Carrier Power Estimation Accuracy

J. Lesh
DSIF Operations Section

In this article estimation theoretic techniques are used to derive expressions for the accuracy of the digital instrumentation subsystem (DIS) and telemetry and command processor (TCP) computer methods of carrier power estimation. Upon evaluation of these expressions it is found that the TCP method is presently far more accurate than the DIS method. A procedure by which the DIS accuracy can be greatly improved is also presented.

I. Introduction

At present there are two automated methods for estimating the incoming carrier signal power at the DSIF receivers. Both of these methods involve evaluation of polynomial expressions of the receiver automatic gain control (AGC) voltage. The coefficients of these polynomials are determined during a pre-track calibration period by applying least squares curve fitting techniques to a set of carrier power versus AGC voltage data pairs. The two estimation methods differ only in the number of calibration data pairs supplied, the range over which calibration data pairs are obtained, the degree of curve fitting attempted and the sampling scheme used to measure the AGC voltage during actual operation.

The first of these methods resides in the DIS monitor computer. Calibration of this method involves establishing ten signal power levels covering the full dynamic range of the receiver. For each of these levels the DIS computer samples the resulting receiver AGC voltage, takes 1000 samples within a one-second period and forms a sample mean. The least squares fitting algorithm operates on the resulting ten pairs of data to determine either a third-order or second-order fitted polynomial. During operation the AGC voltage is sampled at a sampling interval of 1.0 seconds and a mean is computed after five samples. The calibration polynomial is then evaluated using this mean to produce an estimate of the received carrier power.

The second method utilizes the TCP computer in essentially the same way as the DIS computer is used in the above method. Calibration in this case consists of establishing three power level–AGC voltage pairs covering an 8-dB range which is centered around the expected power level for the pass. These three pairs of data are then approximated by a first order least squares polynomial. During operation the AGC voltage is also sampled using a 1.0-second sampling interval. In this case, however, each AGC voltage sample is used to produce a signal power estimate.

To study the errors associated with these methods we shall first consider the errors resulting from the calibration (i.e., curve fitting) process. At first this may appear to be a classical problem of parameter estimation well documented in the literature. For example, if we define the parameter ρ to be a vector whose components are the ideal polynomial coefficients, the matrix H to be the transformation taking ρ into the noise-free observables (calibration power levels), and the vector n to be the vector of observation noise then it is a well-known result (see, for example, Ref. 1) that the minimum variance estimate of ρ and the resulting mean square error are functions of the matrix H and the covariance matrix of the noise vector n. However, in our problem there are three deviations from the classical model. These are:

(1) The dimension of the ideal parameter is unknown.

(2) The dimension of the estimated parameter will be less than (or at most equal to) the ideal parameter.

(3) Over the set of observations the noise variance may vary over several orders of magnitude, yet the curve fitting algorithms in the DIS and TCP assume equal uncertainty in these observations.

The only deviation which causes any significant difficulty is the first. Consequently, the dimension of the ideal parameter will be approximated using as much insight as possible.

After evaluating the calibration errors the operational errors will be considered and the total estimation accuracies computed. In doing this we will find that the TCP method of estimation is more accurate than the DIS method, provided the received power level is within the range of the TCP calibration. Techniques by which the accuracy of the DIS method can be improved will also be presented.

II. Calibration Accuracy

A. Determination of the Noise-Free Model

In order to begin the analysis, we need some model for the ideal (noise free) carrier power versus AGC voltage curve. To determine this model, extensive data were taken at each of the six Deep Space Stations. Using the station operating noise temperature, the variance of each AGC voltage was computed. These data, consisting of from 17 data pairs (for DSS 14) to 40 data pairs (for DSS 12), were then exercised by a curve fit subroutine available in the Univac 1108. This subroutine (Ref. 2) examines polynomials of degree less than or equal to some specified maximum (NMAX) which minimize the weighted mean square error, where the weighting is performed in accordance with the user's *a priori* estimate of the observation errors. The coefficients of the highest degree polynomial which produces a significant decrease in this weighted mean square error are then outputted from the subroutine. Note that the degree of this polynomial will not necessarily be NMAX since the remaining higher order polynomials may produce only an insignificant decrease in the mean square error.

The above program was executed for each set of data and for NMAX $= 2,3,4,\cdots,10$. The following observations were made on the results:

(1) In all cases a sizable reduction in the mean square error occurred when NMAX (and subsequently the degree of the fitted polynomial) was allowed to increase to three.

(2) In all cases where NMAX exceeded three, the subroutine either retained the third-order polynomial or fitted higher order polynomials with only a slight decrease in the mean square error.

Based on these observations we can model the ideal signal power y given the AGC voltage (x) by

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \qquad (1)$$

where a_i, $i = 0, 1, 2, 3$ are the coefficients of the third-order polynomial determined by the subroutine and will be different for each receiver (data set).

B. Derivation of the Mean Square Estimation Error

Using this noise-free model we can now derive the expressions for the mean-square estimation error for each of the curve-fitting algorithms.

