An Analytical Study of the Advantages Which Differenced Tracking Data May Offer for Ameliorating the Effects of Unknown Spacecraft Accelerations

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Using the six parameter representation of the range-rate observable, arguments are presented to show why differenced data may more effectively diminish the effects of unmodelable spacecraft accelerations than the conventional tracking data. For a Viking spacecraft experiencing unknown constant accelerations, the orbit determination solution using differenced data may be two orders of magnitude better than the solution obtained from conventional tracking data.

I. Introduction

In the previous article,[1] some preliminary analysis was performed to examine the advantages of using data taken simultaneously, or nearly simultaneously, from two widely separated tracking stations. In particular, it was shown that the deleterious effect of unmodeled accelerations on the estimate of the spacecraft state may be substantially reduced by differencing the data obtained in this manner. To further illustrate the reasons why differenced data may be superior to conventional data, and, in addition, to obtain some idea of the degree of this superiority, conventional and differenced data were separately used to compute estimates of the position and velocity of a *Viking* spacecraft subject to unmodeled constant accelerations. Since the primary purpose of undertaking this investigation is to gain an increased understanding of the orbit determination process, the range rate observable will be represented by an analytical model involving six parameters.

[1]Rourke, K. H., and Ondrasik, V. J., "Application of Differenced Tracking Data Types to the Zero Declination and Process Noise Problem" (this volume).

II. The Six Parameter Model

As explained in Ref. 1, this six parameter model is developed by first expanding the range-rate observable, in terms of the ratio between the geocentric distances of the observing station and spacecraft, to obtain the following equation:

$$\dot{\rho} = \dot{r} - z_s \dot{\delta} \cos \delta + r_s (\dot{\phi} - \dot{\alpha}) \cos \delta (\phi - \alpha) + r_s \dot{\delta} \sin \delta \cos (\phi - \alpha) \qquad (1)$$

where

r = spacecraft geocentric range

δ = spacecraft declination

α = spacecraft right ascension

r_s = station's distance off the Earth's spin axis

z_s = station's distance above the Earth's equator

ϕ = station's right ascension

$\dot{a} = \dfrac{da}{dt}$

The six parameter model results from assuming that the time-varying quantities involved in Eq. (1) may be represented by the following first-order expansions in time:

$$
\begin{aligned}
\dot{r} &= \dot{r}_0 + \ddot{r}_0 t \\
\delta &= \delta_0 + \dot{\delta}_0 t \\
\alpha &= \alpha_0 + \dot{\alpha}_0 t \\
\dot{\delta} &= \dot{\delta}_0 + \ddot{\delta}_0 t \\
\dot{\alpha} &= \dot{\alpha}_0 + \ddot{\alpha}_0 t \\
\phi &= \phi_0 + \dot{\theta} t
\end{aligned} \tag{2}
$$

where a_0 denotes that the quantity a is evaluated at $t = 0$. Substituting Eq. (2) into Eq. (1) yields

$$
\begin{aligned}
\dot{\rho}(t) = {} & a + b \sin(\phi_0 - \alpha_0 + \dot{\theta} t) + c \cos(\phi_0 - \alpha_0 + \dot{\theta} t) \\
& + dt + et \sin(\phi_0 - \alpha_0 + \dot{\theta} t) \\
& + ft \cos(\phi_0 - \alpha_0 + \dot{\theta} t)
\end{aligned} \tag{3}
$$

where

$$
\begin{aligned}
a &= \dot{r}_0 - z_s \dot{\delta}_0 \cos\delta_0 \\
b &= r_s (\dot{\theta} - \dot{\alpha}_0) \cos\delta_0 \\
c &= r_s \dot{\delta}_0 \sin\delta_0 \\
d &= \ddot{r}_0 + z_s (\dot{\delta}_0^2 \sin\delta_0 - \ddot{\delta}_0 \cos\delta_0) \\
e &= r_s [-(\dot{\theta} - 2\dot{\alpha}_0)\dot{\delta}_0 \sin\delta_0 - \ddot{\alpha}_0 \sin\delta] \\
f &= r_s [-(\dot{\theta} - \dot{\alpha}_0)\dot{\alpha}_0 \cos\delta_0 + \dot{\delta}_0^2 \cos\delta_0 + \ddot{\delta}_0 \sin\delta_0] \\
\dot{\theta} &= 0.729 \times 10^{-4}\ \text{rad/s}
\end{aligned} \tag{4}
$$

