Accuracy of Telemetry Signal Power Loss in a Filter as an Estimate for Telemetry Degradation

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When telemetry data is transmitted through a communication link, some degradation in telemetry performance occurs as a result of the imperfect frequency response of the channel. The term telemetry degradation as used in this article is the increase in received signal power required to offset this filtering. The usual approach to assessing this degradation is to assume that it is equal to the signal power loss in the filtering, which is easily calculated. However, this approach neglects the effects of the nonlinear phase response of the filter, the effect of any reduction of the receiving system noise due to the filter, and intersymbol interference. This article compares an "exact" calculation of the telemetry degradation, which includes all of the above effects, with the signal power loss calculation for RF filtering of NRZ data on a carrier. The signal power loss calculation is found to be a reasonable approximation when the filter follows the point at which the receiving system noise is introduced, especially if the signal power loss is less than 0.5 dB. The signal power loss approximation is less valid when the receiving system noise is not filtered.

I. Introduction

When telemetry data is transmitted through a communication link, some degradation in telemetry performance occurs as a result of the imperfect frequency response of the channel. The term telemetry degradation as used in this article is the increase in received signal power required to offset this filtering. The usual approach to assessing this degradation is to assume that it is equal to the signal power loss in the filtering, which is easily calculated.

The disadvantage of this approach is that it neglects some potentially significant effects. First, it neglects the effect of the nonlinear phase response of the filtering. Second, it neglects the reduction in the effective noise bandwidth of the telemetry detector caused by the filtering when the filtering follows the point at which the receiving system noise is introduced. This reduction in effective noise bandwidth normally offsets a portion of the telemetry degradation caused by the signal distortion. Finally, it neglects the intersymbol interference which occurs when the duration of the channel impulse response is greater than the duration of a telemetry symbol.

This article assesses the accuracy of approximating telemetry degradation by signal power loss for a telemetry channel in which uncoded non-return-to-zero (NRZ) data directly phase-modulate a carrier at a modulation level less than 90 degrees, and the resulting signal is distorted by a band-pass filter. In the receiving system a discrete carrier-tracking phase-locked loop tracks the carrier component of the received signal and coherently demodulates the telemetry data stream. The bits in the telemetry data stream are detected by an integrate-and-dump circuit, which would be a matched filter for undistorted bits, followed by a decision device which decides a 0 was transmitted if the integrate-and-dump circuit output is positive at the end of a bit transmission and a 1 was transmitted if the integrate-and-dump circuit output is negative at the end of a bit transmission. An analysis of such a system was described previously in [1]. For this article, the analysis of [1] is considered "exact"; however, that analysis does contain some approximations. The most important of these is the assumption that the bit-synchronizer timing is always adjusted to minimize the telemetry degradation.

This article compares the telemetry degradation calculated using the "exact" method of [1] with the corresponding signal power loss in the band-pass filter as a function of the band-pass filter 3-dB bandwidth. Results are presented here for band-pass filters whose low-pass equivalents are

(1) a single-pole filter

(2) a five-pole Butterworth filter

(3) a five-pole Tchebychev filter with 0.5-dB ripple factor, and

(4) a five-pole Bessel or linear-phase filter

In each case the filter resonant frequency is equal to the carrier frequency and the 3-dB bandwidth of the band-pass filter is varied between one and seven times the telemetry channel data rate. For each filter, the signal power loss is compared with the "exact" telemetry degradation, calculated using the analysis of [1], for the four possible combinations of

(1) telemetry degradation considering only signal distortion,

(2) telemetry degradation considering both signal distortion and noise bandwidth reduction,

and

(1) a 10^{-3} allowable bit error probability, and

(2) a 10^{-5} allowable bit error probability

Note that the telemetry degradation estimate which considers only signal distortion is the valid estimate when the filtering precedes the point at which the receiving system noise is introduced, while the telemetry degradation estimate which considers both the signal distortion and the noise bandwidth reduction is appropriate when the filtering follows the point at which the receiving system noise is introduced.

II. Calculation of Signal Power Loss in a Filter

The normal approach to calculating the power loss in a filter is to integrate the product of the power spectral density of the input signal and the square of the filter amplitude response over all frequencies. The ratio of this integral to the input signal power is the factor by which the filter reduces the available signal power. As this integration is usually performed numerically, errors occur as a result of the finite step size and limits of the integration.