1. Third-order curve fitting. For a specific value of AGC voltage x let the corresponding power level be given by Eq. (1) and let the estimated power level be given by

$$\hat{y} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 \qquad (2)$$

Then, the mean-square error conditioned on a value of x is given by

$$
\begin{aligned}
E\{(y-\hat{y})^2/x\} = \\
& E\{(a_0-b_0)^2\} + 2E\{(a_0-b_0)(a_1-b_1)\}x \\
& + [2E\{(a_0-b_0)(a_2-b_2)\} + E\{(a_1-b_1)^2\}]x^2 \\
& + 2[E\{(a_1-b_1)(a_2-b_2)\} + E\{(a_0-b_0)(a_3-b_3)\}]x^3 \\
& + [2E\{(a_1-b_1)(a_3-b_3)\} + E\{(a_2-b_2)^2\}]x^4 \\
& + 2E\{(a_2-b_2)(a_3-b_3)\}x^5 + E\{(a_3-b_3)^2\}x^6 \qquad (3)
\end{aligned}
$$

Now, if we define the ideal coefficient vector $\mathbf{a}$ and the estimated coefficient vector $\mathbf{b}$ by

$$
\mathbf{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad (4)
$$

and consider the coefficient estimation error covariance matrix

$$
G = E\{(\mathbf{a}-\mathbf{b})(\mathbf{a}-\mathbf{b})^T\} = [g_{ij}]; \quad i,j = 1,2,3,4 \qquad (5)
$$

then Eq. (3) can be more compactly expressed as

$$
\begin{aligned}
E\{(y-\hat{y})^2/x\} &= g_{11} + 2g_{12}x + [2g_{13} + g_{22}]x^2 \\
&+ 2[g_{23} + g_{14}]x^3 + [2g_{24} + g_{33}]x^4 \\
&+ 2g_{34}x^5 + g_{44}x^6 \qquad (6)
\end{aligned}
$$

This sixth-order polynomial yields the desired mean-square error for a specific value of AGC voltage once we have determined the matrix G.

2. ***Second-order curve fitting.*** For a given value of x the ideal power level is again given by Eq. (1). However, we now estimate this power level by

$$
\hat{y} = b_0' + b_1'x + b_2'x^2
$$

The conditional mean-square error is then given by

$$
\begin{aligned}
E\{(y-\hat{y})^2/x\} &= E\{(a_0-b_0')^2\} + 2E\{(a_0-b_0')(a_1-b_1')\}x \\
&+ [2E\{(a_0-b_0')(a_2-b_2')\} + E\{(a_1-b_1')^2\}]x^2 \\
&+ 2[a_3E\{a_0-b_0'\} + E\{(a_1-b_1')(a_2-b_2')\}]x^3 \\
&+ [2a_3E\{a_1-b_1'\} + E\{(a_2-b_2')^2\}]x^4 \\
&+ 2a_3E\{a_2-b_2'\}x^5 + a_3^2x^6 \qquad (7)
\end{aligned}
$$

If we define the vectors

$$
\mathbf{a}' = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} \text{ and } \mathbf{b}' = \begin{bmatrix} b_0' \\ b_1' \\ b_2' \end{bmatrix} \qquad (8)
$$

and consider the coefficient estimation error covariance matrix

$$
H = E\{(\mathbf{a}'-\mathbf{b}')(\mathbf{a}'-\mathbf{b}')^T\} = [h_{ij}]; \quad i,j = 1,2,3 \qquad (9)
$$

then the mean-square error polynomial becomes

$$
\begin{aligned}
E\{(y-\hat{y})^2/x\} &= h_{11} + 2h_{12}x + [2h_{13} + h_{22}]x^2 \\
&+ 2[a_0a_3 - a_3E\{b_0'\} + h_{23}]x^3 \qquad (10) \\
&+ [2a_1,a_3 - 2a_3E\{b_1'\} + h_{33}]x^4 \\
&+ 2a_3[a_2 - E\{b_2'\}]x^5 + a_3^2x^6
\end{aligned}
$$

Note that this time we must not only compute the covariance matrix of the coefficient estimation error but also the mean of the estimated coefficients as well.

3. ***First-order curve fitting.*** Defining the power level estimate as $\hat{y} = b_0'' + b_1''x$ and repeating the above steps we have that

$$
\begin{aligned}
E\{(y-\hat{y})^2/x\} &= f_{11} + 2f_{12}x + [2a_0a_2 - 2a_2E\{b_0''\} + f_{22}]x^2 \\
&+ 2[a_0a_3 - a_3E\{b_0''\} + a_1a_2 - a_2E\{b_1''\}]x^3 \\
&+ [a_2^2 + 2a_1a_3 - 2a_3E\{b_1''\}]x^4 \\
&+ 2a_2a_3x^5 + a_3^2x^6 \qquad (11)
\end{aligned}
$$

where f_{ij}, $i,j = 1, 2$ are the elements of the coefficient estimation error covariance matrix

$$
F = E\{(\mathbf{a}''-\mathbf{b}'')(\mathbf{a}''-\mathbf{b}'')^T\} = [f_{ij}] \qquad (12)
$$

and the vectors $\mathbf{a}''$ and $\mathbf{b}''$ are defined by

$$
\mathbf{a}'' = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \quad \mathbf{b}'' = \begin{bmatrix} b_0'' \\ b_1'' \end{bmatrix} \qquad (13)
$$

C. Derivation of the Mean and Error Covariance of the Estimated Coefficients

To evaluate the statistical properties of the estimated coefficient vectors we must determine the effects of the calibration data errors on the curve fitting routines. Let us consider a set of established power levels y_i, $i = 1,2, \cdots, n$.

