Effect of Laser Frequency Noise on Fiber-Optic Frequency Reference Distribution

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This article presents an analysis of the effect of the linewidth of a single-longitudinal-mode laser on the frequency stability of a frequency reference transmitted over single-mode optical fiber. The interaction of the random laser frequency deviations with the dispersion of the optical fiber is considered to determine theoretically the effect on the Allan deviation (square root of the Allan variance) of the transmitted frequency reference. It is shown that the magnitude of this effect may determine the limit of the ultimate stability possible for frequency reference transmission on optical fiber, but is not a serious limitation to present system performance.

I. Introduction

Ultrastable fiber optic transmission of hydrogen maser reference signals is presently operational at the Goldstone facility of the NASA/JPL Deep Space Network [1]. This capability supports radio science experiments such as Connected Element Interferometry (CEI) by enabling phase-coherent arraying of widely separated antennas in real time. Also, distribution of a centralized maser reference throughout the entire complex eliminates the need for a hydrogen maser frequency standard at each Deep Space Station, with substantial cost savings and increased reliability.

A reference signal produced by a hydrogen maser frequency standard is presently distributed over distances up to 29 km, with differential fractional frequency stability $\frac{\delta f}{f} \approx 10^{-15}$ for 1000-second averaging times. Although the present fiber optic distribution capability is as stable as the hydrogen maser frequency standard, the ideal distribution system should be an order of magnitude more stable than the distributed signal. With the promise of trapped-ion frequency standards [2] and superconducting cavity masers [3] that will both provide more stable frequency references, fiber optic link stability of 10^{-18} at 1000 seconds will be required for stable distribution.

The demanding requirements that a frequency reference distribution system must meet necessitate the examination of all sources of instability at levels far beyond the needs of typical analog and digital fiber optic communication systems. Presently, laser source amplitude noise and thermal variations of the optical fiber have been identified as the limiting factors to distribution system performance. Improved lasers with lower intensity noise and single-longitudinal-mode operation will be employed in the near future. A thorough examination of fiber optic systems and components has indicated that laser frequency deviations may limit system performance as lower amplitude noise lasers become available. However, a quantitative analysis of the effect of laser frequency noise on a narrow-band frequency distribution system has not previously been performed.

The present analysis theoretically estimates the effects of laser frequency fluctuations on the amplitude and phase stability of a frequency reference transmitted on single-mode optical fiber. An expression for the phase noise spectral density of the modulation signal due to the frequency noise spectral density of the laser is derived and then used to calculate the expected Allan deviation of the transmitted reference signal. The laser-induced phase noise is shown to depend on the modulation signal frequency, fiber length, and fiber dispersion, as well as the magnitude of the laser frequency fluctuations. It is shown that the differential frequency stability of a single-mode fiber optic link is fundamentally limited by laser frequency noise. Thus, as laser intensity noise is reduced, the laser frequency noise will limit transmission stability.

II. Fiber-Laser Interaction

The dispersion of optical fiber causes various optical frequencies to travel with different velocities. Optical carrier frequency deviations couple with the dispersion of the fiber to produce random phase deviations in the envelope of a modulation signal, thereby degrading its phase stability. Optical fiber acts as a frequency discriminator to translate random frequency deviations of the laser into random phase deviations of the RF modulation envelope. Although every effort is made to operate the laser at the minimum dispersion point of the fiber, the slope of the fiber index of refraction versus wavelength is typically not zero. As the laser frequency deviates, the signal experiences changes in the fiber index of refraction that cause phase shifts of the modulation envelope.

Under bias current modulation, a semiconductor laser exhibits changes in wavelength, or chirp, that are synchronous with the bias modulation due to the change in refractive index of the laser gain medium [4]. Lasers also exhibit random frequency fluctuations due to the quantum phenomenon of electron-hole recombination in the gain media, with an attendant change of refractive index [5]. Temperature excursions of the laser diode also affect the index of refraction and the lasing wavelength, but with time constants from minutes to hours.

