m	4.2800		
Receiver antenna gain, dB	154.5500	0.0	1.0
Receiver optics efficiency	0.3800	0.3E–01	0.4E–01
Filter transmission	0.5000	0.3E–01	0.3E–01
Filter bandwidth, Å	10.0000		
Detector efficiency	0.3500	0.1	0.5E–01
Detector FOV, μrad	100.0000		
Operational parameters:			
Alphabet size (M = ?)	256.0000		
Data rate, kbps	114.3500		
Link length, AU	9.0000		
Dead time, μsec	67.4010		
Slot width, nsec	10.0000		
Atmospheric transmission factor	0.5000	0.2	0.2
Link BER	0.2000E–01		
Noise count/slot	3.9659	0.5	0.5
Noise sources:			
Saturn receiver to source distance, AU		9.0	

	Factor	Decibels	Variance
Link control tables			
Laser output power, W	1.9250	32.84 dBm	0.03
Minimum required peak power, W	0.1347E+05		
Transmitter antenna gain	0.1983E+13	122.97	0.03
Antenna diameter, m	0.3000		
Obscuration diameter, m	0.6000E–01		
Beamwidth, μrad	3.0680		
Transmitter optics efficiency	0.6350	–1.97	0.02
Transmitter pointing efficiency	0.9799	–0.09	0.00
Bias error, μrad	0.1000		
RMS jitter, μrad	0.1000		
Space loss (9.00 AU)	0.9887E–39	–390.05	
Atmospheric transmission factor	0.5000	–3.01	1.01
Receiver antenna gain	0.2539E+16	154.05	0.03
Antenna diameter, m	10.0000		
Obscuration diameter, m	4.2800		
Field of view, μrad	100.0000		
Receiver optics efficiency	0.3750	–4.26	0.05

Table 3. (contd)

	Factor	Decibels	Variance
Link control tables (continued)			
Narrowband filter transmission	0.5000	−3.01	0.02
Bandwidth, Å	10.0000		
Received signal power, W	0.5588E−12	−92.53 dBm	1.18
Background power, W	0.5015E−09		
Detector quantum efficiency	0.3750	−4.26	0.25
Photons/joule	0.2678E+19	154.28 dB/mJ	0.00
Detected signal PE/second	0.5612E+06	57.49 dB/Hz	1.44
Symbol time, sec	0.6996E−04	−41.55 dB/Hz	
Detected signal PE/symbol	39.2600	15.94	1.44
Detected background PE/slot	3.9660		
Required signal PE/symbol	14.2000	11.52	0.00
Margin	2.7660	4.42	1.44
3σ		±3.60	