Photon Energies of a Cathode-Ray Tube System

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Light emitted from a cathode-ray tube (CRT) is comprised of reflected environmental light incident upon the CRT system and light generated at the phosphor screen and transmitted through the CRT system. The total photon energy which leaves the system and those energies which are dissipated in the various elements of the CRT system are derived. An expression for the contrast ratio of the CRT system is also presented and discussed.

I. Introduction

Photons generated by the impingement of electrons upon the phosphorescent material of the phosphor screen pass through the cathode-ray tube (CRT) system as output light. A fraction of the photons from the environment (ambient light) incident upon the CRT system is reflected from the CRT system and contributes light to the generated output light. The combination of these two outputs makes up the total light emitted from the CRT system. A common CRT system is comprised of a phosphor screen, a glass faceplate, and a filter screen which is utilized to enhance the contrast ratio of the CRT system.

When light, either generated at the phosphor screen or entering from the ambient, encounters an element of the CRT system, its energy is partially transformed into heat due to the energy-absorbing nature of the CRT element. The remaining energy is partially transmitted if the element is not opaque, and the residual portion is reflected. Conservation of the total energy for this process can be expressed by

$$\alpha(\lambda) + \tau(\lambda) + \rho(\lambda) = 1 \tag{1}$$

where $\alpha(\lambda)$, $\tau(\lambda)$, and $\rho(\lambda)$ are, respectively, the absorptivity, transmissivity, and reflectivity coefficients at wavelength λ of the element (Ref. 1).

Since light undergoes an infinite number of reflections in a CRT system, it follows that there occurs an infinite number of sequential events differentiated by absorption, transmission, and reflection mechanics. These series describe the total energy transformations within the CRT system.

The analysis in this article is divided into two sections, the energies resulting from the environmental light incident upon the system, and the energies resulting from the light generated by the system. In these analyses, all the energies transmitted and reflected from the system, and all the energies dissipated by absorption processes by the phosphor screen, the glass faceplate, and the filter screen elements are considered.

II. Energies From Environmental Light

Consider a coherent beam of light at wavelength λ incident upon the CRT system. The energy $E(\lambda)$ of this light is

$$E(\lambda) = n(\lambda)\, h\, \nu \tag{2}$$

where $n(\lambda)$ represents the number of photons at λ, h is Planck's constant, and ν is the frequency of light. The frequency may be expressed as the velocity of light in the medium divided by the wavelength. Using c and λ as representing the velocity and wavelength of light, respectively, under vacuum conditions, the frequency of light may then be written as

$$\nu = \frac{c}{\lambda} \tag{3}$$

Since the velocity of light in standard air is so nearly equal to the velocity of light in vacuum, for practical purposes, the wavelength of light in the two mediums shall be considered equal and shall not be differentiated in this study.

The fraction of the total light energy transmitted through the filter screen and incident upon the faceplate is $\tau_f(\lambda)\, E(\lambda)$, where $\tau_f(\lambda)$ is the transmissivity coefficient at λ of the screen and the direction of incidence is taken to be normal. The remaining energy $\alpha_f(\lambda)\, E(\lambda)$ is absorbed by the screen. Since the reflectivity coefficient of a filter screen is negligibly low, the energy absorbed by the screen can be rewritten as $[1 - \tau_f(\lambda)]\, E(\lambda)$.

Continuing with the light incident upon the glass faceplate, the portion $\rho_g(\lambda)\, \tau_f(\lambda)\, E(\lambda)$ is reflected from the outer surface of which $\rho_g(\lambda)\, \tau_f^2(\lambda)\, E(\lambda)$ is transmitted back through the screen as the first term of the total light output from the system contributed by the environment light. While $[1 - \rho_g(\lambda)]\, \tau_f(\lambda)\, E(\lambda)$ enters into the faceplate, $[1 - \tau_g(\lambda)]\, [1 - \rho_g(\lambda)]\, \tau_f(\lambda)\, E(\lambda)$ is absorbed by the faceplate and $\tau_g(\lambda)\, [1 - \rho_g(\lambda)]\, \tau_f(\lambda)\, E(\lambda) = E_1$ is incident upon the surface of the inner faceplate (phosphor screen side).