In digital transmission systems, laser chirp is the predominant limit on transmission distance [6, 7]. These wide-band systems are sensitive to phase deviations of the modulation envelope at all frequencies. However, in such systems, spontaneous emission noise is ignored since chirp is the overwhelming effect [7].

In contrast to digital or wide-band analog transmission systems, frequency distribution systems employ a narrow-band loop filter at the output of the fiber receiver. Therefore, high-frequency deviations of the modulation envelope phase are averaged out, leaving only the laser noise within the loop bandwidth. An analysis of the effect of close-to-carrier laser frequency noise on a long-distance frequency reference transmission system has not been published (to our knowledge), so the effects of laser frequency noise have not been known. Also, at the levels of signal to noise ratio (SNR) and frequency stability of the frequency distribution systems under consideration, it has been unclear what role laser frequency noise plays in determining the ultimate system stability attainable. The present analysis provides an estimate of the role of intrinsic laser frequency noise in a narrow-band frequency distribution system to determine the level at which system performance might become limited.

Since high-frequency laser chirp can be neglected in a narrow-band system, the present analysis considers only the intrinsic laser frequency noise within the bandwidth of the output filter. As such, the analysis applies to any type of laser system, although semiconductor lasers are typically used. Externally modulated Nd:YAG lasers at 1318 nm may be an attractive alternative to semiconductor lasers for long-haul analog signal transmission. The present analysis applies equally well to these types of lasers by substitution of the appropriate parameters.

III. Analysis

Intrinsic laser frequency noise has its origins in the discrete random photons spontaneously emitted into the lasing mode that cause random frequency changes of the laser wavelength [8–10]. The high-frequency character of this noise is well-known. It is basically flat within the modulation bandwidth, peaking at the relaxation oscillation resonance of the laser diode cavity, usually in the tens-of-gigahertz region [8, 9]. Within tens of kHz of the carrier, the frequency noise exhibits a $1/f$ character [10]. Close-to-carrier measurements of laser noise are limited to within about 10 kHz, due to the physical difficulty of fabricating frequency discriminators with sufficient resolution at optical frequencies. It is this low-frequency FM noise that is of interest for the analysis of narrow-band frequency distribution systems.

We desire an expression for the phase noise spectral density of the modulation signal as a function of the spectral density of laser frequency fluctuations. The resultant phase noise density may then be used to calculate the expected Allan deviation of the reference signal, provided the character of the laser frequency noise is known.

Consider a single-longitudinal-mode laser coupled to a single-mode fiber. The laser output is amplitude modulated at RF frequency Ω. The phase delay for the modulation signal envelope along the fiber is given by

$$\phi = \frac{2\pi\, n\, L\, \Omega}{c} \qquad \text{(rad)} \tag{1}$$

where n is the fiber index of refraction, L is the fiber length, and c is the speed of light in a vacuum. It is assumed that the laser operates in a single longitudinal mode at $\lambda = 1.3$ μm. Now, consider the effect of a perturbation, such as a change in ambient temperature, on the refractive index of the fiber. This causes a phase change

$$d\phi = \frac{dn\, 2\pi\, L\, \Omega}{c} \qquad \text{(rad)} \tag{2}$$

Multiplying the numerator and denominator on the right-hand side by $d\lambda$ gives

$$d\phi = \frac{2\pi\, L\, \Omega\, d\lambda}{c}\left(\frac{dn}{d\lambda}\right) \qquad \text{(rad)} \tag{3}$$

By writing $d\lambda$ in terms of the laser frequency ν, the phase deviations may be expressed in terms of the laser frequency deviations, which have the same (random) time dependence. Thus

$$d\phi(t) = d\nu(t)\frac{2\pi\, L\, \Omega\, \lambda^2}{c^2}\left(-\frac{dn}{d\lambda}\right) \qquad \text{(rad/sec)} \tag{4}$$

For the analysis of frequency stability, it is more convenient to look at the last expression in the frequency domain by Fourier transforming as follows:

$$S_\phi(f) = S_\nu(f)\left[\frac{-2\pi\, L\, \Omega\, \lambda^2}{c^2}\frac{dn}{d\lambda}\right]^2 \qquad (\text{rad}^2/\text{Hz}) \tag{5}$$

In this expression, $S_\phi(f)$ is the spectral density of the phase fluctuations at an offset frequency f from the RF signal; the fluctuations are induced by the spectrum of random frequency deviations, $S_\nu(f)$, of the laser.

