Effects of the *Pioneer 10* Antenna Polarization and Spacecraft Rotation as Seen in the Radio Metric Data

A. L. Berman
DSN Engineering and Operations Section

The Pioneer 10 antenna polarization and spacecraft rotation introduce a signature into the radio metric doppler data, which, unless otherwise noted, might be confused with a degradation of the tracking system. This signature, especially in regard to effects as seen in doppler residuals and doppler noise, is here analyzed in detail.

I. Introduction

Certain unusual effects are introduced into the *Pioneer 10* radio metric data due to the fact that the spacecraft antenna is circularly polarized and that the spacecraft itself is rotating, combined with the fact that the spin axis of the spacecraft is aligned at various angles with the Earth/spacecraft line of sight.

The Network Analysis Team/Tracking Group (NAT TRK) is charged with continuously monitoring the *Pioneer 10* radio metric data in order to assess the performance of the tracking system. It was therefore necessary for the Network Operations Analysis Tracking Group to analyze the polarization and rotation effects so that the spacecraft effects can be separated from possible system contributions. Briefly, the antenna polarization adds a bias to the radio metric data, while the spacecraft rotation adds a sinusoidal "ripple" to the data, which, of course, increases the data noise. These two effects are now examined in detail.

II. *Pioneer 10* Antenna Polarization Effects

The antenna on *Pioneer 10* is both rotating and right-hand circularly polarized. In this case, the received signal can be thought of as a rotating vector with the frequency being equal to the number of rotations of this vector per unit time. For each complete rotation of the antenna, the number of counts lost from the frequency in two- or three-way is:

$$2 + \frac{19}{221}$$

This is due to one count being lost when the signal is received, and 240/221 of a count being lost after the spacecraft multiplies the signal by 240/221 and transmits it:

$$\text{Total}\begin{cases} 1+\dfrac{240}{221}=1+\dfrac{221}{221}+\dfrac{19}{221} \\ \qquad\quad =2+\dfrac{19}{221} \end{cases}$$

For one way, the number of counts lost from the frequency for each complete rotation is 1.

Since the antenna rotation rate is approximately 4.85 rev/min, the number of counts in the data over 1 min will be

$$-4.85\left(2+\frac{19}{221}\right) \qquad 2,3 \text{ way}$$

$$-4.85 \qquad 1 \text{ way}$$

and the equivalent biases to be observed in the data will be

$$-4.85\left(2+\frac{19}{221}\right)\Big/60 = -0.168616 \text{ Hz for 2,3 way}$$

and

$$-4.85/60 = -0.08083 \text{ Hz for 1 way}$$

III. *Pioneer 10* Spacecraft Rotation Effects

The *Pioneer 10* spacecraft has three conditions which cause a sinusoidal "ripple" in the radio metric data (i.e., the spacecraft antenna is alternately moving with a small velocity toward and away from the ground observer). These three conditions are:

(1) The spacecraft is rotating.

(2) The spacecraft spin axis does not coincide with the Earth line of sight.

(3) The antenna center point is not on the spacecraft spin axis.

The antenna geometry is presented in Fig. 1 where

$r = 8$ in. $= 0.2032$ m

$\omega =$ rotation rate (nominally 4.85 rev/min)

ϕ MAX $\cong 24$ deg

As per the above figure, a ground observer sees a back-and-forth antenna movement (velocity) of:

$$V = r\omega' \sin\phi \sin\omega' t$$

Converting from rev/min to rad/s, we have

$$\omega' = \frac{\omega \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{\text{rev}} \cdot \frac{\text{min}}{60 \text{ s}}$$

$$\omega' = \frac{2\pi\omega}{60}\,\frac{\text{rad}}{\text{s}}$$

so that the velocity is

$$V = r\sin\phi\left\{\frac{2\pi\omega}{60}\right\}\sin\frac{2\pi\omega}{60}t$$

Since the standard radio metric sample rate is 60 s, we are most interested in how much effect this sinusoidal movement will have over a 60-s doppler sample. To do this, we integrate the velocity over a 60-s interval and divide by the sample rate:

$$\frac{1}{T}\int_{\gamma}^{\gamma+T} V(t)\,dt = \frac{1}{60}\int_{\gamma}^{\gamma+60} V(t)\,dt$$

$$= \frac{1}{60}\int_{\gamma}^{\gamma+60} r\sin\phi\left\{\frac{2\pi\omega}{60}\right\}\sin\frac{2\pi\omega}{60}t\,dt$$

We also need to find the particular time γ which causes the integral to be a maximum. To accomplish this, we require the following condition:

$$\frac{\partial}{\partial\gamma}\left\{\frac{1}{60}\int_{\gamma}^{\gamma+60} r\sin\phi\left\{\frac{2\pi\omega}{60}\right\}\sin\frac{2\pi\omega}{60}t\,dt\right\} = 0$$

