A Reanalysis Program for Antenna Member Size Changes

R. Levy
DSIF Engineering Section

An efficient procedure is described for reanalysis of space-truss structural frameworks. The procedure has been programmed to operate as a post-processor to determine response changes from sets of displacements developed for the initial structure by an independent structural analysis system. Examples given show substantial savings in computation time when operating in conjunction with the NASTRAN structural analysis system.

I. Introduction

Efficient methods of reanalysis are useful to assess the changes in structural response as a function of modifications to the properties of the individual member components of the structure. The applications encompass all design phases through inception and future alteration, particularly in conjunction with the implementation of the techniques for structural optimization.

Currently, a number of diverse procedures for reanalysis have evolved to operate within the context of prevalent approaches to the overall formulation chosen for analytical solution of the response problem. Some have been designed to be used in conjunction with force method, displacement method, or mixed formulations. Additional alternatives depend for their attractiveness upon whether or not the number of members for which modifications are considered form a relatively large or a relatively small set, or possibly whether or not the modified members can be readily localized to occur within restricted regions of the structure.

Some of the procedures are mathematically exact in the sense that the solutions will be exactly equivalent to solutions that would be obtained from analysis of the modified structure *ab initio*. Other procedures develop approximate solutions for which exact eventual convergence can be demonstrated iteratively. All methods usually appear to fall within either of two classes:

(1) Given the inverse of the stiffness matrix for the initial structure, the inverse for the modified structure is developed by perturbation.

(2) The solution for the modified structure is developed as a linear combination of solutions for the initial structure.

II. Proposed Method

In the following, an exact method of solution will be described that develops displacements and member forces for the modified structure from linear combinations of displacement function solutions developed for the initial structure. Conceptually, the method is based upon a parallel element approach. For each component member to be modified, a hypothetical parallel member is postulated. Cross-sectional properties of the parallel member are taken as exactly equal to the changes in properties of the parent (original) member and the connectivity is duplicated. The final member forces for the modified structure are the sums of the member forces on parent and parallel members. A condition of the solution is to maintain compatibility of the parent and parallel member distortions. Entirely new members can be added by parallel members and original members can be completely removed by assigning duplicate properties of negative magnitude to the parallel member.

Reanalysis is performed by a post-processor computer program that uses input displacement functions developed previously by an independent structural analysis program. Because the only computed data that are input are the displacement functions, the analytical formulation procedure used to process the initial structure and develop these displacements is immaterial. The present implementation of the program is designed to accept input specifically in the format of the NASTRAN analysis system. A present limitation is the restriction to process only changes in the cross-sectional area property for one-dimensional bars. This limitation, however, is compatible with the design requirements of space truss structures, such as the frameworks of antenna reflectors. The advantages of this method are the simplicity of program operation and input requirements, and the efficiency with which parameter studies can be developed for the response as a function of a spectrum of property changes for particular bars.

III. Mathematical Formulation

The displacements of the modified structure U_M are obtained by superposition of displacements of the initial structure U_I and the displacements ΔU caused by the internal forces of the parallel members acting as loads on the initial structure. That is,

$$[U_M] = [U_I] + [\Delta U] \tag{1}$$

The order of the matrices in Eq. (1) is $m \times k$, where m is the number of unconstrained degrees of freedom, and k is the number of external loading vectors.

To evaluate ΔU, it is convenient to express these displacements as the product of the displacements for unit values U_S of the parallel member forces post-multiplied by the forces R of the parallel members, or

$$\underset{(m\times k)}{[\Delta U]} = \underset{(m\times b)}{[U_S]}\,\underset{(b\times k)}{[R]} \tag{2}$$

In Eq. (2), the index b is equal to the number of property changes summed over all the members.

To enforce compatibility, let

e_F = final distortions of parent members = distortions of parallel members

e_I = initial distortion of parent member for the external loads

e_S = distortions of parent member for unit values of forces of the parallel members

e_0 = distortions of parallel members for unit forces

Therefore, from superposition

$$\underset{(b\times k)}{[e_F]} = \underset{(b\times k)}{[e_I]} + \underset{(b\times b)}{[e_S]}\,\underset{(b\times k)}{[R]} \tag{3}$$

But e_F can be determined directly as the distortions of the parallel members,

$$[e_F] = [e_0]\,[R] \tag{4}$$

After combining Eqs. (3) and (4) and rearranging, R can be obtained by solving

$$([e_0] - [e_S])\,[R] = [e_I] \tag{5}$$

The solution of Eq. (5) for R can be obtained readily in many instances because the order of the coefficient matrix is equal only to the total number of property changes.

Examination of Eq. (5) shows that relatively little additional computational effort is necessary to process more than one property change for a given member. This can be done by saving the e_S and e_I matrices in Eq. (5) and repeating only the relatively minor effort in regenerating new e_0 matrices. In this manner, the responses for sequences of property changes for given members can be obtained efficiently.