The variation of the effective fiber index of refraction with wavelength $\frac{dn}{d\lambda}$ depends on the waveguide parameters and material composition of the fiber. The measured result for typical single-mode fiber at 1300 nm is [11]

$$\frac{dn}{d\lambda} = 270.1 \quad \text{m}^{-1} \tag{6}$$

Inserting this value into Eq. (5) and substituting the appropriate constants produces the simple relation

$$S_\phi(f) = S_\nu(f)\, L^2\, \Omega^2\, (1.02 \times 10^{-51}) \quad (\text{rad}^2/\text{Hz}) \tag{7}$$

The above quantity is the mean-square phase-noise spectral density at an offset f from the modulation signal, Ω. This is the spectrum which would be observed if a perfect oscillator (i.e., an oscillator with no phase noise) modulated the laser and if the output of the photodetector were compared to a second perfect oscillator, as in a phase noise measurement system.

IV. Numerical Estimates

The FM noise spectrum of distributed feedback-type (DFB) single-mode lasers typically used in fiber optic distribution systems exhibits a power-independent $1/f$ character at low frequencies (below about 1 MHz). In the modulation band, the FM noise is white and inversely proportional to optical power [10]. The physical mechanism responsible for the $1/f$ behavior is thought to be the trapping of carriers due to impurities and interfacial boundaries, but it is not fully understood. The white portion of the spectrum is due to spontaneous-emission events and is adequately modeled by theory [5, 8].

The frequency noise spectrum of typical DFB lasers has been measured experimentally [10]. The above-mentioned physical mechanisms may be modeled as

$$S_\nu(f) = \frac{C}{P} + \frac{K}{f} \tag{8}$$

where P is the average output power of the laser and f is the frequency offset from the optical carrier. C and K are empirically determined constants. For the Fujitsu DFB laser diodes measured [10], $C = 1.5 \times 10^4$ (Hz $\cdot$ W), and $K = 5.8 \times 10^{11}$ (Hz^2).

The frequency noise spectrum of a typical DFB laser is illustrated in Fig. 1. As laser power is increased, the white portion of the noise spectrum decreases proportional to P^{-1}. A numerical estimate of the additive RF phase noise requires that only the $1/f$ portion of $S_\nu(f)$ be inserted into Eq. (7), which gives

$$S_\phi(f) = \frac{5.8 \times 10^{11}}{f}\Omega^2 L^2 (1.05 \times 10^{-51}) \ \ (\mathrm{rad}^2/\mathrm{Hz})$$

$$= L^2 \Omega^2 \frac{5.9 \times 10^{-40}}{f} \ \ (\mathrm{rad}^2/\mathrm{Hz}) \qquad (9)$$

The $1/f$ laser frequency noise is converted to $1/f$, or "flicker," phase noise in the fiber optic distribution system. This level of $1/f$ phase noise depends on the inherent quantum fluctuations of the laser frequency and represents the ultimate phase noise floor of the system.

For flicker phase noise, the Allan deviation (square root of the Allan variance) may be calculated from the following relation [12]:

$$\sigma_y(\tau) = \sqrt{\frac{3}{(2\pi)^2 \tau^2} \frac{S_\phi(f)\, f}{\Omega^2} \ln(8.88\, f_h\, \tau)} \qquad (10)$$

where f_h is the frequency cutoff of the phase noise. In this case, f_h is one-half the bandwidth of the output filter.