Or, since in regard to the integration, the term

$$\frac{r\sin\phi}{60}\left\{\frac{2\pi\omega}{60}\right\}$$

is constant, we have

$$\frac{\partial}{\partial\gamma}\int_{\gamma}^{\gamma+60}\sin\frac{2\pi\omega}{60}t\,dt = 0$$

As regards differentiation under the integral sign, we have the general relationship:

If

$$H(\alpha) = \int_{g_0(\alpha)}^{g_1(\alpha)} F(x,\alpha)\,dx$$

we then have

$$\frac{\partial H}{\partial \alpha} = \int_{g_0}^{g_1} \frac{\partial F(x,\alpha)}{\partial \alpha}\, dx - F(g_0,\alpha)\frac{\partial g_0}{\partial \alpha} + F(g_1,\alpha)\frac{\partial g_1}{\partial \alpha}$$

Substituting into the above equation, we have

$$\begin{aligned}
\frac{\partial}{\partial \gamma}\int_{\gamma}^{\gamma+60} \sin\frac{2\pi\omega}{60}\, t\, dt
&= \int_{\gamma}^{\gamma+60} \frac{\partial}{\partial \gamma}\left\{\sin\frac{2\pi\omega}{60}\, t\right\} dt - \sin\frac{2\pi\omega}{60}\,\gamma\,\frac{\partial}{\partial \gamma}(\gamma) \\
&\quad + \sin\frac{2\pi\omega}{60}(\gamma+60)\frac{\partial}{\partial \gamma}(\gamma+60) \\
&= \sin\frac{2\pi\omega}{60}(\gamma+60) - \sin\frac{2\pi\omega}{60}\,\gamma \\
&= \sin\left(\frac{2\pi\omega\gamma}{60} + 2\pi\omega\right) - \sin\left(\frac{2\pi\omega}{60}\gamma\right) \\
&= 0
\end{aligned}$$

using the trigonometric identity

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

we have

$$\sin\frac{2\pi\omega}{60}\gamma\cos 2\pi\omega + \cos\frac{2\pi\omega}{60}\gamma\sin 2\pi\omega - \sin\left(\frac{2\pi\omega\gamma}{60}\right) = 0$$

Let

$$\sin\frac{2\pi\omega}{60}\gamma = x$$

$$\cos 2\pi\omega = a$$

Then, since

$$\cos^2 A + \sin^2 A = 1$$

we have

$$\cos\frac{2\pi\omega}{60}\gamma = \sqrt{1 - \left(\sin\frac{2\pi\omega}{60}\gamma\right)^2} = \sqrt{1-x^2}$$

and

$$\sin 2\pi\omega = \sqrt{1-(\cos 2\pi\omega)^2} = \sqrt{1-a^2}$$

so that

$$\begin{aligned}
xa + \sqrt{1-x^2}\sqrt{1-a^2} - x &= 0 \\
x(a-1) + \sqrt{(1-x^2)(1-a^2)} &= 0 \\
x(1-a) &= \sqrt{(1-x^2)(1-a^2)} \\
x^2(1-a)^2 &= (1-x^2)(1-a^2) \\
x^2(1-a) &= (1-x^2)(1+a) \\
x^2(1-a+1+a) &= 1+a \\
x^2 &= \frac{1+a}{2} \\
x &= \pm\sqrt{\frac{1+a}{2}} \\
x &= \pm\sqrt{\frac{1+\cos 2\pi\omega}{2}} \\
\sin\frac{2\pi\omega}{60}\gamma &= \pm\sqrt{\frac{1+\cos 2\pi\omega}{2}}
\end{aligned}$$

and

$$\begin{aligned}
\left(\sin\frac{2\pi\omega}{60}\gamma\right)^2 &= \frac{1+\cos 2\pi\omega}{2} \\
1 - \left(\cos\frac{2\pi\omega}{60}\gamma\right)^2 &= \frac{1+\cos 2\pi\omega}{2} \\
\left(\cos\frac{2\pi\omega}{60}\gamma\right)^2 &= \frac{1-\cos 2\pi\omega}{2} \\
\cos\frac{2\pi\omega}{60}\gamma &= \pm\sqrt{\frac{1-\cos 2\pi\omega}{2}}
\end{aligned}$$

