A Second/Third-Order Hybrid Phase-Locked Receiver for Tracking Doppler Rates

R. C. Tausworthe
Telecommunications Division

This article describes a stable phase-locked receiver configuration for tracking frequency ramp signals. A usual second-order receiver is used for lockup; subsequently a very simple modification is made to the loop filter, altering the loop to one of the third order. The altered loop then tracks the incoming signal with zero static phase error. The receiver bandwidth is practically unchanged; the damping factor lies in the region 0.5 and 0.707, and the design point is 12 dB in gain margin above instability.

I. Introduction

The phase-locked loops used in the DSN and spacecraft receivers are severely restricted in design by several requirements relating to their response to dynamic signal conditions. For example, the loop must not oscillate at any signal level above design point. Further, it must be capable of tracking frequency offsets and rates within specifiable static phase error limits.

To rule out oscillations at low signal levels, the accepted practice has been to design second-order phase-lock systems. Such a design, however, limits the doppler-rate tracking capability to spacecraft missions with a very low acceleration profile. Consideration of the spacecraft accelerations expected during a Jupiter flyby (Ref. 1) shows that there is such an excessive static phase error in the loop that lock probably cannot be maintained without operational aid of some sort.

This paper presents a technique which totally relaxes loop stress and thereby eliminates operational problems and any data degradation which might otherwise accompany loop detuning. The method allows the ordinary existing receiver to be used during lock-up. Once locked then, the loop filter is augmented by an integrator to remove any static-phase-error buildup. The analysis presented here shows that the loop bandwidth in this alteration is negligibly changed.

II. The Second-Order Loop for Acquisition

The design equations for a second-order phase-locked receiver are well documented (Ref. 2) and thus are not to be derived here. To introduce what follows, however, certain conventions and definitions are in order. We shall assume that the usual loop filter is to be used:

$$F_a(s) = \frac{1 + \tau_2 s}{1 + \tau_1 s}, \qquad \tau_1 >> \tau_2 \tag{1}$$

The rms input signal level and loop gain K_a, along with the time constants τ_1 and τ_2 fix the loop bandwidth w_{L_a} and damping factor ζ, by way of the parameter r:

$$\begin{aligned} w_{L_a} &= \frac{r+1}{2\tau_2} = \frac{r+1}{r_0+1} w_{L_0} \\ \zeta_a &= \frac{1}{2}\sqrt{r} = \zeta_0 \sqrt{\frac{r}{r_0}} \\ r &= \frac{AK_a\tau_2^2}{\tau_1} \end{aligned} \tag{2}$$

The zero subscripts here refer to values at design-signal level. We have affixed a subscript a to other loop parameters in this acquisition mode. If the input phase-doppler function relative to the VCO at rest has a frequency offset Ω_0 and rate Λ_0, i.e.,

$$d(t) = \theta_0 + \Omega_0 t + \frac{1}{2}\Lambda_0 t^2 \tag{3}$$

then there is an ultimate steady-state buildup of loop stress

$$\begin{aligned} \phi_{ss} &\approx \left(\frac{\Omega_0}{AK} + \frac{\Lambda_0\tau_1}{AK}\right) + \frac{\Lambda_0}{AK}t \\ &\approx \left(\frac{\Omega_0}{\beta^2\tau_1} + \frac{\Lambda_0}{\beta^2}\right) + \left(\frac{\Lambda_0}{\beta^2}\right)\left(\frac{t}{\tau_1}\right) \end{aligned} \tag{4}$$

The latter expression gives the stress in terms of the loop natural frequency, β. The linear buildup in time only becomes of interest when t becomes appreciable to τ_1, but the constant part of the static error is there as soon as the lock-up transient disappears. Control of the VCO tuning can make the Ω_0 term disappear, whereas that with Λ_0 can only be reduced by sweeping the VCO at the proper rate.