Of E_1, the portion $\rho_g(\lambda)\, E_1$ is reflected and $[1 - \rho_g(\lambda)]\, E_1$ is the energy finally incident upon the phosphorescent material.

The phosphorescent material absorbs and reflects incident energy since its transmissivity coefficient is essentially zero. Thus, for every beam incident upon its surface, a fraction $[1 - \rho_s(\lambda)]$ is absorbed, and $\rho_s(\lambda)$ is reflected. Subsequently, a portion of the reflected beam is re-reflected from the inner surface of the faceplate back for another reflection from the phosphorescent material. Thus, an infinite series of subsequent reflection and absorption energy sequential events occurs. It can be shown that the series of energy reflections and absorptions can be expressed in the form such that the sum total energy of E_1 which is reflected from the faceplate and phosphor screen into the faceplate at the inner side is $F_g(\lambda)\, E_1$, where

$$F_g(\lambda) = \rho_g(\lambda) + \frac{\rho_s(\lambda)\, [1 - \rho_g(\lambda)]^2}{1 - \rho_g(\lambda)\, \rho_s(\lambda)} \tag{4}$$

The total amount of energy E_1 which is absorbed by the phosphorescent material is $F_s(\lambda)\, E_1$, where

$$F_s(\lambda) = \frac{[1 - \rho_g(\lambda)]\, [1 - \rho_s(\lambda)]}{1 - \rho_g(\lambda)\, \rho_s(\lambda)} \tag{5}$$

and $F_g(\lambda) + F_s(\lambda) = 1$.

As the reflected beam $F_g(\lambda)\, E_1$ passes through the faceplate, $[1 - \tau_g(\lambda)]\, F_g(\lambda)\, E_1$ is absorbed by the faceplate, and $[1 - \rho_g(\lambda)]\, \tau_g(\lambda)\, F_g(\lambda)\, E_1$ emerges from the outer surface of the faceplate toward the filter screen. However, $\rho_g(\lambda)\, \tau_g(\lambda)\, F_g(\lambda)\, E_1$ is the portion of energy reflected from the outer surface of the faceplate toward the phosphor screen for another series of reflections and dissipations. By continuing similar arguments for the reflected beam, series expressions for these energies which are dissipated and transmitted in the various elements of the CRT system can be developed. These series are given below for a monochromatic beam of light.

A. Light Energy Output

The total energy $L_0(\lambda)$ emerging from the CRT system is the series of energies formulated from the analysis above, as follows:

$$\begin{aligned} L_0(\lambda) = {} & \tau_f^2(\lambda)\,\rho_g(\lambda)\,E(\lambda) + \tau_f^2(\lambda)\,[1-\rho_g(\lambda)]^2\,\tau_g^2(\lambda)\,F_g(\lambda)\,E(\lambda) \\ & + \rho_g(\lambda)\,\tau_f^2(\lambda)\,[1-\rho_g(\lambda)]^2\,\tau_g^4(\lambda)\,F_g^2(\lambda)\,E(\lambda) \\ & + \rho_g^2(\lambda)\,\tau_f^2(\lambda)\,[1-\rho_g(\lambda)]^2\,\tau_g^6(\lambda)\,F_g^3(\lambda)\,E(\lambda) + \cdots \end{aligned} \tag{6}$$

which can be written as

$$L_0(\lambda) = \tau_f^2(\lambda)\,E(\lambda)\left\{\rho_g(\lambda) + \frac{[1-\rho_g(\lambda)]^2\,\tau_g^2(\lambda)\,F_g(\lambda)}{1-\rho_g(\lambda)\,F_g(\lambda)\,\tau_g^2(\lambda)}\right\} \tag{7}$$