For each y_i there corresponds some ideal AGC voltage x'_i such that

$$y_i = a_0 + a_1 x'_i + a_2(x'_i)^2 + a_3(x'_i)^3$$

However, when y_i is established during the calibration phase, the computer (DIS or TCP) measures the AGC voltage $x_i = x'_i + \Delta_i$ where it is assumed that Δ_i is a zero mean gaussian random variable with $E(\Delta_i^2) = \sigma_i^2$. Furthermore, we assume $E\{\Delta_i \Delta_j\} = 0$ for $i \neq j$. In order to consider the calibration error problem as a parameter estimation problem we need to translate the error in the AGC voltage into an equivalent error in the carrier power level. Therefore, let us consider some ideal carrier power y_{0i} which corresponds to the measured value of AGC voltage; that is

$$y_{0i} = a_0 + a_1 x_i + a_2 x_i^2 + a_3 x_i^3$$

Substituting $x_i = x'_i + \Delta_i$

$$y_{0i} = y_i + [a_1 + 2a_2 x'_i + 3a_3(x'_i)^2] \times \Delta_i + (a_2 + 3a_3 x'_i)\Delta_i^2 + a_3 \Delta_i^3$$

Substituting again for x'_i and solving for y_i yields

$$y_i = y_{0i} - (a_1 + 2a_2 x_i + 3a_3 x_i^2) \times \Delta_i + (a_2 + 3a_3 x_i)\Delta_i^2 - a_3 \Delta_i^3 \qquad (14)$$

Thus, the power level supplied to the computer can be considered as consisting of an ideal power level y_{0i} plus an error term where both depend on the known (sampled) value of the AGC voltage.

When all of the data have been entered into the computer the curve fitting program first places the AGC voltage values in a matrix of the form (Ref. 3)

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^m \\ 1 & x_2 & x_2^2 & \cdots & x_2^m \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 1 & x_n & x_n^2 & \cdots & x_n^m \end{bmatrix} \qquad (15)$$

where the number of rows equals the number of data pairs entered and the number of columns m is one larger than the degree of polynomial to be fitted. The program also arranges the signal power levels in a vector of the form

$$\mathbf{Y} = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \qquad (16)$$

The coefficients of the fitted polynomial are then determined by the expression

$$\mathbf{b} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y} \qquad (17)$$

where $\mathbf{b}$ is the estimated coefficient vector

$$\mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_m \end{bmatrix}$$

and b_i are the fitted coefficients of x^i. We can now consider the statistics of $\mathbf{b}$ for the three curve fitting algorithms. For notational simplicity, it is assumed that the $\mathbf{X}$ matrix has been restricted to the proper dimension for the degree of polynomial to be estimated (i.e., the number of columns is one larger than the degree of the fitted curve).

1. Third-order curve fitting. For the third-order algorithm we only need to compute the covariance matrix of the coefficient estimation error G. From Eqs. (14) and (16) we see immediately that the vector $\mathbf{Y}$ can be expressed as

$$\mathbf{Y} = \mathbf{Y}_0 - \boldsymbol{\alpha} \qquad (18)$$

where

$$\mathbf{Y}_0 = \begin{bmatrix} y_{01} \\ y_{02} \\ \cdot \\ \cdot \\ \cdot \\ y_{0n} \end{bmatrix} \qquad (19)$$

and the error vector $\boldsymbol{\alpha}$ is given by

$$\boldsymbol{\alpha} = \begin{bmatrix} (a_1 + 2a_2x_1 + 3a_3x_1^2)\Delta_1 - (a_2 + 3a_3x_1)\Delta_1^2 + a_3\Delta_1^3 \\ (a_1 + 2a_2x_2 + 3a_3x_2^2)\Delta_2 - (a_2 + 3a_3x_2)\Delta_2^2 + a_3\Delta_2^3 \\ \cdot \qquad\qquad \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \qquad\qquad \cdot \\ \cdot \qquad\qquad \cdot \qquad\qquad \cdot \\ (a_1 + 2a_2x_n + 3a_3x_n^2)\Delta_n - (a_2 + 3a_3x_n)\Delta_n^2 + a_3\Delta_n^3 \end{bmatrix} \qquad (20)$$

Now, if we use the definitions of **a** and **b** given in Eq. (4) and note that

$$\mathbf{Y}_0 = \mathbf{X}\,\mathbf{a} \tag{21}$$

then Eq. (17) becomes

$$\mathbf{b} = \mathbf{a} - (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\boldsymbol{\alpha} \tag{22}$$

and the covariance matrix is given by

$$G = E\{(\mathbf{a}-\mathbf{b})(\mathbf{a}-\mathbf{b})^T\} = (X^TX)^{-1}X^T\,E\{\boldsymbol{\alpha}\boldsymbol{\alpha}^T\}X(X^TX)^{-1} \tag{23}$$

Now consider the components of the matrix $E\{\boldsymbol{\alpha}\boldsymbol{\alpha}^T\}$. From Eq. (20) if we let α_i be the i^{th} component of $\boldsymbol{\alpha}$ then we have for $i \neq j$

$$E\{\alpha_i\alpha_j\} = (a_2 + 3a_3x_i)(a_2 + 3a_3x_j)\,\sigma_i^2\,\sigma_j^2 \tag{24}$$

and for $i = j$

$$\begin{aligned} E\{\alpha_i^2\} = {} & (a_1 + 2a_2x_i + 3a_3x_i^2)\sigma_i^2 \\ & + 3[(a_2 + 3a_3x_i)^2 + 2(a_1 + 2a_2x_i + 3a_3x_i^2)]\,(\sigma_i^2)^2 \\ & + 15\,a_3^2\,(\sigma_i^2)^3 \end{aligned} \tag{25}$$