Since $\ddot{r}_0$, $\ddot{\delta}_0$, and $\ddot{\alpha}_0$ are not independent of r_0, δ_0, α_0, $\dot{r}_0$, $\dot{\delta}_0$, and $\dot{\alpha}_0$, the expressions in Eq. (4) for the coefficients a–f are not suitable for analysis. However, as shown in Ref. 1, the relationships between these quantities may be found, and result in the following equations:

$$
\begin{aligned}
a &= \dot{r}_0 - z_s (\dot{\delta}_0 \cos\delta_0) \\
b &= r_s (\dot{\theta}_0 - \dot{\alpha}_0) \cos\delta_0 \\
c &= r_s \dot{\delta}_0 \sin\delta_0 \\
d &= \ddot{r}_{g0} + r_0 (\dot{\delta}_0^2 + \dot{\alpha}_0^2 \cos^2\delta_0) \\
& \quad + z_s \left(\dot{\delta}_0^2 \sin\delta_0 + \dot{\alpha}_0^2 \cos^2\delta_0 \sin\delta_0 + 2\frac{\dot{r}_0}{r_0} \cos\delta_0 - \ddot{\delta}_{g0} \cos\delta_0 \right) \\
e &= r_s \left(-\dot{\theta}_0 \dot{\delta}_0 \sin\delta_0 + 2\frac{\dot{r}_0}{r_0} \dot{\alpha}_0 \cos\delta_0 - \ddot{\alpha}_{g0} \right) \\
f &= r_s \left(-\dot{\theta}_0 \dot{\alpha}_0 \cos\delta_0 + \dot{\alpha}_0^2 \cos^3\delta_0 + \dot{\delta}_0^2 \cos\delta_0 - 2\frac{\dot{r}_0}{r_0} \dot{\delta}_0 \sin\delta_0 + \ddot{\delta}_{g0} \sin\delta_0 \right) \\
\ddot{r}_g &= -\mu \left[\frac{r}{r_p^3} - r_e \left(\frac{1}{r_p^3} - \frac{1}{r_e^3} \right) \times < \cos\delta \cos\delta_s \cos(\alpha - \alpha_s) + \sin\delta \sin\delta_s > \right] \\
\ddot{\delta}_g &= -\mu \frac{r_e}{r} \left(\frac{1}{r_p^3} - \frac{1}{r_e^3} \right) \times < \sin\delta \cos\delta_s \cos(\alpha - \alpha_s) - \cos\delta \sin\delta_s > \\
\ddot{\alpha}_g &= -\mu \frac{r_e}{r} \left(\frac{1}{r_p^3} - \frac{1}{r_e^3} \right) \cos\delta \sin(\alpha - \alpha_s) \\
r_p &= \{ r^2 + r_e^2 - 2 r r_e [\cos\delta \cos\delta_s \cos(\alpha - \alpha_s) + \sin\delta \sin\delta_s] \}^{1/2}
\end{aligned}
$$

r_e = distance from Earth to Sun

δ_s = declination of the Sun

α_s = right ascension of the Sun

μ = gravitational constant of the Sun (5)

Any error analysis based upon this model proceeds by treating the coefficients a–f as data points which describe the range-rate observable. However, these "data" points are not independent, and in fact may be highly correlated. The correlations and appropriate weights associated with these coefficients may be expressed by the following information matrix:

$$
J_a = \frac{N}{\sigma_{\dot{\rho}}^2} \frac{1}{\int_\rho d\psi}
\begin{bmatrix}
\int_\rho d\psi & \int_\rho \sin\psi \, d\psi & \int_\rho \cos\psi \, d\psi & \int_\rho \psi \, d\psi & \int_\rho \psi \sin\psi \, d\psi & \int_\rho \psi \cos\psi \, d\psi \\
 & \int_\rho \sin^2\psi \, d\psi & \int_\rho \sin\psi \cos\psi \, d\psi & \int_\rho \psi \sin\psi \, d\psi & \int_\rho \psi \sin^2\psi \, d\psi & \int_\rho \psi \sin\psi \cos\psi \, d\psi \\
 & & \int_\rho \cos^2\psi \, d\psi & \int_\rho \psi \cos\psi \, d\psi & \int_\rho \psi \sin\psi \cos\psi \, d\psi & \int_\rho \psi \cos^2\psi \, d\psi \\
 & & & \int_\rho \psi^2 \, d\psi & \int_\rho \psi^2 \sin\psi \, d\psi & \int_\rho \psi^2 \cos\psi \, d\psi \\
 & & & & \int_\rho \psi^2 \sin^{-2}\psi \, d\psi & \int_\rho \psi^2 \sin\psi \cos\psi \, d\psi \\
 & & & & & \int_\rho \psi^2 \cos^2\psi \, d\psi
\end{bmatrix}
\tag{6}
$$

where

$\psi = \dot{\theta} t$

$\sigma_{\dot{\rho}}$ = variance of the white noise associated with the range-rate measurements

N = number of range-rate data points

$\int_\rho$ indicates that the integral extends over the full tracking interval, but has a non-zero contribution only when data is being taken