Substituting Eq. (9) into the last expression, the modulation frequency cancels, and the expression for the Allan deviation reduces to

$$\sigma_y(\tau) = \sqrt{\frac{3}{(2\pi)^2\, \tau^2} L^2\, 6 \times 10^{-40} \ln(8.88\, f_h\, \tau)} \qquad (11)$$

The laser-induced flicker phase noise thus sets the minimum bias level of the $1/\tau$ section of the Allan deviation plot. The fact that the last expression does not depend on the RF modulation frequency, Ω, is significant. This implies that moving to higher or lower modulation frequencies for reference signal distribution will not alter the laser frequency noise "floor" of the Allan deviation.

For purposes of comparison, we consider an actual frequency distribution link in use at the NASA/JPL Goldstone Deep Space Communications Complex. The longest frequency distribution run is 29 km. Assuming that the output filter bandwidth, f_h, is 10 Hz, the Allan deviation, calculated from Eq. (11), is

$$\sigma_y(\tau) \simeq \frac{4.1 \times 10^{-16}}{\tau} \qquad (12)$$

The relation between the laser-frequency noise-limited Allan deviation of the 29-km link and the Allan deviation of a typical hydrogen maser is plotted in Fig. 2. This represents the ultimate frequency stability attainable with such a link, provided all other noise sources are negligible.

V. Present State of the Art

The ultimate link stability plotted in Fig. 2 will only be attained if all other noise sources are negligible. In reality, other noise sources do contribute to the link stability. This is illustrated in Fig. 3, where an actual measurement of the 29-km Goldstone link is plotted in addition to the maser and the ultimate-stability-link curves of Fig. 2.

In present-day systems, the Allan deviation $1/\tau$ intercept is set by the SNR of the fiber link, which is determined by the laser intensity noise. The SNR of a typical high-performance analog link is 120 dB/Hz. Figure 4 illustrates the 1-sec Allan deviation as a function of fiber length. It is immediately apparent from Fig. 4 that the laser frequency noise does not limit frequency distribution system performance at this time, since the laser SNR dominates. As lower amplitude-noise lasers become available, the laser frequency noise floor of the Allan deviation may begin to limit frequency reference distribution system performance.

Figure 5 depicts fiber link Allan deviation at 1 second as a function of link SNR. The laser relative intensity noise (RIN) sets the SNR of the fiber link for short distances [13]. Also shown is the Allan deviation floor due to laser frequency noise for a 29-km link. This plot shows clearly that laser frequency noise limits the frequency stability "floor" of the fiber link to $4 \times 10^{-16}/\tau$ for SNR $\geq$ 145 dB/Hz. Systems with as high as 150 dB/Hz SNR may be achievable with externally modulated high-power semiconductor-diode-pumped Nd:YAG solid-state lasers, or through the use of squeezed light generated directly from semiconductor lasers. As these system improvements are realized, the low-frequency $1/f$ FM noise of the laser may begin to limit system performance.

A final observation: Since the fiber optic transmission system converts laser frequency noise to RF phase noise, it may be the case that a stabilized fiber optic link comprises a very accurate system for measuring the frequency deviations of lasers close to the optical carrier. This approach is under consideration for future research.

VI. Conclusion

At present, the noise floor of fiber optic distribution systems is determined by the laser signal to noise ratio

(SNR) in the RF modulation band. However, lasers with lower relative intensity noise (RIN) or those which use squeezed light promise increases in link SNR, and passive and active temperature-stabilization schemes can improve link stability at long averaging times. As these improvements in components and systems are realized, the fundamental limit for frequency stability due to laser frequency noise may be reached.

The present analysis provides the contribution to the phase noise of a transmitted frequency reference due to single-mode laser frequency deviations. Through interaction with the dispersion of the fiber, the $1/f$ FM noise close to the optical carrier is converted to $1/f$ phase noise close to the RF reference signal. The additive $1/f$ laser-induced phase noise is a function of the fiber dispersion and length and determines the ultimate Allan deviation floor of the fiber optic distribution system in the 1- to 100-second region.