After solving, and combining Eqs. (1) and (2), the final displacements of the modified structure are obtained from

$$\underset{(m\times k)}{[U_M]} = \underset{(m\times k)}{[U_I]} + \underset{(m\times b)}{[U_S]}\,\underset{(b\times k)}{[R]} \tag{6}$$

The final member forces can be found either by summing forces on parent and parallel bars, or else by expanding the indices of Eq. (3) to cover all of the members of interest and then applying the internal force–distortion relationship for these members.

In the limited case of one-dimensional bar members, the member distortion is the extension of a bar along its axis and is given by

$$e_i = \frac{S_i L_i}{A_i E_i} \tag{7}$$

where S_i, L_i, A_i, and E_i are, respectively, the member force, length, area, and modulus of elasticity for the ith member. As an alternative to Eq. (7), the present implementation computes the extension from the grid coordinates and displacements associated with the bar as follows:

$$e_i = \sum_{\alpha=1}^{3} \frac{q_i^\alpha H_i^\alpha}{L_i}, \qquad i = 1, 2, \cdots, b \tag{8}$$

where q_i^α is the difference in displacements of the terminal nodes of the bar in the direction of the αth axis of Cartesian coordinates, H_i^α is the projection of the bar axis along the αth axis,

and

$$L_i = \left(\sum_{\alpha=1}^{3} [H_i]^2\right)^{1/2} \tag{9}$$

The computations for Eqs. (8) and (9) are performed by using the member connection cards to define the terminal nodes and the grid cards that give the nodal coordinates. These data can be duplicates of the input for the original structural analysis program. The displacements used to generate the e_I set are the responses to the applied loading. The displacements that generate the e_S set are obtained by applying a pair of unit loads directed towards each other at the terminal nodes associated with each parallel member; each pair of loads forms one loading vector and contributes one column of displacements to the U_S matrix, which is used to compute one column of distortions in the e_S matrix.

The unit loads that produce the e_S set are equilibrated by corresponding unit loads that extend the parallel member. Consequently, the e_0 matrix is a diagonal matrix of distortions for unit tensile loads, with each diagonal element of the form

$$e_0 = \frac{L_i}{A_i E_i} \tag{10}$$

where A_i is the change in area for the ith member and this can be positive or negative, depending upon whether the area of the ith member is to be increased or decreased.

IV. Program Execution and Conclusions

A flow chart of the reanalysis program described is shown in Fig. 1. The program is developed either to process property changes for individual bars or common property changes to groups of bars.

Table 1 contains a summary of the central processing unit times for program operation (Univac 1108 Exec 8 computer) for two different structures that have been reanalyzed by this program. For both structures, the input displacements were read from a tape which was created by suppressing NASTRAN output punch card images by means of the executive breakpoint feature of the 1108 computer. The central processing unit (CPU) times shown include, in addition to computation time, the time used for data input (card and tape reading) and a substantial amount of printed output.

Two NASTRAN runs were made in each case of Table 1. The first run was made independently of reanalysis requirements. The purpose was to check stability of the analytical model and possibly to provide some

Table 1. CPU times[a] for Univac 1108 Exec 8 computer

Parameter	Structure I	Structure II
Matrix order m	48	1350
Total number of bars	60	1492
Number of loading vectors k	2	2
Number of bars in change group b	6	37
NASTRAN CPU times[a]		
Original structure: for U_I	32 s	12 min 44 s
Original structure: for U_I and U_S	45 s (cold start)	11 min 41 s (restart)
Reanalysis program CPU times[a]		
Generate U_M for first area change	2.7 s	37.0 s
Generate U_M for each additional area change	0.3 s	2.3 s

[a]CPU times are central processing unit times for all operations of computations plus input/output, exclusive of compilation time.