In using the six coefficients a–f as data points, the estimation filter accepts residuals in a–f, which have been generated in some manner, and modifies the six elements of the spacecraft state such that the residuals in $\dot{\rho}(t)$ are minimized. If the residuals in a–f are generated by an error source (e.g., unmodeled non-gravitational accelerations), there will be a resulting error in the spacecraft state. Using the classical least-squares technique, this solution procedure may be written as

$$
\Delta \mathbf{x}_{\dot{\rho}} = \Lambda_{\dot{\rho}} A^T J_a \Delta \mathbf{a} \tag{7}
$$

where

$\Delta \mathbf{x}_{\dot{\rho}}$ = solution vector for the spacecraft state resulting from the use of range-rate data only

$\Delta \mathbf{a}$ = a vector representing changes in the coefficients a–f which have been generated in some manner

$\Lambda_{\dot{\rho}} = (A^T J_a A)^{-1}$ = state covariance resulting from the use of range-rate data only

$$
A = \frac{\partial (a, b, c, d, e, f)}{\partial (r_0, \delta_0, \alpha_0, \dot{r}_0, \dot{\delta}_0, \dot{\alpha}_0)} \tag{8}
$$

The effect of including range data in the solution may be represented by supplying *a priori* information to the information matrix as shown below:

$$
\Delta \mathbf{x}_r = \Lambda_r A^T J_a \Delta \mathbf{a} \tag{9}
$$

where

$\Delta \mathbf{x}_r$ = solution vector for the spacecraft state resulting from the use of range rate and range data

$$
\Lambda_r = [A^T J_a A + J_r(\mathrm{ap})]^{-1}
$$

$$
J_r(\mathrm{ap}) = \left[\begin{array}{c|c} \sigma_r^2(\mathrm{ap}) & 0 \\ \hline 0 & 0 \end{array}\right]
$$

$\sigma_r(\mathrm{ap})$ = *a priori* standard deviation of the geocentric range

$$
\tag{10}
$$

III. Specification of Tracking Patterns and Trajectory Information

The possible advantages inherent in the differenced range-rate data will be illustrated by comparing (1) the covariances, and (2) the solution error produced by constant unknown accelerations, when these quantities are computed separately, using the differenced data and the conventional range-rate data. The particular example that will be chosen involves the *Viking* trajectory described in Table 1 and the use of tracking patterns shown in Fig. 1. These tracking passes are essentially horizon to horizon and since the epoch has been chosen to occur at the meridian crossing of DSS 14, only the DSS 14 tracking pattern will be symmetric.

The standard deviation of the coefficients a–f, for data arcs containing pass 1, passes 1–3, passes 1–5, and passes 1–7 of Fig. 1 are easily calculated from Eq. (7) and are shown in Fig. 2. In this figure the standard deviations resulting from the symmetric passes of DSS 14 are labeled with (SYM) and those resulting from the non-symmetric passes which will be used for the differenced data are labeled by (NON-SYM). It should be noted that when more than one pass of data is used the standard deviations resulting from the symmetric passes (which will be used with the conventional data) are approximately an order of magnitude lower than those resulting from the use of non-symmetric passes (which will be used with the differenced data).

IV. Spacecraft State Standard Deviations and Errors Resulting From the Use of Conventional Data

If the components of the unknown, constant, non-gravitational acceleration are expressed in the r_0, δ_0, α_0 coordinate system, it is easily seen from Eq. (4) that these accelerations produce errors in the coefficients describing the conventional range rate of an amount given below:

$$\Delta a = 0$$

$$\Delta b = 0$$

$$\Delta c = 0$$

$$\Delta d = \Delta \ddot{k}_r - \frac{z_s}{r} \cos \delta_0 \, \Delta \ddot{k}_\delta$$

$$\Delta e = -\frac{r_s}{r} \cos \delta_0 \, \Delta \ddot{k}_\alpha$$

$$\Delta f = \frac{r_s}{r} \sin \delta_0 \, \Delta \ddot{k}_\delta \qquad (11)$$

where

$\Delta \ddot{k}_{r,\delta,\alpha}$ = components of the unknown, constant, non-gravitational accelerations

The errors in the coefficients, a–f, produced by a constant non-gravitational acceleration of amount 10^{-12} km/s^2 in all three components are shown in Table 2.

The errors in the estimate of the spacecraft state and the associated computed standard deviations may now be obtained by using Eqs. (7) and (8) for conventional range-rate data only and by using Eqs. (9) and (10) for the conventional range-rate data supplemented by a range point. The results are shown in Figs. 3 and 4, where the quantities resulting from the use of conventional range-rate data only and conventional range-rate data plus a range point are labeled by ($\dot{\rho}$) and ($\dot{\rho}, r_0$), respectively. To avoid numerical difficulties the solutions involving range data were performed by starting with the *a priori* information listed in Table 3.