For the types of filters considered in [1], a closed-form solution for the signal power loss in the filter can be obtained. This approach avoids the problem of selecting an integration step size and integration limits that reduce the integration error to an acceptable value. The analysis of [1] assumes that the filter has only N simple poles and a finite response Q_0 at infinite frequency, and that the filter pole locations and the residues at these poles are known. P_k was the kth pole and Q_k was the residue at that pole. Under these circumstances, the filter impulse response will be

$$h(\tau) = Q_0\delta(\tau) + \sum_{k=1}^{N} Q_k \exp(P_k \tau) \qquad (1)$$

where $\delta(\tau)$ is the Dirac delta function.

For NRZ telemetry data directly phase modulated on a carrier, the input to the filter will have the form

$$x(t) = d(t)\, 2^{1/2} \sin(\omega t + \phi)$$

where $d(t)$ is a sequence of statistically independent, equiprobable ± 1-valued symbols of equal duration T, and ω and ϕ are the carrier frequency and phase. As long as ϕ is statistically independent of $d(t)$, the autocorrelation function of the input signal will be

$$R_x(\tau) = R_d(\tau) \cos(\omega\tau) \qquad (2)$$

where

$$R_d(\tau) = \begin{cases} 1 - \dfrac{|\tau|}{T}, & |\tau| < T \\ 0, & |\tau| > T \end{cases} \qquad (3)$$

is the autocorrelation function of the NRZ symbol stream. As the filter input signal has unit power, the factor by which the filter reduces the signal power will be

$$\overline{y^2(t)} = \int_0^\infty \int_0^\infty d\tau_1\, d\tau_2\, h(\tau_1)\, h(\tau_2)\, R_x(\tau_2 - \tau_1) \tag{4}$$

where

$$y(t) = \int_0^\infty h(\tau)\, x(t-\tau)\, d\tau$$

is the filter output.

Now define $g(\tau)$ such that

$$h(\tau) = Q_0 \delta(\tau) + g(\tau) \tag{5}$$

where

$$\lim_{\tau \to \infty} g(\tau) = 0 \tag{6}$$

Then, using Eq. (5) in Eq. (4) and simplifying where possible,

$$\overline{y^2(t)} = Q_0^2 R_x(0) + 2Q_0 \int_0^\infty g(\tau)\, R_x(\tau)\, d\tau$$

$$+ 2\int_0^\infty d\tau_1 \int_{\tau_1}^\infty d\tau_2\, g(\tau_1) g(\tau_2) R_x(\tau_2 - \tau_1)$$

Now, using Eqs. (2) and (3) and again simplifying where possible,

$$\overline{y^2(t)} = Q_0^2 + 2Q_0 \int_0^T g(\tau)\left(1 - \frac{\tau}{T}\right) \cos(\omega\tau)\, d\tau$$

$$+ 2\int_0^T du \int_0^\infty d\tau\, g(\tau) g(\tau + u) \left(1 - \frac{u}{T}\right) \cos(\omega u) \tag{7}$$

Note that in Eq. (7) the only restriction on $g(\tau)$ is that of Eq. (6).

At this point, examining Eqs. (1) and (5), it is found that

$$g(\tau) = \sum_{k=1}^{N} Q_k \exp(P_k \tau) \tag{8}$$

is the form of $g(\tau)$ of interest here. Substituting Eq. (8) in Eq. (7) yields, after some algebraic manipulation,

$$\overline{y^2(t)} = Q_0^2 - \sum_{h=1}^{N} Q_h \left[Q_0 - \sum_{k=1}^{N} \frac{Q_k}{(P_h + P_k)}\right]$$

$$\times \left[\frac{1}{P_h + i\omega}\left(1 + \frac{1 - \exp[(P_h + i\omega)T]}{(P_h + i\omega)T}\right)\right.$$

$$\left. + \frac{1}{P_h - i\omega}\left(1 + \frac{1 - \exp[(P_h - i\omega)T]}{(P_h - i\omega)T}\right)\right] \tag{9}$$