For the longest fiber optic frequency distribution link in the NASA/JPL Deep Space Network (29 km), using data for commercially available DFB lasers, the analysis indicates that the link Allan deviation is limited to $4 \times 10^{-16}/\tau$ (for averaging times between 1 second and 100 seconds). This stability limit will be reached at link SNR of 145 db/Hz, which is 25 dB better than the present system.

Further increases in SNR will not yield higher link stability unless laser frequency noise is decreased as well. The laser FM noise stability limit is two orders of magnitude higher stability than the best current frequency standard, which indicates that laser frequency noise will not limit fiber optic frequency distribution capability in the foreseeable future.

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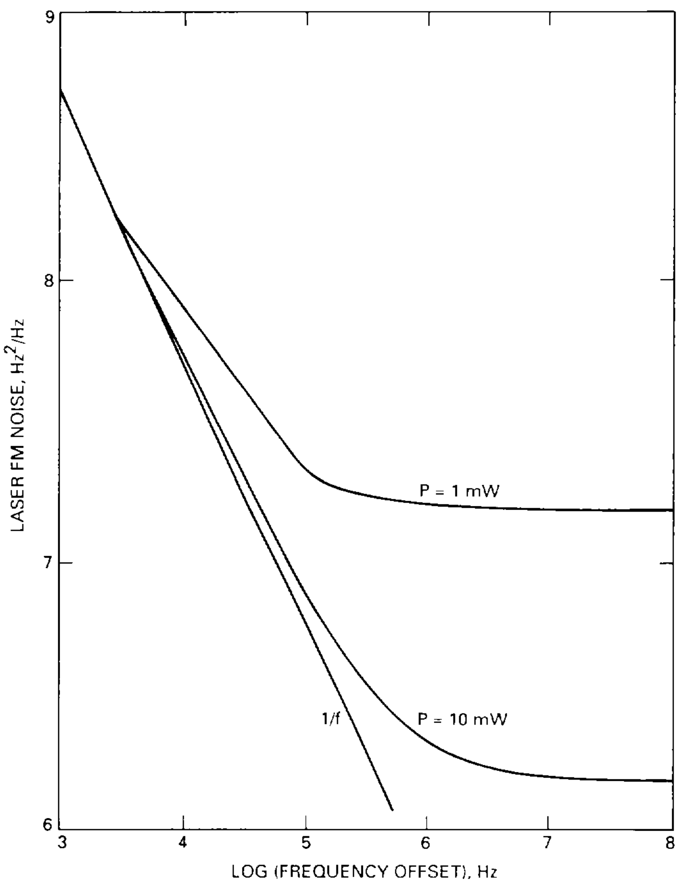

Fig. 1. Typical DFB laser frequency noise spectrum.