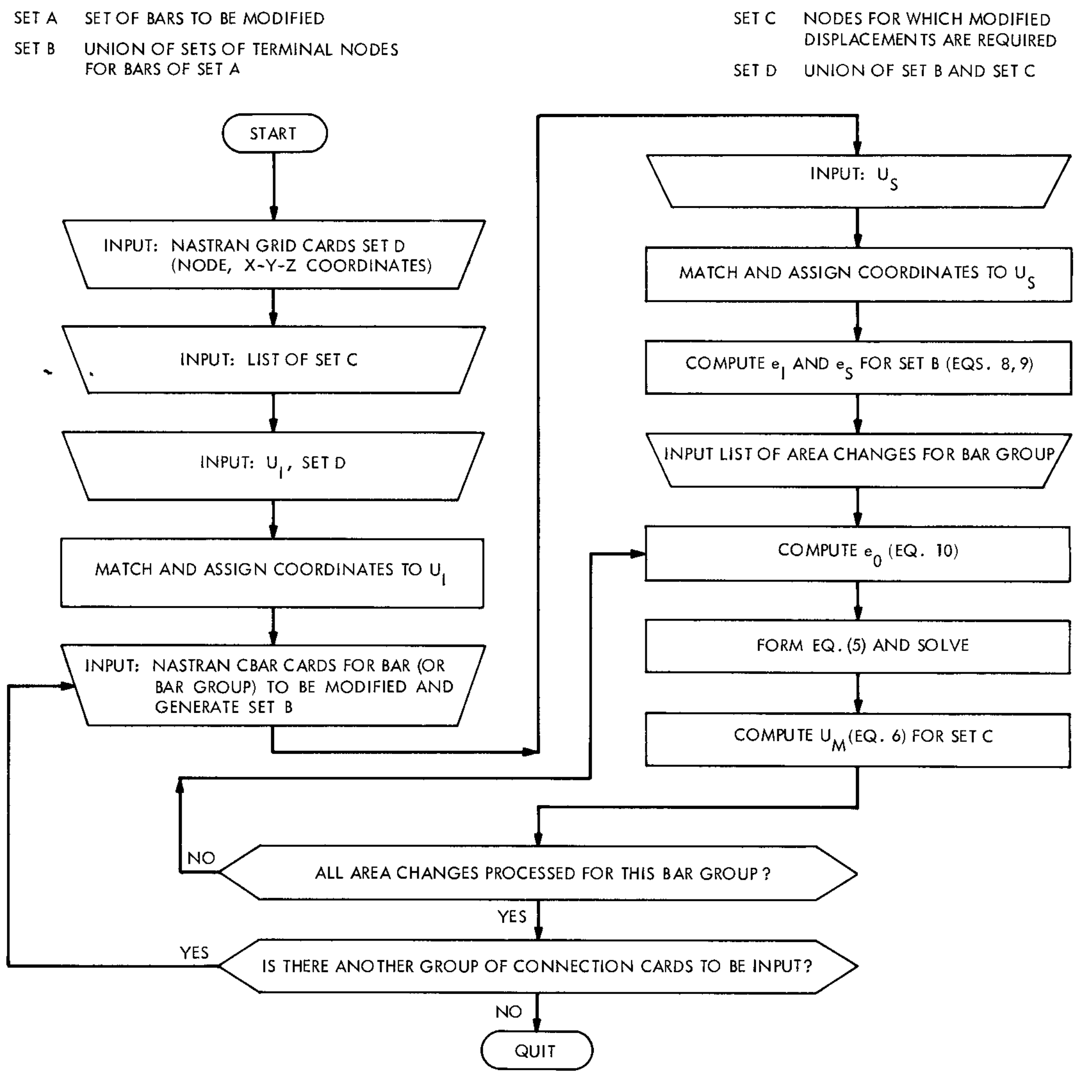

Fig. 1. Flow chart for computations of modified structure displacements (NASTRAN post-processor)

insight into which members were the most desirable candidates for revision. When the checking can be eliminated and the members to be changed are known in advance, reanalysis could start from the second run, which generates all needed information. The time used for the first run, however, gives an indication of the time required to process one additional group change of member properties in lieu of using the reanalysis post-processor program.

Table 2 illustrates the advantages in computation total time that can be obtained by using the reanalysis procedure to process up to four hypothetical property changes for a given bar group for the two structures. The tabulations show the time to analyze each modification as a new structure by using NASTRAN only and the time to process the modifications by the reanalysis program. The "NASTRAN only" column does not consider program restarts, because experience does not show significant economies in restarting when the basic structure is modified. Although Table 2 shows substantial advantages for reanalysis post-processing when at least two changes are considered, the benefits would be even more pronounced whenever the first NASTRAN run is eliminated and only the second NASTRAN run (Table 1) is used.

Computational time savings are most pronounced when the number of property changes is a relatively small proportion of all of the properties because the number of computations depends upon the number of changes. Whenever a large number of members are jointly to be given a small number of changes, the advantages of re-

Table 2. CPU times[a] for reanalysis of multiple property changes of a bar group

Structure	Number of property changes	Time, s: NASTRAN only	Time, s: NASTRAN + post-processor reanalysis
I	1	32	47.7
	2	64	48.0
	3	96	48.3
	4	128	48.6
II	1	768	738
	2	1536	740
	3	2304	743
	4	3072	746

[a]CPU times are central processing unit times for all operations of computations plus input/output, exclusive of compilation time.

analysis would tend to diminish. For example, if all members were to be changed, the time to generate the U_S displacements would be equivalent to the generation of the full flexibility matrix. In such cases, it might be most efficient to analyze the modified structures independently.

In addition to the operations described in the foregoing discussion, it has been found convenient and efficient within the reanalysis program to perform required additional computations in terms of the modified displacements that are developed. For example, the present version of the program contains a subroutine to compute the rms deviations from the best-fitting paraboloid, using the U_M displacements for a set of specified nodes (set C, Fig. 1) on the reflector surface.