Examining Eq. (2), one notes that by setting ω equal to zero in Eq. (9) one obtains the signal power loss factor for direct filtering of the baseband telemetry stream. Setting ω equal to zero in Eq. (9) yields

$$\overline{y^2(t)} = Q_0^2 - 2\sum_{h=1}^{N} \left(\frac{Q_h}{P_h}\right) \left[Q_0 - \sum_{k=1}^{N} \frac{Q_k}{(P_h + P_k)}\right]$$

$$\left[1 + \frac{1 - \exp(P_h T)}{P_h T}\right] \tag{10}$$

For particular cases Eqs. (9) or (10) can be simplified further. For example, for baseband filtering of the telemetry stream by a single-pole filter with unit response at zero frequency and 3-dB bandwidth, $f_0 = \omega_0/(2\pi)$, $N = 1$, $Q_0 = 0$, $Q_1 = \omega_0$, and $P_1 = -\omega_0$, and Eq. (10) simplifies to

$$\overline{y^2(t)} = \frac{\exp(-\omega_0 T) - 1 + \omega_0 T}{\omega_0 T}$$

However, for more complex cases it is usually simpler to evaluate Eqs. (9) or (10) numerically using complex arithmetic.

III. Numerical Results

Figures 1 through 4 of this article show telemetry degradation for a 100-kbps telemetry channel as a function of the band-pass filter 3-dB bandwidth for four different types of band-pass filters.

(1) Figure 1 shows results for a band-pass filter whose low-pass equivalent is a single-pole filter.

(2) Figure 2 shows results for a band-pass filter whose low-pass equivalent is a five-pole Butterworth filter.

(3) Figure 3 shows results for a band-pass filter whose low-pass equivalent is a five-pole Tchebychev filter with 0.5-dB ripple factor.

(4) Figure 4 shows results for a band-pass filter whose low-pass equivalent is a five-pole Bessel or linear-phase filter.

In each case the number of band-pass filter poles is twice the number of poles in its low-pass equivalent.

In Figs. 1 through 4, telemetry degradation is plotted as a function of the band-pass filter 3-dB bandwidth for five different methods of calculating the degradation. For the curve labeled SIGNAL POWER LOSS the telemetry degradation is assumed to be the factor by which the band-pass filter reduces the available signal power. This was calculated using the equations derived in the preceding section. The other four telemetry degradation curves in these figures were calculated using the analysis described in [1]. The curves labeled $P_B = 10^{-3}$, SIGNAL FILTERED and $P_B = 10^{-5}$, SIGNAL FILTERED assume the band-pass filter precedes the point at which the receiving system noise is introduced. Thus, the filter distorts the signal, but does not affect the receiving system noise. These curves are appropriate for band-pass filtering in the transmit section of a communication link. The curves labeled $P_B = 10^{-3}$, SIGNAL AND NOISE FILTERED and $P_B = 10^{-5}$, SIGNAL AND NOISE FILTERED assume the band-pass filter follows the point at which the receiving system noise is introduced and therefore filters both signal and noise. These curves are appropriate for band-pass filters within the receiving system. In each case the 10^{-3} or 10^{-5} refers to the allowable bit error probability.

The results in Fig. 1 are for a band-pass filter whose low-pass equivalent has a single pole. For small degradations the SIGNAL POWER LOSS curve lies between the two SIGNAL FILTERED curves and the two SIGNAL AND NOISE FILTERED curves. The choice of allowable bit error probability makes some difference, but not a large difference. At 700-kHz 3-dB bandwidth, seven times the bit rate, the SIGNAL AND NOISE FILTERED curve degradations are about 0.1 dB, the SIGNAL POWER LOSS curve degradation is about 0.2 dB, and the SIGNAL FILTERED curve degradations are about 0.3 dB. The degradation estimate based on signal power loss is conservative for band-pass filtering in the receiver, but is optimistic for band-pass filtering in the transmitter.