One of the most notable features of Fig. 3 is that the errors generated by the range-rate data only solutions are constant, while the errors generated by the range-rate data supplemented by a range point depend upon the data arc. The doppler only solutions are constant because the six data points and six solve-for parameters are related in such a manner that allows the residuals to be reduced to zero. However, when the range is deleted from the solution, there are only five solve-for parameters and the residuals cannot be set to zero, only minimized. This minimization is dependent upon the correlations, which are a function of time, and hence the solution will be a function of time. A further examination of Fig. 3 shows that an unknown acceleration will produce errors primarily in the range estimate, if range-rate data only is used and in the estimates of the declination and right ascension rates if a range measurement is also used. These results may be easily explained. From Eq. (10) it is apparent that unknown accelerations in the radial direction are about four orders of magnitude more important than accelerations perpendicular to the radial direction. The solution filter will account for this spurious radial acceleration by adjusting the gravitational, and centrifugal accelerations. Since the range enters most strongly into the range-rate observable through the gravitational acceleration, if it is available for estimation, almost all of the error will emerge in this quantity. A very good approximation to a range error produced by a constant acceleration may often be obtained by using the following equation:

$$\Delta r = \frac{\Delta \ddot{k}}{\partial d / \partial r} \qquad (12)$$

where

$\Delta \ddot{k}$ = constant acceleration error

$\partial d/\partial r \approx m/r_p^3 (2 - 3 \sin^2 \psi) + (\dot{\alpha}^2 \cos^2 \delta + \dot{\delta}^2)$ (Ref. 1)

ψ = Earth–spacecraft–Sun angle

For the example under consideration, Eq. (12) gives an approximation to the range error of 50.8 km, which is very close to the result shown in Fig. 3. If the range has been essentially deleted from the solution by the *a priori* information, the radial acceleration will be absorbed in the centrifugal acceleration term, because any change in the gravitational acceleration would now require changes in δ and α which are well determined by the b and c coefficients. An error in the centrifugal accelerations will manifest itself as errors in $\dot{\alpha}$ and $\dot{\delta}$. Simple equations for $\Delta\dot{\alpha}$ and $\Delta\dot{\delta}$ comparable to the Δr equation above cannot be written down because $\dot{\delta}$ and $\dot{\alpha}$ are also strongly involved in the e and f coefficients. Although the errors in the δ and α directions are less than a kilometer, they are included because the results scale directly with the magnitude of the accelerations and for those typical of solar electric spacecraft the errors could be three orders of magnitude larger than those shown in Fig. 3.

V. The Six Parameter Model for Differenced Data

The six parameter model representing the differenced range-rate data may be obtained by first using Eq. (3) to express separately the topocentric range rate from two stations as shown below:

$$\dot{\rho}_1(t) = a_1 + b_1 \sin(\dot{\theta}t) + c_1 \cos(\dot{\theta}t) + d_1 t + e_1 t \sin(\dot{\theta}t) + f_1 t \cos(\dot{\theta}t)$$

$$\dot{\rho}_2(t) = a_2 + b_2 \sin(\lambda_{21} + \dot{\theta}t) + c_2 \cos(\lambda_{21} + \dot{\theta}t) + d_2 t + e_2 t \sin(\lambda_2 + \dot{\theta}t) + f_2 t \cos(\lambda_{21} + \dot{\theta}t)$$

where

$\lambda_{21} = \lambda_2 - \lambda_1$

$t = 0$ occurs at meridian crossing of station 1

Clearly the differenced range rate, $\nabla\dot{\rho}$, may be represented by the difference between these two equations as shown below:

$$\nabla\dot{\rho} = a_d + b_d \sin(\dot{\theta}t) + c_d \cos(\dot{\theta}t) + d_d t + e_d t \sin(\dot{\theta}t) + f_d t \cos(\dot{\theta}t) \tag{13}$$

where

$$a_d = -(z_{s_1} - z_{s_2})\,\dot{\delta}\cos\delta$$
$$b_d = b_1 - (b_2 \cos\lambda_{21} - c_2 \sin\lambda_{21})$$
$$c_d = c_1 - (c_2 \cos\lambda_{21} + b_2 \sin\lambda_{21})$$
$$d_d = (z_s - z_{sz})(\dot{\delta}_0^2 \sin\delta_0 - \ddot{\delta}_0 \cos\delta_0)$$
$$= (z_s - z_{sz})\left(\dot{\delta}_0^2 \sin\delta_0 + \dot{\alpha}_0^2 \cos\delta_0 \sin\delta_0 + 2\frac{\dot{r}_0}{r_0}\dot{\delta}_0 \cos\delta_0 - \ddot{\delta}_{g0}\cos\delta_0\right)$$

$$e_d = e_1 - (e_2 \cos\lambda_{21} - f_2 \sin\lambda_{21})$$
$$f_d = f_1 - (f_2 \cos\lambda_{21} + e_2 \sin\lambda_{21}) \tag{14}$$

The a_d and d_d terms have been written explicitly to show that the geocentric range rate and accelerations have cancelled out and no longer appear in the coefficients. The covariance and solutions using differenced range-rate data only and differenced range-rate data supplemented by a range point may be obtained by using Eqs. (7)–(10) with the a–f coefficients replaced by the a_d–f_d coefficients of Eq. (14). As was pointed out in the previous article (Footnote 1), the differencing procedure introduces problems associated with the frequency standard. However, for the sake of clarity, these oscillator-induced problems will be ignored.