Or, by defining the matrix $C = [C_{ij}];\ i,j = 1,2,\cdots,n$ where

$$C_{ij} = E\{\alpha_i\alpha_j\}$$

then the final form of the covariance matrix is

$$G = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,C\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1} \tag{26}$$

2. Second-order curve fitting. To evaluate the errors associated with the second-order algorithm we must compute the mean of the estimated coefficient vector as well as the error covariance matrix. We still have the relationship

$$\mathbf{Y} = \mathbf{Y}_0 - \boldsymbol{\alpha}$$

However, due to the decreased dimension of the **X** matrix, Eq. (21) is no longer valid. We can nevertheless use the definitions given in Eq. (8) and show that

$$\mathbf{Y} = \mathbf{X}\mathbf{a}' + a_3\mathbf{x}^3 - \boldsymbol{\alpha} \tag{27}$$

where the vector $\mathbf{x}^3$ is given by

$$\mathbf{X}^3 = \begin{bmatrix} x_1^3 \\ x_2^3 \\ \cdot \\ \cdot \\ \cdot \\ x_n^3 \end{bmatrix} \tag{28}$$

Making this substitution Eq. (17) becomes

$$\mathbf{b}' = \mathbf{a}' + (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,(a_3\mathbf{x}^3 - \boldsymbol{\alpha}) \tag{29}$$

From this equation we see that the expected value of $\mathbf{b}'$ is

$$E\{\mathbf{b}'\} = \mathbf{a}' + (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,(a_3\mathbf{x}^3 - E\{\boldsymbol{\alpha}\}) \tag{30}$$

or by defining

$$\alpha_i' = a_3x_i^3 + (a_2 + 3a_3x_i)\sigma_i^2 \tag{31}$$

then

$$E\{b_j'\} = a_j + \sum_{k=1}^{n} U_{j+1,\,k}\,\alpha_k', \qquad j = 0,1,2 \tag{32}$$

where $U_{i,j}$ is the $(i,j)^{\text{th}}$ element of the matrix $(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$.

For the error covariance matrix we have from Eq. (9)

$$\begin{aligned} H &= E\{(\mathbf{a}'-\mathbf{b}')(\mathbf{a}'-\mathbf{b}')^T\} \\ &= (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,E\{(a_3\mathbf{x}^3 - \boldsymbol{\alpha})(a_3\mathbf{x}^3 - \boldsymbol{\alpha})^T\}\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1} \end{aligned} \tag{33}$$

Considering the components of this expectation we have

$$\begin{aligned} & E\{(a_3x_i^3 - \alpha_i)(a_3x_j^3 - \alpha_j)\} \\ & \quad = a_3^2x_i^3x_j^3 - a_3x_j^3\,E\{\alpha_i\} - a_3x_i^3\,E\{\alpha_j\} + E\{\alpha_i\alpha_j\} \end{aligned} \tag{34}$$

Using Eq. (24) we see that for $i \neq j$

$$\begin{aligned} & E\{(a_3x_i^3 - \alpha_i)(a_3x_j^3 - \alpha_j)\} \\ & \qquad = a_3^2x_i^3x_j^3 + a_3x_i^3\,(a_2 + 3a_3x_j)\,\sigma_j^2 \\ & \qquad + a_3x_j^3\,(a_2 + 3a_3x_i)\,\sigma_i^2 \\ & \qquad + (a_2 + 3a_3x_i)(a_2 + 3a_3x_j)\,\sigma_i^2\,\sigma_j^2 \end{aligned} \tag{35}$$

and with the aid of Eq. (25) we have for $i = j$

$$E\{(a_3 x_i^3 - \alpha_i)^2\}$$
$$= a_3^2 x_i^6 + [2a_3 x_i^3 (a_2 + 3a_3 x_i) + (a_1 + 2a_2 x_i + 3a_3 x_i^2)]\,\sigma_i^2$$
$$+ 3[(a_2 + 3a_3 x_i)^2 + 2a_3(a_1 + 2a_2 x_i + 3a_3 x_i^2)]\,(\sigma_i^2)^2$$
$$+ 15\,a_3^2 (\sigma_i^2)^3 \tag{36}$$

As in the previous case we will define a matrix

$$C' \doteq [C'_{ij}]; \qquad i,j = 1,2,\cdots, n$$

where

$$C'_{ij} = E\{(a_3 x_i^3 - \alpha_i)(a_3 x_j^3 - \alpha_j)\}$$

so that the error covariance matrix can be more compactly given by

$$H = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\, C'\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1} \tag{37}$$

3. First-order curve fitting. The procedure for the first order algorithm is essentially the same as the second order case above. That is, we use the definitions of $\mathbf{a}''$ and $\mathbf{b}''$ in Eq. (13), recognize that

$$\mathbf{Y}_0 = \mathbf{X}\,\mathbf{a}'' + a_2\mathbf{x}^2 + a_3\mathbf{x}^3 - \boldsymbol{\alpha} \tag{38}$$

and use Eq. (17) to obtain

$$\mathbf{b}'' = \mathbf{a}'' + (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,(a_2\mathbf{x}^2 + a_3\mathbf{x}^3 - \boldsymbol{\alpha})$$

where

$$\mathbf{x}^2 = \begin{bmatrix} x_1^2 \\ x_2^2 \\ \cdot \\ \cdot \\ \cdot \\ x_n^2 \end{bmatrix} \tag{39}$$