B. Energy Dissipated Into the Filter Screen

The total energy $L_f(\lambda)$ dissipated into the filter screen in the form of thermal energy is

$$\begin{aligned} L_f(\lambda) = {} & [1-\tau_f(\lambda)]\,E(\lambda) + \rho_g(\lambda)\,\tau_f(\lambda)\,[1-\tau_f(\lambda)]\,E(\lambda) \\ & + [1-\rho_g(\lambda)]^2\,\tau_g^2(\lambda)\,F_g(\lambda)\,[1-\tau_f(\lambda)]\,\tau_f(\lambda)\,E(\lambda) \\ & + [1-\rho_g(\lambda)]^2\,\tau_g^4(\lambda)\,F_g^2(\lambda)\,\rho_g(\lambda)\,[1-\tau_f(\lambda)]\,\tau_f(\lambda)\,E(\lambda) + \cdots \end{aligned} \tag{8}$$

resolving to

$$L_f(\lambda) = [1-\tau_f(\lambda)]\,E(\lambda)\left\{1 + \rho_g(\lambda)\,\tau_f(\lambda) + \frac{[1-\rho_g(\lambda)]^2\,\tau_g^2(\lambda)\,F_g(\lambda)\,\tau_f(\lambda)}{1-\rho_g(\lambda)\,F_g(\lambda)\,\tau_g^2(\lambda)}\right\} \tag{9}$$

C. Energy Absorbed by the Glass Faceplate

The total energy $L_g(\lambda)$ absorbed by the glass faceplate is written as

$$\begin{aligned} L_g(\lambda) = {} & [1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,E(\lambda) \\ & + [1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g^2(\lambda)\,\rho_g(\lambda)\,F_g(\lambda)\,E(\lambda) \\ & + [1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g^4(\lambda)\,\rho_g^2(\lambda)\,F_g^2(\lambda)\,E(\lambda) + \cdots \\ & + [1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g(\lambda)\,F_g(\lambda)\,E(\lambda) \\ & + [1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g^3(\lambda)\,\rho_g(\lambda)\,F_g^2(\lambda)\,E(\lambda) + \cdots \end{aligned} \tag{10}$$

which becomes

$$L_g(\lambda) = \frac{[1-\tau_g(\lambda)]\,[1-\rho_g(\lambda)]\,[1+F_g(\lambda)\,\tau_g(\lambda)]\,\tau_f(\lambda)\,E(\lambda)}{1-\rho_g(\lambda)\,F_g(\lambda)\,\tau_g^2(\lambda)} \tag{11}$$

D. Energy Absorbed by the Phosphor Screen

The total energy $L_s(\lambda)$ absorbed by the phosphor screen is similarly a series of energies which, when combined, results in the following equation:

$$L_s(\lambda) = \frac{F_s(\lambda)\,\tau_g(\lambda)\,\tau_f(\lambda)\,[1 - \rho_g(\lambda)]\,E(\lambda)}{1 - \rho_g(\lambda)\,F_g(\lambda)\,\tau_g^2(\lambda)} \tag{12}$$

Energy is conserved since

$$E(\lambda) = L_0(\lambda) + L_f(\lambda) + L_g(\lambda) + L_s(\lambda) \tag{13}$$

III. Energies From Generated Light

Let a coherent beam of light having wavelength λ be generated at the surface of the CRT phosphor screen by the impingement of electrons upon the phosphorescent material. The energy $E_g(\lambda)$ of this beam of light may be expressed by

$$E_g(\lambda) = n_g(\lambda)\,h\,\nu \tag{14}$$

where $n_g(\lambda)$ is the number of photons generated.

Of the beam of light which impinges upon the inner surface of the glass faceplate, the portion $\rho_g(\lambda)\,E_g(\lambda)$ is reflected and the remaining portion $[1 - \rho_g(\lambda)]\,E_g(\lambda)$ passes into the faceplate. Direction to the faceplate surface is assumed to be normal. The reflected portion incident upon the phosphor screen is partially absorbed by an amount equal to $[1 - \rho_s(\lambda)]\,\rho_g(\lambda)\,E_g(\lambda)$ and the other portion $\rho_s(\lambda)\,\rho_g(\lambda)\,E_g(\lambda)$ is reflected back upon the inner faceplate surface.