VI. Spacecraft State Variances and Errors Resulting From the Use of Differenced Data

The unknown constant non-gravitational accelerations of 10^{-12} km/s^2 considered previously will produce errors in the a_d–f_d coefficients of the amount shown in Table 2. It is readily apparent from this table that the error in d_d is now of the same size as the errors in the e_d and f_d coefficients.

The formal covariance, and errors in spacecraft state due to unknown constant accelerations, may now be computed for the data arcs shown in Fig. 1, and are illustrated in Figs. 3 and 4. In these figures, the quantities which result from the use of differenced range-rate data only are labeled by (DIFF $\dot{\rho}$) and those which result from the use of differenced range-rate supplemented by a range point are labeled by (DIFF $\dot{\rho}$, r_0). An examination of Fig. 3 shows that, as was the case for conventional data, the acceleration errors are absorbed by either the range or $\dot{\delta}$ and $\dot{\alpha}$.

VII. Comparison Between the Conventional and Differenced Data Results

To obtain a clear comparison between the spacecraft states standard deviations and errors generated by the two data types under consideration, each quantity in Fig. 3 or 4 computed from differenced data was divided by the same quantity computed from conventional data. The results of following this procedure for the standard deviations and errors computed from five passes are shown in Table 4.

Bearing in mind the assumptions upon which this analysis has been based, an examination of Table 4 and

Figs. 3 and 4 leads to the following tentative conclusions regarding the use of conventional and differenced range-rate data to obtain a spacecraft state solution in the presence of unknown constant accelerations:

(1) The differenced data cannot determine the range or range rate.

(2) The formal standard deviations for δ, α, $\dot{\delta}$, and $\dot{\alpha}$ are generally slightly better using the conventional data if more than one pass of data is used.

(3) For solutions involving range-rate data only, unknown constant accelerations produce errors in the spacecraft state which are generally about the same size, irrespective of whether conventional or differenced data is used.

(4) If a range point is included in the data set, errors in δ, α, $\dot{\delta}$, and $\dot{\alpha}$ produced by unknown constant accelerations of equal magnitude in three orthogonal directions, are at least 100 times smaller if differenced range-rate data is used rather than the conventional range-rate data.

The fact that the differenced data cannot estimate the geocentric range or range rate is not surprising because the portions of the range-rate observable which is most effective in determining these quantities have been intentionally eliminated in the differencing process. This is not a serious matter because the range and range rate can be determined from the conventional data.

The main advantage of differenced range-rate data over conventional range-rate data is that state estimates obtained from the differenced data are not degraded nearly as much by unmodeled radial accelerations as estimates obtained from conventional range-rate data as was mentioned above. It is this feature that raises the promise that using differenced doppler data may be at least a partial solution to the process noise problem.

Before leaving this section it should be mentioned once more that the analysis performed here is representative of a real physical situation only to the degree that the six parameter model is representative of the range-rate observable and that the unknown accelerations are constant.

VIII. Summary and Discussion

The purpose of the analysis carried out in the previous sections was motivated by the desire to increase our understanding of how the differencing techniques ameliorates the effect of unmodelable accelerations, and also to obtain some idea of how effective these techniques may be. By making use of the six parameter representation of the range-rate observable, it was shown, once again, that the unmodeled accelerations which severely degrade the solution are those occurring in the radial direction. It appears that the effect of these accelerations can be substantially reduced by differencing the data taken simultaneously from two tracking stations. For the *Viking* trajectory, which was used as an example, the unmodeled constant accelerations degraded the conventional data solution two orders of magnitude more than the differenced data solution.

Although the analysis presented in this article indicates that differenced data may be very useful in diminishing the effects of unmodelable accelerations, before any real confidence may be acquired in this technique it will be necessary to perform an uncompromised accuracy analysis study. Such a study is currently underway and will be reported on in the near future.

Reference

1. Ondrasik, V. J., and Curkendall, D. W., "A First-Order Theory for Use in Investigating the Information Content Contained in a Few Days of Radio Tracking Data," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. III, pp. 77–93. Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1971.