The expected value of $\mathbf{b}''$ is

$$E\{\mathbf{b}''\} = \mathbf{a}'' + (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\,(a_2\mathbf{x}^2 + a_3\mathbf{x}^3 - E\{\boldsymbol{\alpha}\}) \tag{40}$$

or in terms of its components

$$\mathrm{E}\{b_j''\} = a_j + \sum_{k=1}^{n} U'_{j+1,\,k}\,\alpha_k'' \tag{41}$$

where $U_{i,j}$ is the $(i,j)^{\text{th}}$ component of $(\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T$ and

$$\alpha_k'' = a_2 x_k^2 + a_3 x_k^3 + (a_2 + 3a_3 x_k)\,\sigma_k^2 \tag{42}$$

Likewise, the covariance matrix F is given by

$$F = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\, E\{(a_2\mathbf{x}^2 + a_3\mathbf{x}^3 - \boldsymbol{\alpha})$$
$$\times\,(a_2\mathbf{x}^2 + a_3\mathbf{x}^3 - \boldsymbol{\alpha})^T\}\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1} \tag{43}$$

where we have for $i \neq j$

$$E\{(a_2 x_i^2 + a_3 x_i^3 - \alpha_i)(a_2 x_j^2 + a_3 x_j^3 - \alpha_j)\}$$
$$= x_i^2 x_j^2\,[a_2^2 + a_3^2\, x_i x_j + a_2 a_3 (x_i + x_j)]$$
$$+ (a_2 x_j^2 + a_3 x_j^3)(a_2 + 3a_3 x_i)\,\sigma_i^2$$
$$+ (a_2 x_i^2 + a_3 x_i^3)(a_2 + 3a_3 x_j)\,\sigma_j^2$$
$$+ (a_2 + 3a_3 x_i)(a_2 + 3a_3 x_j)\,\sigma_i^2\sigma_j^2 \tag{44}$$

and for $i = j$

$$E\{(a_2 x_i^2 + a_3 x_i^3 - \alpha_i)^2\}$$
$$= (a_2 + a_3 x_i)^2 x_i^4$$
$$+ [2x_i^2(a_2 + a_3 x_i)(a_2 + 3a_3 x_i) + (a_1 + 2a_2 x_i + 3a_3 x_i^2)]\,\sigma_i^2$$
$$+ 3[(a_2 + 3a_3 x_i)^2 + 2a_3(a_1 + 2a_2 x_i + 3a_3 x_i^2)]\,(\sigma_i^2)^2$$
$$+ 15\,a_3^2\,(\sigma_i^2)^3 \tag{45}$$

Using a more compact notation

$$\mathrm{H} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\, C''\mathbf{X}(\mathbf{X}^T\mathbf{X})^{-1} \tag{46}$$

where

$$C'' = [C''_{ij}]; \qquad i,j = 1,2,\cdots, n$$

and

$$C''_{ij} = E\{(a_2 x_i^2 + a_3 x_i^3 - \alpha_i)(a_2 x_j^2 + a_3 x_j^3 - \alpha_j)\} \tag{47}$$

D. Calculations

A computer program was written to evaluate the coefficients of Eqs. (6), (10) and (11) using the appropriate covariance matrices (Eqs. 26, 37 and 46) and coefficient mean expressions (Eqs. 32 and 41). To determine the DIS computer calibration errors, calibration data sets for each of six Deep Space Stations were compiled. Each data set consisted of ten uniformly spaced values of AGC voltage covering the full dynamic range of the associated receiver as well as the variance for each value of voltage. For the sake of definition the "full dynamic range" was defined as AGC voltages corresponding to signal power levels from -110 dBmW to 4 dB above threshold. The resulting sixth-order polynomials in x (AGC voltage) were then plotted for AGC voltages in the range from 0 to -8.0 V. Figure 1 shows these results where the number associated with each curve indicates the degree of the curve-fitting algorithm. Similar plots were obtained for each of the other data sets.

To better understand the behavior of the DIS calibration errors, the above quantities were recomputed using semi-full and restricted range calibration data sets. The semi-full range consisted of ten uniformly spaced values covering power levels from -110 dBmW to -160 dBmW. The restricted range data set consisted of ten uniformly spaced values covering the power range from -130 dBmW to -150 dBmW. The results are shown in Figs. 2 and 3.

The calibration accuracy expressions (Eqs. 10 and 11 only) were also evaluated for the TCP calibration method. To accomplish this, data sets of three AGC values covering an 8-dB signal power range and centered at -144 dBmW were formed. A typical result is shown in Fig. 4.

Figures 1 to 4 may be of interest in showing how the calibration errors behave but are of limited use in making quantitative comparisons of calibration techniques. What is needed is some kind of a figure of merit for each calibration technique. The most logical quantity is to consider the integral mean square error defined by

$$I = \frac{1}{x_u - x_v} \int_{x_v}^{x_u} E\{(y - \hat{y})^2/x\}\, dx \qquad (48)$$

x_u and x_v are normally taken to be the extreme values of AGC voltage associated with the calibration data range. This quantity was computed for each of the six stations and each of the calibration techniques discussed above. Then an average over the six stations was taken to determine the figure of merit (actually figure of error) for each technique. A comparison of these quantities for the DIS computer is given in Table 1.