To produce a maximum, we require

$$-\frac{\pi}{2} < \frac{2\pi\omega}{60}\gamma < 0$$

so that the conditions for a maximum become

$$\sin\frac{2\pi\omega}{60}\gamma = -\sqrt{\frac{1+\cos 2\pi\omega}{2}}$$

$$\cos\frac{2\pi\omega}{60}\gamma = +\sqrt{\frac{1-\cos 2\pi\omega}{2}}$$

We can now solve for the maximum velocity in a 60-s sample:

$$\frac{1}{T}\int_{\gamma}^{\gamma+T} V(t)\,dt$$

$$= \frac{1}{60}\int_{\gamma}^{\gamma+60} r \sin\phi \left(\frac{2\pi\omega}{60}\right) \sin\frac{2\pi\omega}{60} t\,dt$$

$$= \frac{r\sin\phi}{60}\left(\frac{2\pi\omega}{60}\right)\int_{\gamma}^{\gamma+60} \sin\frac{2\pi\omega}{60} t\,dt$$

$$= \frac{r\sin\phi}{60}\left(\frac{2\pi\omega}{60}\right)\left[-\frac{60}{2\pi\omega}\cos\frac{2\pi\omega}{60} t\right]_{\gamma}^{\gamma+60}$$

$$= -\frac{r\sin\phi}{60}\left[\cos\frac{2\pi\omega}{60} t\right]_{\gamma}^{\gamma+60}$$

$$= -\frac{r\sin\phi}{60}\left[\cos\frac{2\pi\omega}{60}(\gamma+60) - \cos\frac{2\pi\omega}{60}\gamma\right]$$

Using the trigonometric identity

$$\cos(A+B) = \cos A\cos B - \sin A \sin B$$

we have

$$= -\frac{r\sin\phi}{60}\left[\cos\frac{2\pi\omega}{60}\gamma\cos 2\pi\omega - \sin\frac{2\pi\omega}{60}\gamma \sin 2\pi\omega - \cos\frac{2\pi\omega}{60}\gamma\right]$$

and substituting the conditions for a maximum into the above equations, we have

$$\frac{1}{T}\int_{\gamma}^{\gamma+T} V(t)\,dt = -\frac{r\sin\phi}{60}\left[\sqrt{\frac{1-\cos 2\pi\omega}{2}}\cos 2\pi\omega + \sqrt{\frac{1+\cos 2\pi\omega}{2}}\sin 2\pi\omega - \sqrt{\frac{1-\cos 2\pi\omega}{2}}\right]$$

Using $\omega = 4.85$ we have

$$\cos 2\pi\omega = 0.58779$$

$$\sin 2\pi\omega = -0.80902$$

$$\sqrt{\frac{1-\cos 2\pi\omega}{2}} = 0.45399$$

$$\sqrt{\frac{1+\cos 2\pi\omega}{2}} = 0.89101$$

Therefore

$$\frac{1}{T}\int_{\gamma}^{\gamma+T} V(t)\,dt = -\frac{r\sin\phi}{60}\{(0.45399)(0.58779) + (0.89101)(-0.80902) - 0.45399\}$$

$$= -\frac{r\sin\phi}{60}\{0.26685 - 0.72084 - 0.45399\}$$

$$= -\frac{r\sin\phi}{60}\{-0.90798\}$$

$$\frac{1}{T}\int_{\gamma}^{\gamma+T} V(t)\,dt = -\frac{r\sin\phi}{60}\{0.90798\}$$

We wish to know the period of this sinusoidal "ripple" in the data. Since we are sampling at 60 s and the frequency is 4.85 rev/min, the new period will be

$$\frac{60\text{ s}}{5.00 - 4.85} = 400\text{ s}$$

$$= 6.67\text{ min}$$

We therefore expect to see in the data a sine wave of amplitude

$$\frac{r\sin\phi}{60}\{0.90798\}$$

and of period

$$400\text{ s}$$

We would now like to verify this in the actual pseudo-residual output. On March 13, 1972, the Earth line of sight/spin axis angle was approximately 24° and $r = 8$ in. $= 0.2032$ m.

This would give us an amplitude of

$$\frac{r\sin\phi}{60}\{0.90789\} = \frac{(0.2032)(0.40674)}{60}(0.90798)$$

$$= 0.0012507\text{ m/s}$$

and using 15.28 Hz/m/s, we have

$$= 0.019111\text{ Hz}$$

Figure 2 consists of pseudo-residual two-way doppler residuals from DSS 11 on March 13, through which an arbitrary sine wave of amplitude 0.020 Hz and of period 400 s has been fit. As can be seen, the data is in good agreement with the modeled effect. With a sine wave of this magnitude, one would expect a doppler noise figure of approximately 0.012 Hz, and this is quite close to the average noise value of 0.011 Hz observed in the two-way doppler data during the period when the Earth line of sight/spin axis was 24 deg and the doppler sample rate was 60 s.