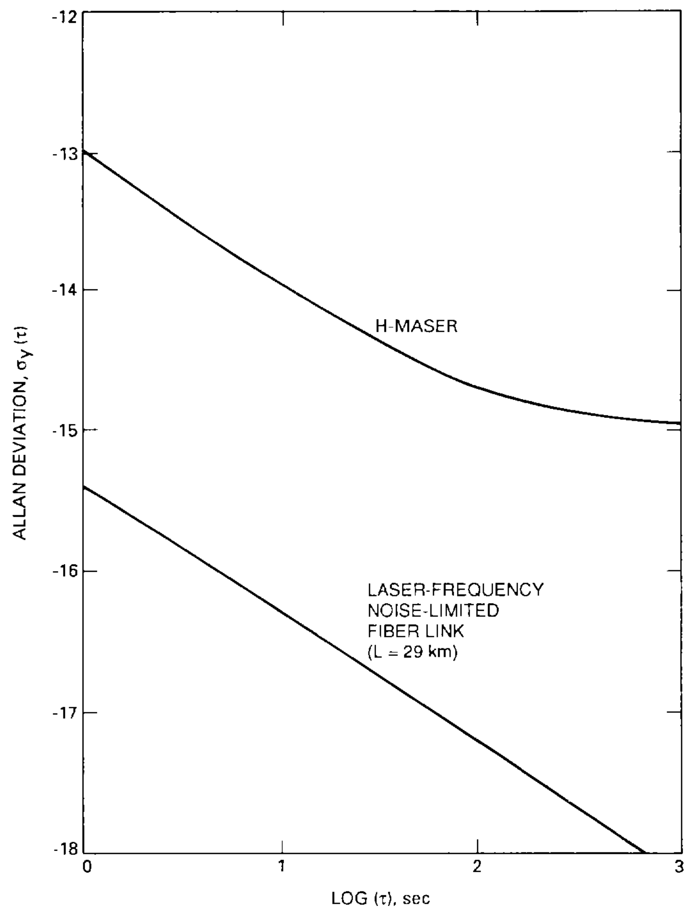

Fig. 2. Hydrogen maser and laser FM noise-limited fiber optic link frequency-stability comparison.

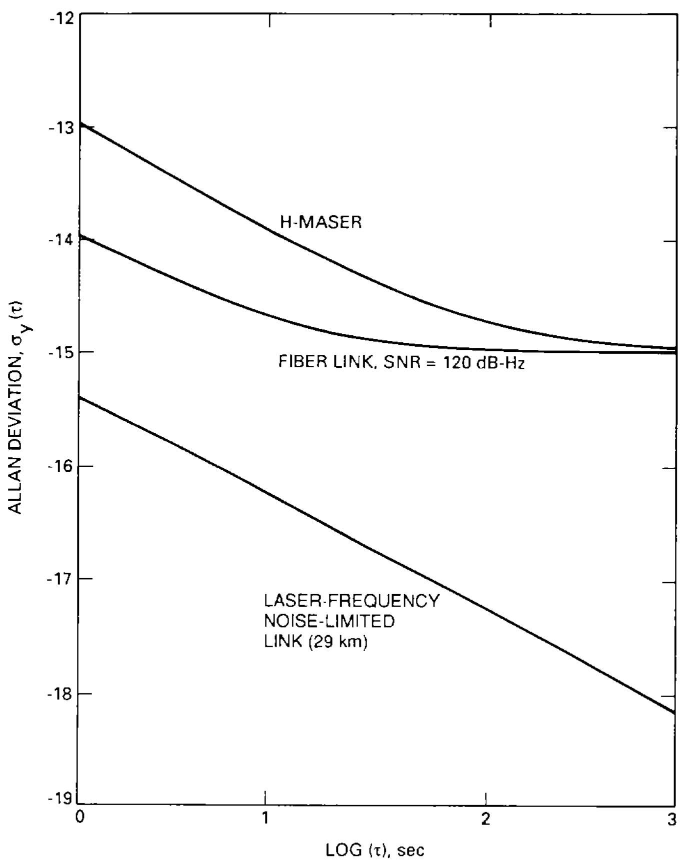

Fig. 3. Comparison of Allan deviation: H-maser, actual 29-km link, and theoretical FM noise-limited link.