Comparison of these quantities for TCP and DIS computers might be somewhat misleading due to the large differences in calibration ranges. Consequently, Eq. (48) was reevaluated for the DIS using values of x_u and x_v which correspond to the calibration limits of the TCP. The results are given in Tables 2, 3 and 4.

We see from Table 1 that the present method of DIS calibration is quite inaccurate and in fact produces a calibration error standard deviation in excess of 0.3 dB. Comparing the DIS and TCP in Table 2 we find that the DIS has a calibration error more than two orders of magnitude larger than the TCP. Also from Tables 1 and 2 we see that the DIS error can be significantly reduced by using the third order algorithm in the DIS.

The DIS error can be further reduced by restricting its calibration range. From Table 3 it is evident that the DIS and TCP have equivalent calibration errors when the DIS uses the third order method and the semi-full calibration range. By further restriction of the DIS range we see from Table 4 that the DIS actually becomes more accurate.

III. Total Carrier Power Estimation Accuracy

The total estimation mean square error will be the sum of the calibration and the operational mean square errors. The operational errors include the effects of AGC variation (from receiver front end noise), operational AGC sampling and A/D converter quantization. Evaluation of the operational errors results in the curves shown in Fig. 5 (for the DIS) and Fig. 6 (for the TCP).

Also shown in Figs. 5 and 6 are certain calibration errors. In Fig. 5, we see that for the present method of calibration, the DIS estimation accuracy is completely dominated by the calibration accuracy and is much more inaccurate than the TCP method of carrier power estimation. Furthermore, the accuracy of the DIS can be significantly improved by using the techniques described above. For example, by using the semi-full calibration range and the third-order algorithm, the standard deviation of the DIS estimation error will drop from approximately 0.3 to about 0.01 dB, allowing the TCP and DIS to have approximately the same accuracies.

IV. Conclusion

In this article the expressions for the mean square calibration errors of the TCP and DIS computer algorithms used in carrier power estimation were presented and evaluated. When these results were combined with the associated operational errors, it was found that for the present calibration methods the TCP method of estimation is far more accurate than the DIS method. Also, it was noticed that the accuracy of the DIS method can be significantly improved by a slight alteration of the DIS calibration method.

References

1. Luenberger, D. G., *Optimization by Vector Space Methods,* Chapt. 4. John Wiley & Sons, Inc., New York, 1969.

2. Lawson, C. L., "Least Squares Polynomial Fit to Data, S.P.," *JPL Fortran V Subroutine Directory,* 900-291, Edition No. 4.0, Section 10.1.2 (JPL internal document).

3. Cornahan, B., Luther, H. A., and Wilkes, J. O., *Applied Numerical Methods,* Chapt. 8. John Wiley & Sons, Inc., New York, 1969.

Table 1. DIS computer mean square calibration error

AGC Band-width	Degree of curve fit	Integral mean square calibration error, (dB)^2		
		Full calibration range	Semi-full calibration range	Restricted calibration range
Narrow	3	4.67×10^{-3}	1.78×10^{-4}	2.97×10^{-5}
	2	$*9.35 \times 10^{-2}$	5.51×10^{-2}	9.47×10^{-4}
Medium	3	2.24×10^{-2}	8.23×10^{-4}	1.38×10^{-4}
	2	$*1.11 \times 10^{-1}$	5.59×10^{-2}	1.05×10^{-3}
Wide	3	3.13×10^{-2}	1.04×10^{-3}	1.67×10^{-4}
	2	$*1.26 \times 10^{-1}$	5.60×10^{-2}	1.08×10^{-3}

* = present method of calibration.

Table 2. Comparison of DIS and TCP computer mean square calibration errors when the DIS is calibrated using the full data range

AGC bandwidth	DIS calibration error, (dB)^2		TCP calibration error, (dB)^2
	NFIT = 3	NFIT = 2	
Narrow	1.88×10^{-3}	$*6.40 \times 10^{-2}$	1.66×10^{-4}
Medium	4.58×10^{-3}	$*6.66 \times 10^{-2}$	1.63×10^{-4}
Wide	5.82×10^{-3}	$*6.75 \times 10^{-2}$	1.62×10^{-4}

* = present method of calibration.

Table 3. Comparison of DIS and TCP computer mean square calibration errors when the DIS is calibrated using the semi-full data range

AGC bandwidth	DIS calibration error, (dB)^2		TCP calibration error, (dB)^2
	NFIT = 3	NFIT = 2	
Narrow	5.31×10^{-5}	9.51×10^{-2}	1.66×10^{-4}
Medium	2.46×10^{-4}	9.73×10^{-2}	1.63×10^{-4}
Wide	3.08×10^{-4}	9.73×10^{-2}	1.62×10^{-4}

Table 4. Comparison of DIS and TCP computer mean square calibration errors when the DIS is calibrated using the restricted data range

AGC bandwidth	DIS calibration error, (dB)^2		TCP calibration error, (dB)^2
	NFIT = 3	NFIT = 2	
Narrow	9.16×10^{-6}	4.98×10^{-4}	1.66×10^{-4}
Medium	4.23×10^{-5}	5.24×10^{-4}	1.63×10^{-4}
Wide	5.28×10^{-5}	5.32×10^{-4}	1.62×10^{-4}