Table 1. *Viking* trajectory information

Quantity	Value
r_0	0.8854×10^8 km
δ_0	20.31 deg
α_0	57.76 deg
$\dot{r}_0$	15.32 km/s
$\dot{\delta}_0$	0.2278×10^{-7} rad/s
$\dot{\alpha}_0$	0.8896×10^{-7} rad/s
t_0	1976 Jan 22 3^h 33^m meridian crossing at DSS 14

Table 2. Errors in a–f produced by a constant acceleration

Conventional coefficients	Error[a]	Differenced coefficients	Error[a]
a	0	a_d	0
b	0	b_d	0
c	0	c_d	0
d	9.9996×10^{-13}	d_d	-7.7869×10^{-17}
e	-5.5120×10^{-17}	e_d	-3.8737×10^{-17}
f	2.0400×10^{-17}	f_d	-3.3122×10^{-17}

[a]Produced by $\Delta\ddot{k}_{r,\delta,\alpha} = 10^{-12}$ km/s^2

Table 3. *A priori* information for the spacecraft state

Spacecraft coordinate	*A priori* value
r	10^{-1} km
δ	10^{-8} rad
α	10^{-8} rad
$\dot{r}$	10^{-2} km/s
$\dot{\delta}$	10^{-10} rad/s
$\dot{\alpha}$	10^{-10} rad/s

Table 4. Comparisons of standard deviations and errors obtained by using conventional and differenced range-rate data

Coordinate	σ (diff)/σ (conv)		Δ (diff)/Δ (conv)[a]	
	Range rate only	Range rate + range	Range rate only	Range rate + range
r	2.34×10^3	1	−0.942	—
δ	4.08	1.65	−3.79	0.0109
α	2.62	0.516	0.00196	0.00244
$\dot{r}$	2.12×10^7	6.07×10^5	4.76×10^6	9.23×10
$\dot{\delta}$	2.78×10	0.0713	0.0000345	0.00626
$\dot{\alpha}$	2.35	1.29	3.76	0.00400

[a]For unmodeled constant acceleration of 10^{-12} km/s^2.