This reflected beam undergoes a similar reflection and absorption sequence between the faceplate and phosphor screen, resulting in an infinite number of sequential events. The combined series of energies passing into the faceplate may be expressed by the form $K_g(\lambda)\,E(\lambda)$, where

$$K_g(\lambda) = \frac{1 - \rho_g(\lambda)}{1 - \rho_s(\lambda)\,\rho_g(\lambda)} \tag{15}$$

Similarly, the total energy absorbed by the phosphorescent material by all these primary sequential reflections and absorptions is $K_s(\lambda)\,E_g(\lambda)$, where

$$K_s(\lambda) = \frac{\rho_g(\lambda)\,[1 - \rho_s(\lambda)]}{1 - \rho_s(\lambda)\,\rho_g(\lambda)} \tag{16}$$

Of $K_g(\lambda)\,E_g(\lambda)$, the portion absorbed by the faceplate is $[1 - \tau_g(\lambda)]\,K_g(\lambda)\,E_g(\lambda)$ and the remaining portion $E_r(\lambda) = \tau_g(\lambda)\,K_g(\lambda)\,E_g(\lambda)$ becomes incident upon the outer surface of the faceplate. Of $E_r(\lambda)$, the portion $[1 - \rho_g(\lambda)]\,E_r(\lambda)$ passes onto the filter screen and $\rho_g(\lambda)\,E_r(\lambda)$ is reflected back toward the inner surfaces of the faceplate. The amount $\tau_g(\lambda)\,\rho_g(\lambda)\,E_r(\lambda)$ incident upon the inner surface is then divided into two light beams, one beam passing through the inner surface to the phosphor screen for secondary reflections back through the system, and one beam reflected to the outer surface.

It can be reasoned that the primary light beam $K_g(\lambda)\,E_g(\lambda)$ entering into the faceplate undergoes an infinite number of reflections from the inner and outer surfaces of the faceplate, and for each sequence of reflections, some fraction of light is passed toward the phosphor screen or toward the filter screen respective of the inner or outer faceplate surface. In the interim of these reflections within the faceplate, a series of energies is absorbed by the faceplate. Thus, three total energies from the primary light beam entering the faceplate from the inner side occur: (1) a total energy reflected from the faceplate toward the phosphor screen $E_{ph}(\lambda)$, (2) a total energy passed from the faceplate toward the filter screen $E_f(\lambda)$, and (3) a total energy absorbed by the glass faceplate $E_a(\lambda)$. These series of energies are combined and formulated as follows:

$$E_{ph}(\lambda) = \frac{[1 - \rho_g(\lambda)]\,\rho_g(\lambda)\,\tau_g^2(\lambda)\,K_g(\lambda)\,E(\lambda)}{1 - \tau_g^2(\lambda)\,\rho_g^2(\lambda)} \tag{17}$$

$$E_f(\lambda) = \frac{[1 - \rho_g(\lambda)]\,\tau_g(\lambda)\,K_g(\lambda)\,E(\lambda)}{1 - \tau_g^2(\lambda)\,\rho_g^2(\lambda)} \tag{18}$$

$$E_a(\lambda) = \frac{[1 - \tau_g(\lambda)]\,[1 + \rho_g(\lambda)\,\tau_g(\lambda)]\,K_g(\lambda)\,E(\lambda)}{1 - \tau_g^2(\lambda)\,\rho_g^2(\lambda)} \tag{19}$$

Of the beam $E_f(\lambda)$, the portion $[1 - \tau_f(\lambda)]\,E_f(\lambda)$ is absorbed by the filter screen, and $\tau_f(\lambda)\,E(\lambda)$ finally emerges from the CRT system as light.

Continuing back at the inner surface of the faceplate, after $E_{ph}(\lambda)$ impinges upon the phosphor screen, a portion $[1 - \rho_s(\lambda)]\,E_{ph}(\lambda)$ is absorbed by the phosphorescent material, and the remainder $\rho_s(\lambda)\,E_{ph}(\lambda)$ is reflected back toward the faceplate. This reflected portion undergoes an infinite number of reflections and absorptions between the faceplate and phosphor screen similar to that undergone by $E_g(\lambda)$, which was described in the above analysis. Thus, $K_s(\lambda)\,\rho_s(\lambda)\,E_{ph}(\lambda)$ is absorbed by the phosphorescent material, and $K_g(\lambda)\,\rho_s(\lambda)\,E(\lambda)$ enters into the faceplate, which in turn undergoes interreflections between the inner and outer surface similar to the above analysis (Eqs. 17–19). Again, a portion of this energy entering the faceplate will be divided into three portions: (1) a portion reflected back to the phosphor screen; (2) a portion absorbed by the faceplate; and (3) a portion passing from the outer surface of the faceplate, being partially absorbed by the filter screen and the remaining energy finally emerging from the CRT system as a term of the series of light output.