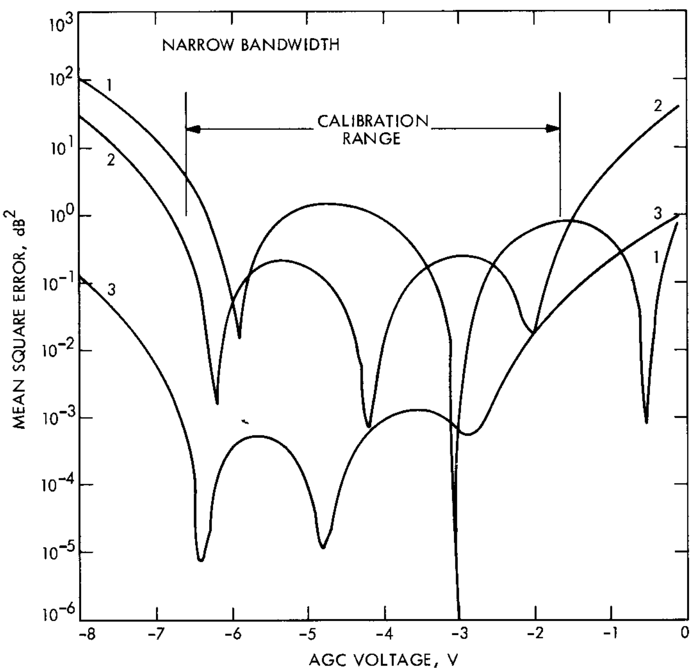

Fig. 1. DIS mean square calibration error when calibrated over full dynamic range for: (1) first-order, (2) second-order, and (3) third-order curve fitting

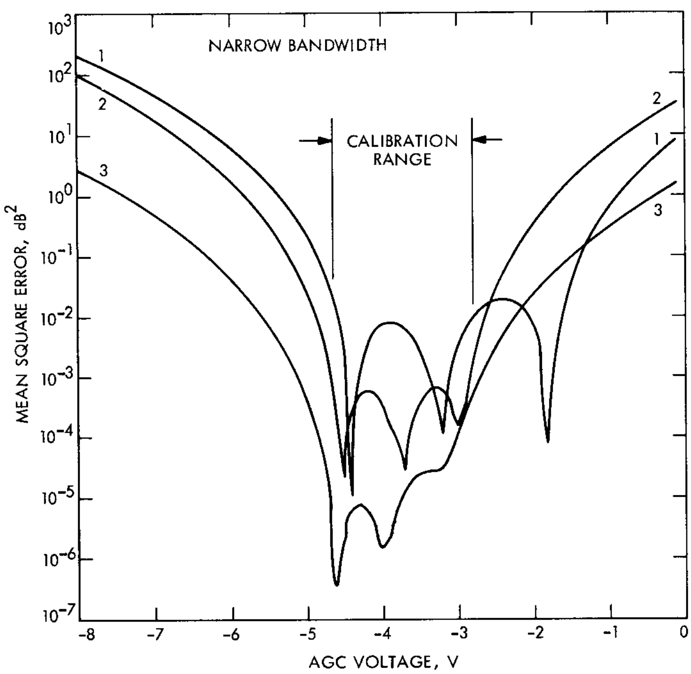

Fig. 3. DIS mean square calibration error when calibrated over restricted range for: (1) first-order, (2) second-order and (3) third-order curve fitting

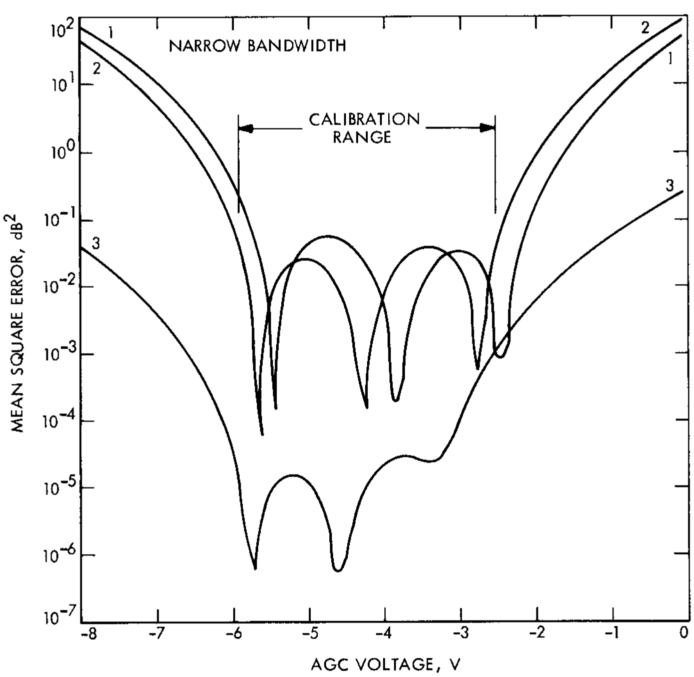

Fig. 2. DIS mean square calibration error when calibrated over semi-full dynamic range for: (1) first-order, (2) second-order, and (3) third-order curve fitting