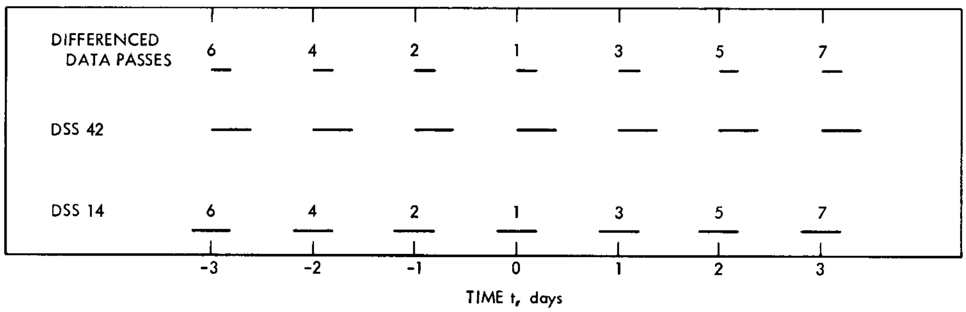

Fig. 1. Tracking patterns and pass numbers

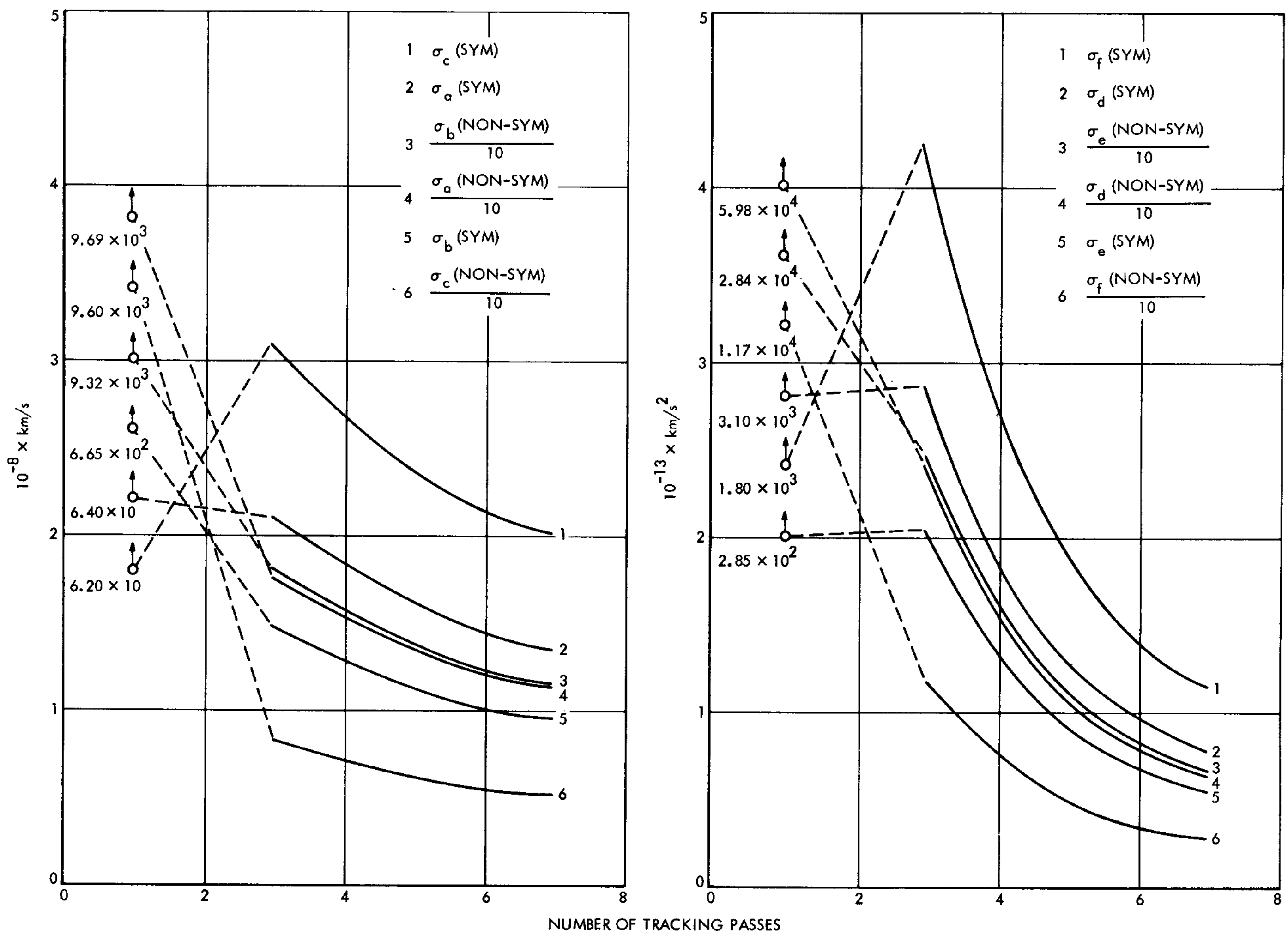

Fig. 2. Standard deviations of $a \rightarrow f$

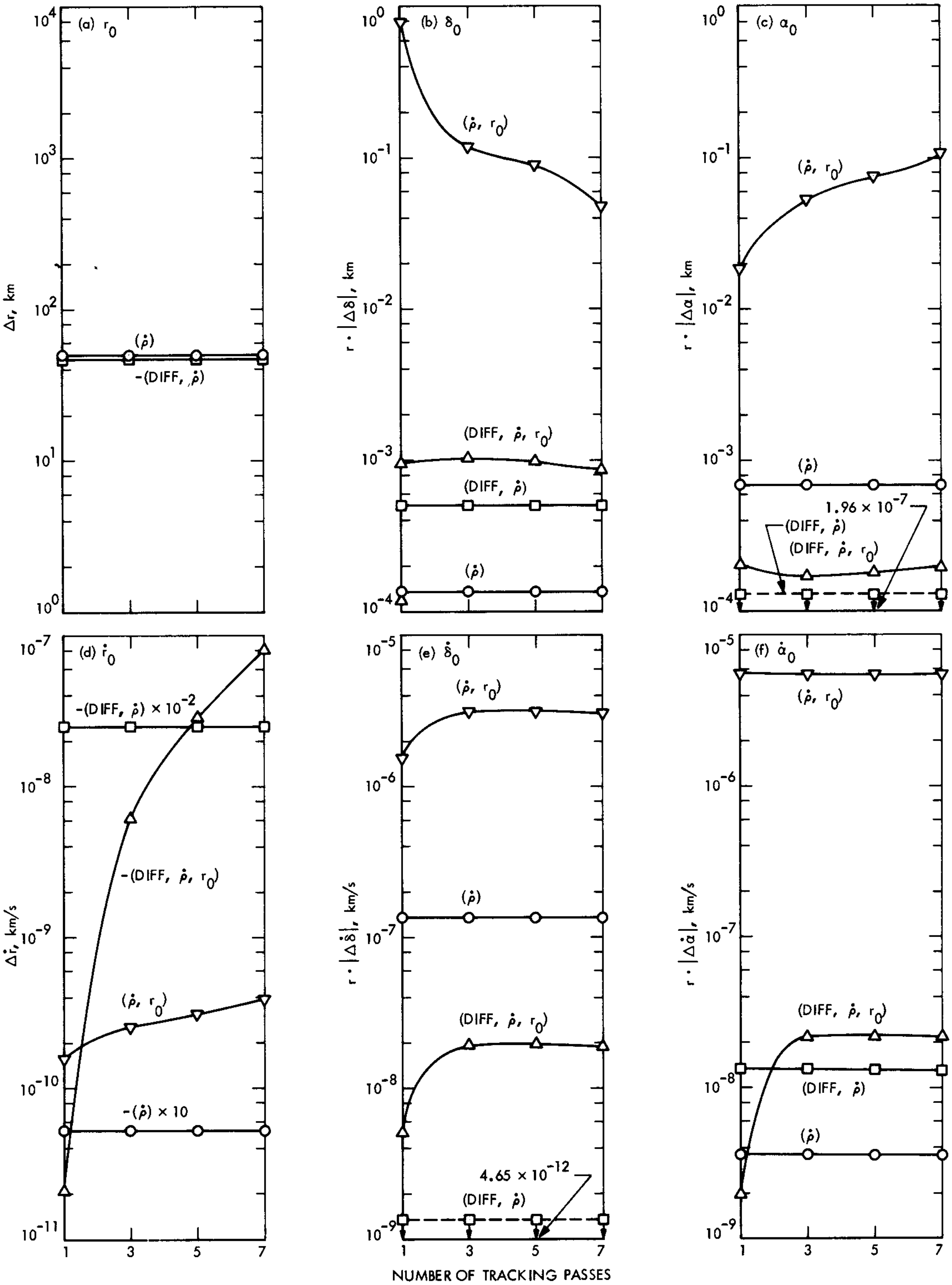

Fig. 3. Spacecraft state errors produced