The results in Fig. 2 are for a band-pass filter whose low-pass equivalent is a five-pole Butterworth filter. In this case the SIGNAL POWER LOSS curve agrees reasonably well with the SIGNAL AND NOISE FILTERED curves for small degradations. The degradations for the SIGNAL FILTERED curves are appreciably worse than that for the other curves. At 700-kHz 3-dB bandwidth, seven times the 100-kbps bit rate, the degradations for the SIGNAL POWER LOSS and SIGNAL AND NOISE FILTERED curves are about 0.15 dB, while those for the SIGNAL FILTERED curves are more than 0.1 dB worse.

The results in Fig. 3 are for a band-pass filter whose low-pass equivalent is a five-pole Tchebychev filter with 0.5-dB ripple factor. As in the five-pole Butterworth case, the agreement between the SIGNAL POWER LOSS and SIGNAL AND NOISE FILTERED curves appears reasonable for small degradations. However, the absolute degradations and the difference between the SIGNAL FILTERED curves and the other three curves are greater for the Tchebychev case than the Butterworth case shown in Fig. 2. At 700-kHz 3-dB bandwidth, seven times the 100-kbps bit rate, the degradations for the SIGNAL POWER LOSS and SIGNAL AND NOISE FILTERED curves are 0.2 to 0.25 dB, while those for the SIGNAL FILTERED curves are about 0.45 dB.

The results in Fig. 4 are for a band-pass filter whose low-pass equivalent is a five-pole Bessel or linear-phase filter. The data in Fig. 4 resemble those in Fig. 1, the single-pole case, much more than the data in Figs. 2 and 3 for the five-pole Butterworth and Tchebychev cases. For small degradations, the SIGNAL POWER LOSS curve lies about halfway between the SIGNAL AND NOISE FILTERED and SIGNAL FILTERED curves. At 700-kHz 3-dB bandwidth, seven times the 100-kbps bit rate, the degradations are about 0.1 dB for the SIGNAL AND NOISE FILTERED curves, 0.2 dB for the SIGNAL POWER LOSS curve, and 0.3 dB for the SIGNAL FILTERED curves.

IV. Conclusion

Figures 1 through 4 show that, for situations where the filtering follows the point at which the receiving system noise is introduced, the signal power loss is a reasonable estimate of the telemetry degradation. The approximation is most appropriate when the signal power loss is less than 0.5 dB. The approximation is also better for filters whose low-pass equivalent is a five-pole Butterworth or Tchebychev filter than for a filter whose low-pass equivalent is a single-pole or five-pole Bessel filter. When the filtering occurs before the point at which the receiving system noise is introduced, such as filtering in the transmitter, the signal power loss is a less accurate estimate of the telemetry degradation.

Reference

[1] M. A. Koerner, "Effect of RF Filtering on the Performance of Uncoded PCM/PM Telemetry Channels," *TDA Progress Report 42-77*, vol. January–March 1984, Jet Propulsion Laboratory, Pasadena, California, pp. 104–125, May 15, 1984.