Another set of terms can be similarly developed to further the expression of the energy terms evolved from the interreflections of the phosphor and faceplate surfaces. The energy series for each element are given below.

A. Light Energy Output

The total energy $\Gamma_0(\lambda)$ of the light emerging from the CRT system is expressed as

$$\Gamma_0(\lambda) = \frac{[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g(\lambda)\,K_g(\lambda)\,E_g(\lambda)}{1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)} + \frac{[1-\rho_g(\lambda)]^2\,\tau_f(\lambda)\,\tau_g^3(\lambda)\,K_g^2(\lambda)\,\rho_g(\lambda)\,\rho_s(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]^2} + \cdots \tag{20}$$

which becomes

$$\Gamma_0(\lambda) = \frac{[1-\rho_g(\lambda)]\,\tau_f(\lambda)\,\tau_g(\lambda)\,K_g(\lambda)\,E_g(\lambda)}{Q(\lambda)} \tag{21}$$

where

$$Q(\lambda) = \frac{1-\rho_s(\lambda)\,\rho_g(\lambda)-\rho_g^2(\lambda)\,\tau_g^2(\lambda)-\rho_s(\lambda)\,\rho_g(\lambda)\,\tau_g^2(\lambda)+2\,\rho_g^2(\lambda)\,\rho_s(\lambda)\,\tau_g^2(\lambda)}{1-\rho_s(\lambda)\,\rho_g(\lambda)} \tag{22}$$

B. Energy Dissipated Into the Filter Screen

The total energy $\Gamma_f(\lambda)$ dissipated into the filter screen is

$$\begin{aligned}\Gamma_f(\lambda) = {} & \frac{[1-\tau_f(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_g(\lambda)\,K_g(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]} \\ & + \frac{[1-\tau_f(\lambda)]\,[1-\rho_g(\lambda)]^2\,K_g^2(\lambda)\,\rho_s(\lambda)\,\tau_g^3(\lambda)\,\rho_g(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]^2} \\ & + \frac{[1-\tau_f(\lambda)]\,[1-\rho_g(\lambda)]^3\,K_g^3(\lambda)\,\rho_s^2(\lambda)\,\tau_g^5(\lambda)\,\rho_g^2(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]^3} + \cdots\end{aligned} \tag{23}$$

which reduces to

$$\Gamma_f(\lambda) = \frac{[1-\tau_f(\lambda)]\,[1-\rho_g(\lambda)]\,\tau_g(\lambda)\,K_g(\lambda)\,E_g(\lambda)}{Q(\lambda)} \tag{24}$$

C. Energy Absorbed by the Glass Faceplate

The total energy $\Gamma_g(\lambda)$ absorbed by the glass faceplate may be formulated in a series of energies as follows:

$$\Gamma_g(\lambda) = \frac{[1-\tau_g(\lambda)]\,[1+\rho_g(\lambda)\,\tau_g(\lambda)]\,K_g(\lambda)\,E_g(\lambda)}{1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)}$$
$$+\frac{[1-\tau_g(\lambda)]\,[1+\rho_g(\lambda)\,\tau_g(\lambda)]\,\rho_s(\lambda)\,\rho_g(\lambda)\,\tau_g^2(\lambda)\,[1-\rho_g(\lambda)]\,K_g^2(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]^2}$$
$$+\frac{[1-\tau_g(\lambda)]\,[1+\rho_g(\lambda)\,\tau_g(\lambda)]\,\rho_s^2(\lambda)\,\rho_g^2(\lambda)\,\tau_g^4(\lambda)\,[1-\rho_g(\lambda)]^2\,K_g^3(\lambda)\,E_g(\lambda)}{[1-\tau_g^2(\lambda)\,\rho_g^2(\lambda)]^3}+\cdots \qquad (25)$$

which resolves to

$$\Gamma_g(\lambda) = \frac{[1-\tau_g(\lambda)]\,[1+\rho_g(\lambda)\,\tau_g(\lambda)]\,K_g(\lambda)\,E_g(\lambda)}{Q(\lambda)} \qquad (26)$$