III. Optimum Doppler-Rate Tracking Loop

Let us suppose that the previous second-order loop has acquired lock by proper sweeping; then at $t = 0$, a new loop filter $F(s)$ is initiated to track, optimized so as to minimize the total phase error by the method of Jaffee and Rechtin (Ref. 3). The optimum loop response is given (Ref. 2) by the Yovits–Jackson formula (Ref. 4).

$$\begin{aligned} L_t(s) &= 1 - \frac{s^3}{\left[-s^6 + \frac{A^2\lambda^2}{N_0}\Lambda_0^2\right]^+} \\ &= \frac{2\xi s^2 + 2\xi^2 s + \xi^3}{s^3 + 2\xi s^2 + 2\xi^2 s + \xi^3} \end{aligned} \tag{5}$$

in which the brackets $[\,]^+$ indicate the left-half-plane square root of the enclosed function, and the parameter ξ is

$$\xi = \left(\frac{A^2\lambda^2\Lambda_0^2}{N_0}\right)^{1/6} \tag{6}$$

λ is the Jaffee–Rechtin Lagrange multiplier, which sets the loop bandwidth as desired. The new loop filter is

$$F_t(s) = \frac{2\xi^2}{AK_t}\left(\frac{1 + \frac{s}{\xi}}{s} + \frac{\xi}{2s^2}\right) \tag{7}$$

By choosing $\xi = 1/\tau_2$ the first term in this new filter resembles the old one, except that there has been a change in level removable by choice of K_t

$$\begin{aligned} F_t(s) &= \frac{1 + \tau_2 s}{\tau_1 s} + \frac{1}{2\tau_2\tau_1 s^2} \\ &\approx F_a(s) + \frac{1}{2\tau_2\tau_1 s^2} \end{aligned} \tag{8}$$

The gain value producing this relationship is

$$K_t = \frac{2K_a}{r} \tag{9}$$

In usual practice, the design value of r is taken as $r_0 = 2$, in accordance with the Jaffee–Rechtin optimization for a Ω_0-term only. Hence, we shall also specify that the constants in the acquisition loop be chosen so that $r_0 = 2$. That is, the acquisition loop is identical to the present mechanization. This rationale is a fortunate choice, because it implies that the gains for both the second- and third-order loops are equal:

$$K_t = \left(\frac{2}{r_0}\right)K_a = K_a = K \tag{10}$$

Therefore, the optimum rate-tracking loop merely augments the existing second-order loop used for acquisition by another integration in the loop filter.

The form of the loop filter immediately suggests the synthesis shown in Fig. 1. The isolation amplifiers shown are assumed to have a very high input impedance so as not to degrade τ_1.

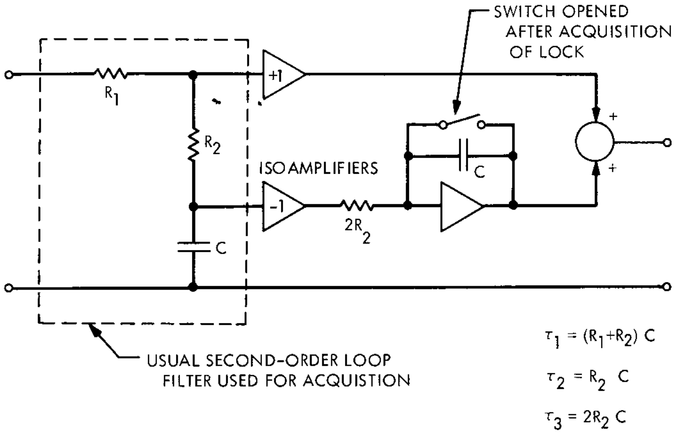

Fig. 1. The second/third-order system loop filter

IV. Performance of the Tracking Loop

At any arbitrary signal level A, the rate-tracking loop transfer function, assuming $\tau_1 >> \tau_2$ for the filter shown in Fig. 1, is

$$L_t(s) = \frac{AK\tau_2\left(s^2 + \dfrac{1}{\tau_2}s + \dfrac{1}{s\tau_2^2}\right)}{s^3 + AK\tau_2 s^2 + AKs + \dfrac{AK}{2\tau_2}} \qquad (11)$$

and the corresponding loop noise bandwidth is

$$w_{L_t} = \frac{r}{2\tau_2}\left(\frac{2r+1}{2r-1}\right) \qquad (12)$$

The ratio of the noise bandwidth during tracking to that of the acquisition loop is

$$\frac{w_{L_t}}{w_{L_a}} = \frac{r}{r+1}\left(\frac{2r+1}{2r-1}\right) \qquad (13)$$