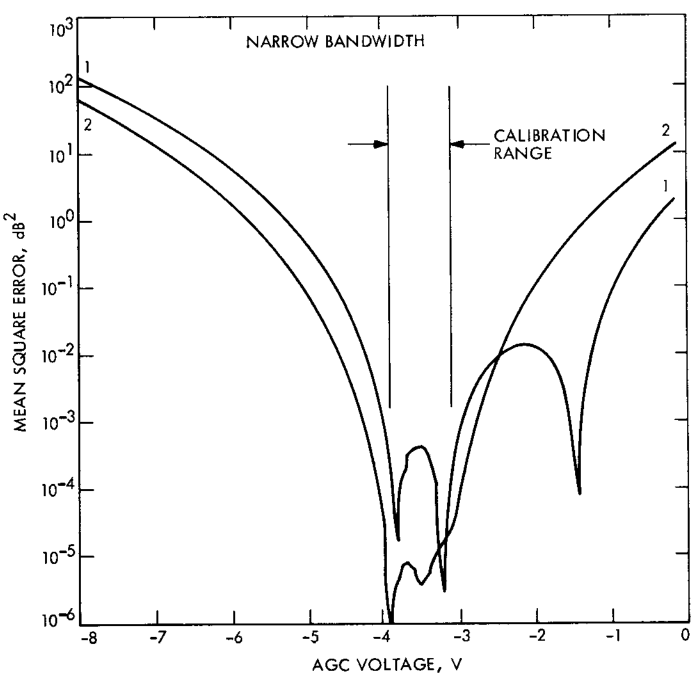

Fig. 4. TCP mean square calibration error for: (1) first-order, and (2) second-order curve fitting

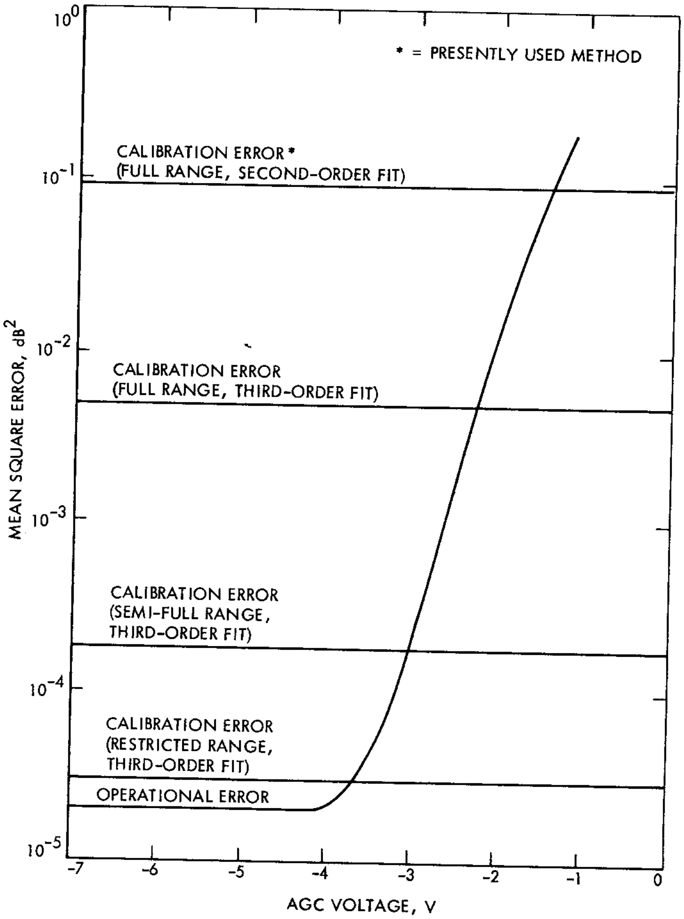

Fig. 5. Comparison of the DIS calibration and operational errors

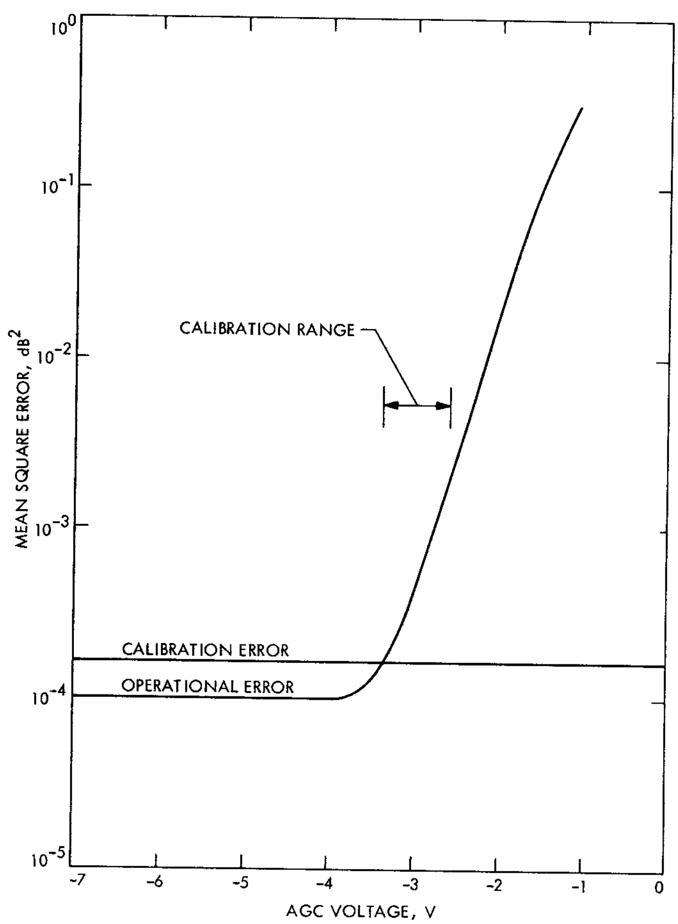

Fig. 6. Comparison of the TCP calibration and operational errors