by unmodeled constant accelerations of 10^{-12} km/s^2 when conventional and differenced data are used

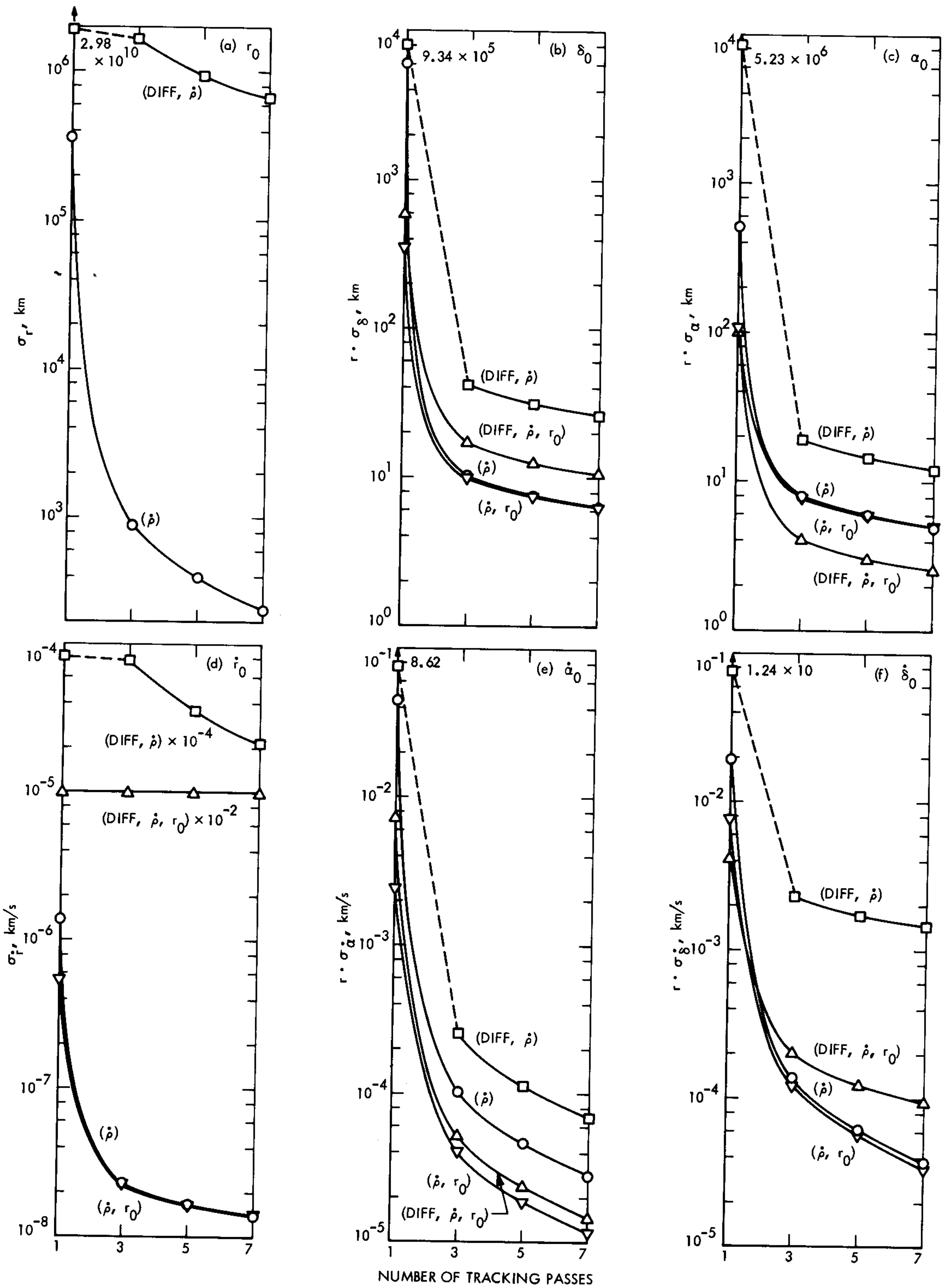

Fig. 4. Spacecraft state standard deviations and errors resulting from the use of conventional and differenced data