As a function of signal margin above design point, as depicted in Fig. 2, this ratio goes from 10/9 to unity. That is, there is a maximum increase in loop bandwidth of 11% at design point, tapering down to only about 1% at 10-dB loop margin, the usual minimum recommended operating level.

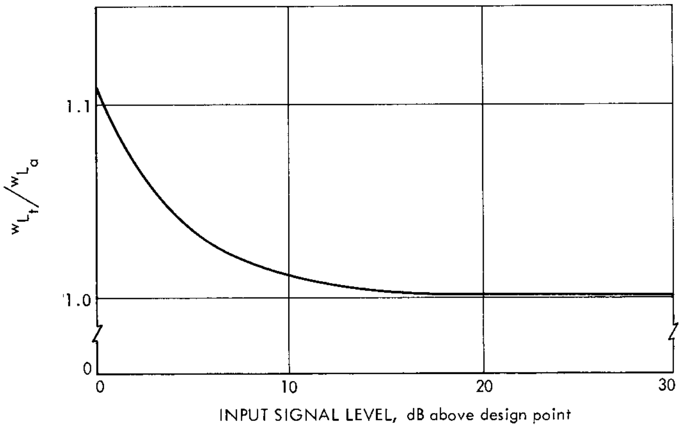

Fig. 2. Comparison of loop noise bandwidths in the two operating modes versus loop signal margin

The system has three poles in its response, one on the negative real axis, and the other two, complex. The system response thus exhibits some underdamping. As shown in the root locus of Fig. 3, for margins above design point, the damping coefficient corresponding to these system poles lies in the region

$$0.5 \leq \zeta < 0.707 \qquad (14)$$

The voltage gain margin between instability and design point is a factor of 4, i.e., a 12-dB power-gain margin.

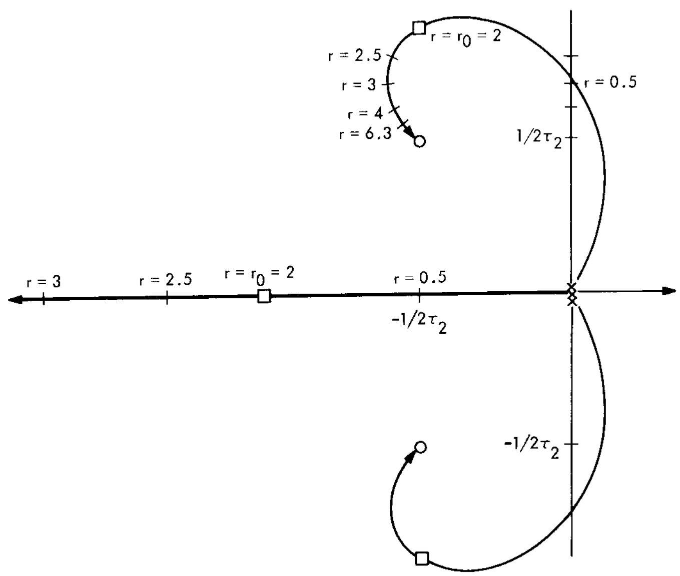

Fig. 3. Root locus of the third-order loop. System poles at design point ($r_0 = 2$) are shown in squares. Usual minimum recommended operating condition is at $r = 6.3$

V. Conclusion

This article shows that a minor modification to presently designed phase-locked receivers can result in a very enhanced doppler-rate tracking capability, without degrading noise performance to any significant degree, especially when operating at or above minimum recommended conditions. The switch from acquisition to tracking mode can be actuated automatically by an AGC sensing relay, if desired.

References

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