For 1-s doppler samples, a sinusoid with the full amplitude V

$$
\begin{aligned}
V &= r \sin \phi \left\{ \frac{2\pi\omega}{60} \right\} \\
&= (0.2032)\,(0.4067)\,(0.5079) \\
&= 0.04197 \text{ m/s} \\
&= 0.64130 \text{ Hz}
\end{aligned}
$$

should be visible in the data. On March 11, DSS 51 took 2-way, 1-s sample doppler data. These data showed a sinusoid of period 12.364 s and of amplitude 0.55 Hz. The small discrepancy between the calculated amplitude and the observed amplitude in the 1-s data is not explainable at this time.

IV. Conclusion

In summary, the *Pioneer 10* antenna polarization and rotation introduces a bias of -0.168616 Hz into the two-way doppler data. This is quite small and in general is masked by the fact that the predicts usually have an absolute error of greater than 0.1 Hz at any given time. The *Pioneer 10* rotation also causes a sinusoid of period 400 s and of amplitude

$$
-\frac{r \sin \phi}{60} \left[\sqrt{\frac{1 - \cos 2\pi\omega}{2}} \cos 2\pi\omega + \sqrt{\frac{1 + \cos 2\pi\omega}{2}} \sin 2\pi\omega - \sqrt{\frac{1 - \cos 2\pi\omega}{2}} \right]
$$

to be introduced into the 60-s data. During the time when

$$
\begin{aligned}
\phi &= 24 \text{ deg} \\
\omega &= 4.85 \text{ rev/min} \\
r &= 0.2032 \text{ m}
\end{aligned}
$$

the expected amplitude was 0.01911 Hz, and both the calculated period and amplitude agree very well with the observed data as seen in Fig. 2. This also produces an expected noise in the two-way 60-s sample doppler data of approximately 0.012 Hz, which closely agrees with the noise actually observed in the data.

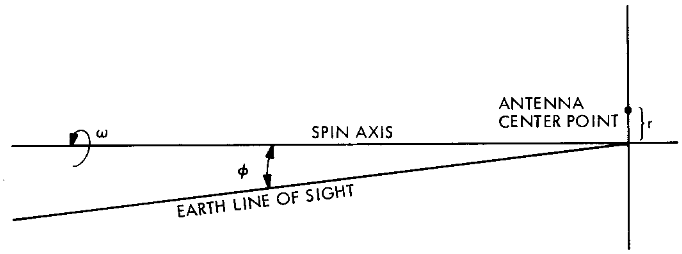

Fig. 1. Antenna geometry

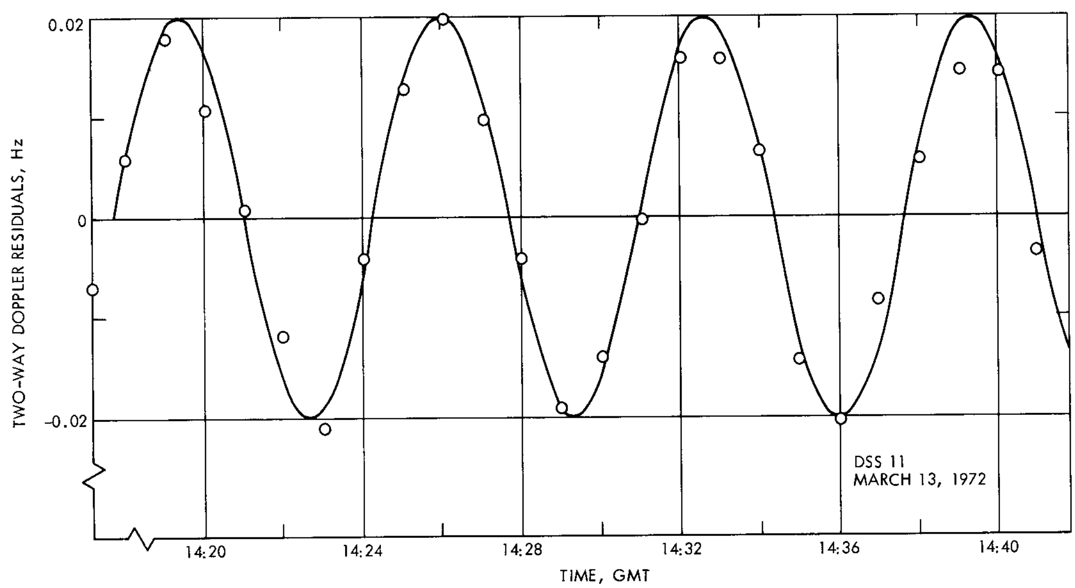

Fig. 2. *Pioneer 10* two-way doppler versus time