D. Energy Absorbed by the Phosphor Screen

The total energy $\Gamma_s(\lambda)$ absorbed by the phosphor screen is similarly reduced from a series to the following form:

$$\Gamma_s(\lambda) = \left[1+\frac{[1-\rho_g(\lambda)]\,\tau_g^2(\lambda)\,K_g(\lambda)}{Q(\lambda)}\right]\left[\frac{[1-\rho_s(\lambda)]\,\rho_g(\lambda)\,E_g(\lambda)}{1-\rho_s(\lambda)\,\rho_g(\lambda)}\right] \qquad (27)$$

Again, energy is conserved since

$$E_g(\lambda) = \Gamma_0(\lambda) + \Gamma_f(\lambda) + \Gamma_g(\lambda) + \Gamma_s(\lambda) \qquad (28)$$

IV. Total Energies

The total energies produced by both the environmental light incident upon the CRT system and the light generated by the CRT system in the forms of light output and energy absorptions by the CRT elements can be obtained by summating the energies produced for each form. Let $d(E_t)_0$ at λ represent the total light energy output for the considered sample in the wavelength region of λ and $\lambda + d\lambda$. Thus, the incremental energy $d(E_t)_0/d\lambda$ at λ is the sum of the monochromatic energies $L_0(\lambda)$ and $\Gamma_0(\lambda)$ (Sections II-A and III-A). By integrating this result over a given spectrum (λ_1 to λ_2), the total light energy output from the CRT system over that spectrum is obtained. Thus, the total light energy output and total energies absorbed in each element can be written as follows.

A. Total Light Energy Output $(E_t)_0$

$$(E_t)_0 = \int_{\lambda_1}^{\lambda_2} [L_0(\lambda) + \Gamma_0(\lambda)]\,d\lambda \qquad (29)$$

B. Total Energy Dissipated Into the Filter Screen $(E_t)_f$

$$(E_t)_f = \int_{\lambda_1}^{\lambda_2} [L_f(\lambda) + \Gamma_f(\lambda)]\,d\lambda \qquad (30)$$

C. Total Energy Absorbed by the Glass Faceplate $(E_t)_g$

$$(E_t)_g = \int_{\lambda_1}^{\lambda_2} [L_g(\lambda) + \Gamma_g(\lambda)]\,d\lambda \qquad (31)$$

D. Total Energy Absorbed by the Phosphor Screen $(E_t)_s$

$$(E_t)_s = \int_{\lambda_1}^{\lambda_2} [L_s(\lambda) + \Gamma_s(\lambda)]\,d\lambda \qquad (32)$$

V. Contrast Ratio

The contrast ratio of the CRT system is defined as the quotient of the maximum luminance divided by the minimum luminance. Luminance output from a CRT system can be varied by modulating the flux of electrons incident upon the phosphor screen. By assigning G to represent the maximum degree of photometric modulation or gain of the CRT system, it follows that the maximum and minimum number of photons generated by the electron beam are related by G according to the equivalent

$$[n_g(\lambda)]_{\max} = G\,[n_g(\lambda)]_{\min} \tag{33}$$

The maximum light energy output can be expressed, from Eq. (29), as

$$[(E_t)_0]_{\max} = \int_{\lambda_1}^{\lambda_2} \{L_0(\lambda) + [\Gamma_0(\lambda)]_{\max}\}\,d\lambda \tag{34}$$

The minimum light energy output from the CRT system is then

$$[(E_t)_0]_{\min} = \int_{\lambda_1}^{\lambda_2} \left\{L_0(\lambda) + \frac{[\Gamma_0(\lambda)]_{\max}}{G}\right\} d\lambda \tag{35}$$

Converting these energies to luminance by introducing the luminous efficacy K, the contrast ratio C_r of the CRT system, including environmental light, can now be written as follows:

$$C_r = \frac{\displaystyle\int_{\lambda_1}^{\lambda_2} K(\lambda)\,\{L_0(\lambda) + [\Gamma_0(\lambda)]_{\max}\}\,d\lambda}{\displaystyle\int_{\lambda_1}^{\lambda_2} K(\lambda)\left\{L_0(\lambda) + \frac{[\Gamma_0(\lambda)]_{\max}}{G}\right\} d\lambda} \tag{36}$$