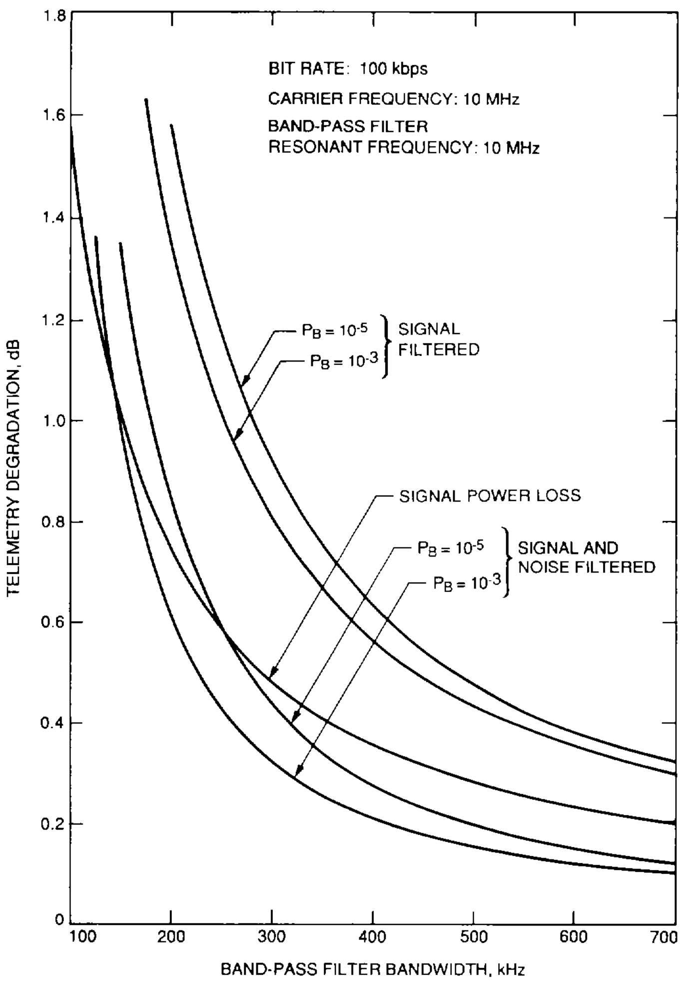

Fig. 1. Comparison of "exact" telemetry degradation and signal power loss for a band-pass filter whose low-pass equivalent is a single-pole filter.

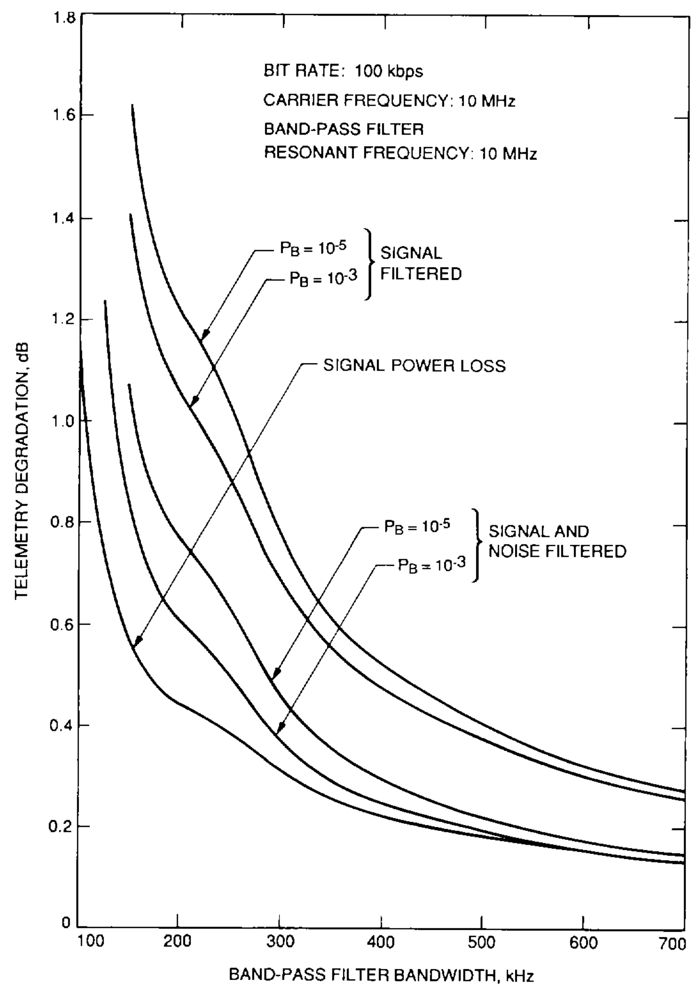

Fig. 2. Comparison of "exact" telemetry degradation and signal power loss for a band-pass filter whose low-pass equivalent is a five-pole Butterworth filter.