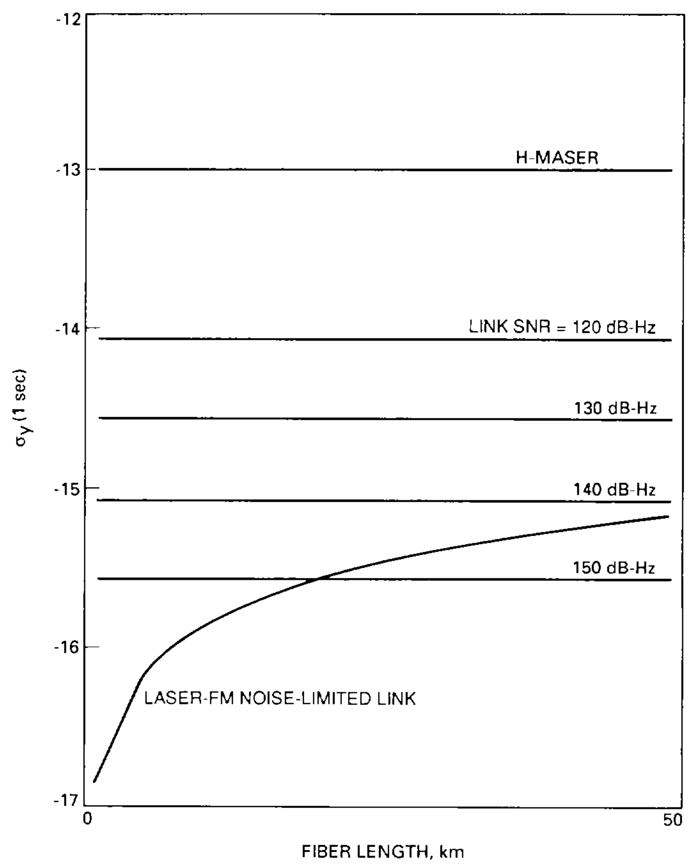

Fig. 4. Comparison of frequency stability at one second versus fiber length for maser, fiber optic links of various SNR, and noise-limited laser FM.

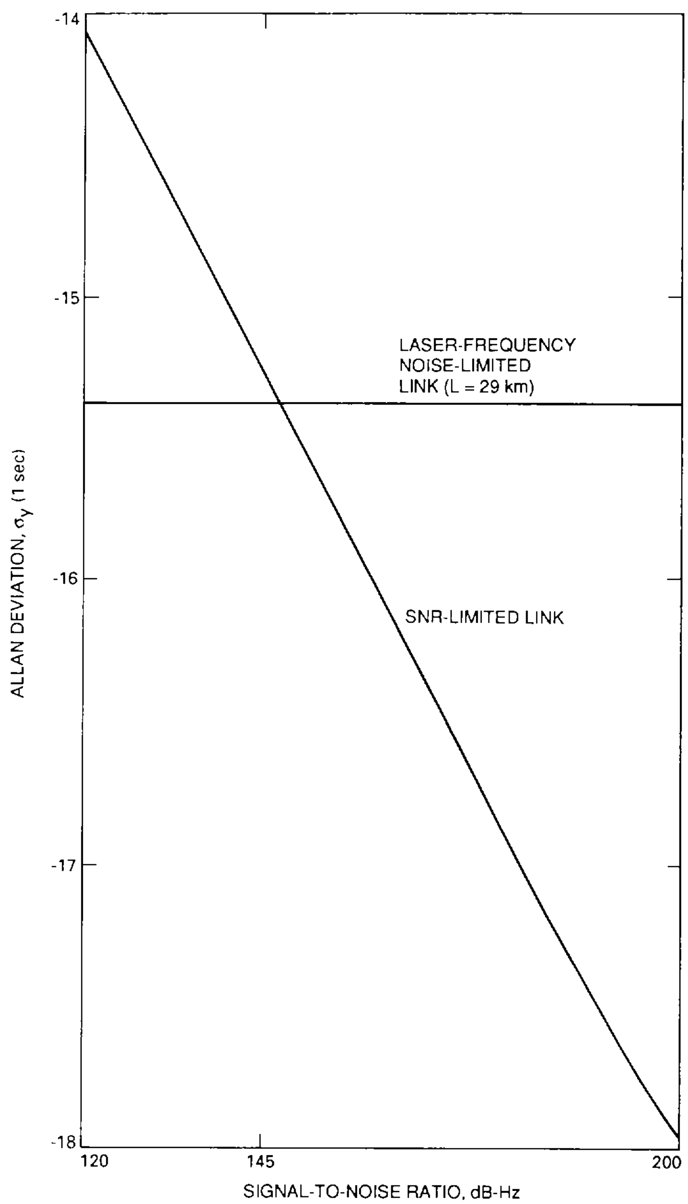

Fig. 5. Comparison of fiber link frequency stability at one second versus SNR with laser FM noise-limited 29-km link.