VI. Discussion and Conclusion

It is apparent that Eq. (36) is too involved to be used to practically estimate the contrast ratio performance of a CRT system. For reasons of simplification, it shall be assumed that the distribution of $n(\lambda)$ and $n_g(\lambda)$ with respect to λ is uniform. This permits the assignment of an average value over the considered spectrum for each optical property of the various elements of the CRT system. For example, the average transmissivity coefficient $\bar{\tau}_f$ of the filter screen under this condition can be written as

$$\bar{\tau}_f = \frac{\displaystyle\int_{\lambda_1}^{\lambda_2} \tau_f(\lambda)\,d\lambda}{\lambda_2 - \lambda_1} \tag{37}$$

Equation (36), under these conditions, can then be modified to the form

$$\bar{C}_r = \frac{\bar{L}_0 + \bar{\Gamma}_0}{\bar{L}_0 + \dfrac{(\bar{\Gamma}_0)_{\max}}{G}} \tag{38}$$

where the bar over each total energy signifies that average values for the optical properties are used.

By including only the primary reflections (first-order terms) associated with the primary beams of the environmental light and generated light of the CRT system, an expression for the contrast ratio $(\bar{C}_r)'$ which includes only these first-order terms can be derived (Ref. 2). This expression is given as

$$(\bar{C}_r)' = \frac{\bar{\tau}_f\,k\,L + \Gamma_{\max}}{\bar{\tau}_f\,k\,L + \dfrac{\Gamma_{\max}}{G}} \tag{39}$$

where

$$k = \frac{\bar{\tau}_g^2\,\bar{\rho}_s\,(1 - \bar{\rho}_g) + \bar{\rho}_g}{\bar{\tau}_g} \tag{40}$$

and L and Γ are the luminance levels of the environmental light and generated light, respectively. Also, the spectral distributions of L and Γ are assumed to be equal.

It is of interest to compare the contrast ratio given by Eq. (38), which includes the series of interreflection effects upon the elements, with the contrast ratio given by Eq. (39), which considers only the first-order effects of the primary beam.

Representative values for the optical properties of a typical CRT configuration which utilizes a P4 phosphor is given in Table 1 (Ref. 2).

Assume that the selected CRT has a filter screen with a mean transmissivity $\bar{\tau}_f = 0.50$. By substituting the values for the properties (Table 1) into Eqs. (38) and (39), estimated contrast ratios, including fractional differences, are obtained. These ratios are shown in Table 2 for three different environmental lighting conditions.

It can be seen from Table 2 that the value of contrast ratio using Eq. (39) results in a slightly higher contrast ratio value than that which results from using Eq. (38).

Table 1. Optical property values of a typical CRT

Property	Value
$\bar{\rho}_s$	0.76
$\bar{\tau}_g$	0.63
$\bar{\rho}_g$	0.04
G	33.00

Table 2. Comparative results of contrast ratio

Ratio	Lighting condition		
	Dark	Dim	Bright
Environment-to-generated-light ratio (L/Γ_{max})	0	0.10	1.00
$\bar{C}_r$ (Eq. 38)	33.0	17.7	4.16
$(\bar{C}_r)'$ (Eq. 39)	33.0	18.2	4.36
$[\bar{C}_r - (\bar{C}_r)']/\bar{C}_r$	0	−0.028	−0.048

In the case where a filter screen is not used, the results obtained from the two equations are about double those shown in Table 2. It may be concluded that Eq. (39) may be used to obtain a reasonably accurate estimate of the contrast ratio capability of a CRT system. The results obtained will be slightly optimistic.

For the CRT configuration described in Table 1, the light output and energies dissipated in the various CRT elements may be estimated from Eqs. 28–31. Consider a practical intensity of environmental light to be 10% as great as the light generated by the CRT. Under these conditions, the total photon energy (incident upon and generated by the CRT system) would be porportioned as follows: 32% is transmitted and reflected from the CRT as light output, 36% is dissipated by the filter screen, 40% is absorbed by the glass faceplate, and 2% is absorbed by the phosphorescent material.

For a CRT system which has no filter screen, under the same light conditions given above, the total photon energy would be porportioned as follows: 65% is light output, 42% is absorbed by the glass faceplate, and 3% is absorbed by the phosphorescent material.

References

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