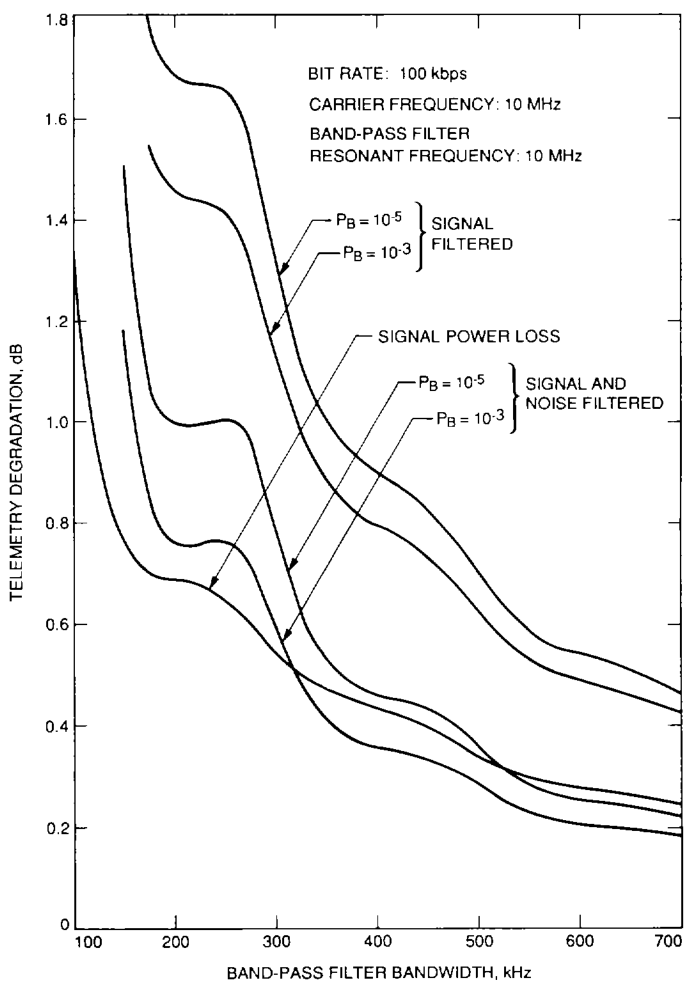

Fig. 3. Comparison of "exact" telemetry degradation and signal power loss for a band-pass filter whose low-pass equivalent is a five-pole Tchebychev filter with a 0.5-dB ripple factor.

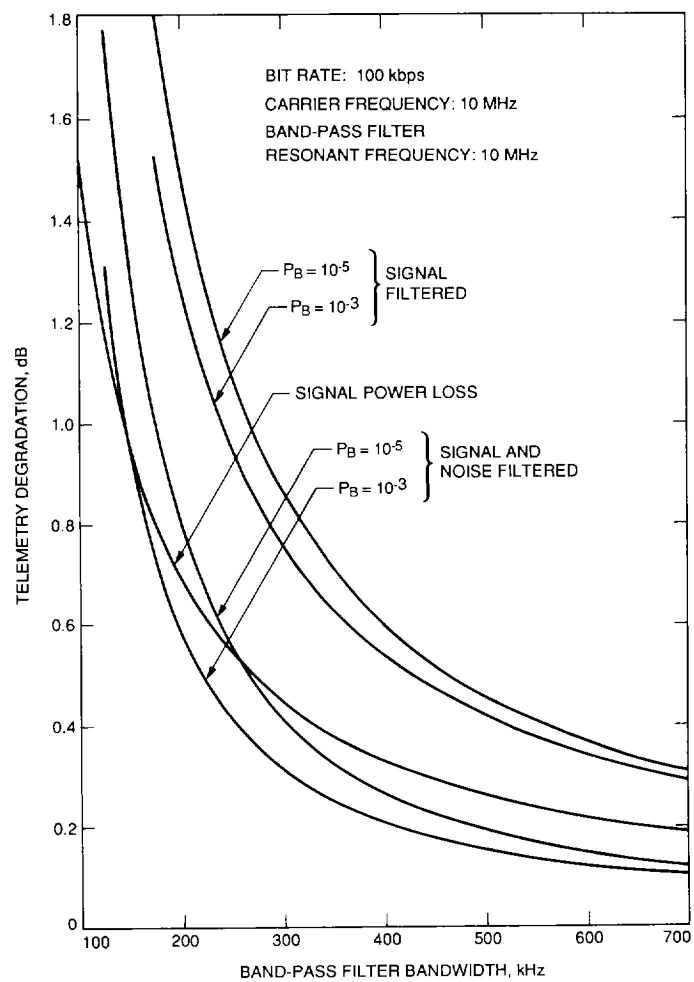

Fig. 4. Comparison of "exact" telemetry degradation and signal power loss for a band-pass filter whose low-pass equivalent is a five-pole Bessel or linear-phase filter.