On A Communications Strategy for Channels With Unknown Capacity

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When the capacity of a channel cannot be determined in advance, or if it can change unpredictably, there is a problem in selecting the rate of transmission. On the one hand, a design for the worst case prevents recovery of large amounts of data when the situation is better than anticipated. On the other hand, a design based on optimistic assumptions is threatened by total failure when the conditions are bad. This article discusses a strategy that covers the in-between range of possibilities, and its implementation on Mariner-type telemetry systems. It is shown that large increases in expected data rate can be obtained at the cost of small reductions in the minimum rate. Final decisions are made on the ground after all the data are received.

I. Introduction

In this article we shall be concerned with the question of how to maximize the expected data return when the channel capacity cannot be predicted in advance. An example of this type is a planetary entry probe to Venus or Jupiter, whose atmospheric transmission characteristics are not yet fully known. And even if they were known, these transmission characteristics would be subject to unpredictable changes due to planet weather and/or entry trajectory.

II. A Two-State Channel

Consider a situation where it is known that the channel capacity will be either 10 bits/s or 100 bits/s. Furthermore, suppose that the probability of 10 is λ so that the probability of 100 is $\overline{\lambda} = 1 - \lambda$. Also, suppose that the above information is all that is known prior to launch of a spacecraft whose mission is to explore the atmosphere of a planet by means of an atmospheric entry probe that burns. At what data rate should the probe transmit? No communication is assumed from Earth to probe, and final data rate decisions must be made on the ground, after all the data are in.

III. The Conservative Strategy

A conservative strategy is based on the knowledge that transmission at a rate greater than channel capacity is impossible. Thus, if a transmission rate of more than 10 bits/s is adopted and the channel capacity turns out to be only 10 bits/s no information will be received. Consequently, to prevent a calamity, the conservative approach is to transmit at 10 bits/s (we define channel capacity as the rate achievably with known coding/decoding schemes). Unfortunately, this scheme provides only 10 bits/s even when the capacity turns out to have been 100 bits/s. The transmitter cannot determine the channel capacity and adapt its rate, as assumed, because of the short duration of the mission and minimum complexity of the probe. The expected rate is the same as the minimum rate: $E[R] = R_{\min} = 10$ bits/s.

IV. The Aggressive Strategy

An aggressive or gambling approach is to hope for the occurrence of the 100 bit/s situation and elect to transmit at the maximum rate. However, even if the probability $\bar{\lambda}$ of this eventuality is high, say 0.9, there is still a 10 per cent chance that the capacity will be 10 bit/s and no data will be received.

In general, the expected data return is

$$E[R] = 100 \text{ bits/s} \times (\text{probability capacity} = 100 \text{ bits/s})$$

$$+ 0.0 \times (\text{probability capacity} = 10 \text{ bits/s}) = 100\bar{\lambda}$$

Thus, if $\lambda = 0.9$, $E[R] = 90$, which is much higher than the guaranteed 10 bits/s of the conservative strategy. Although the expected return is now much higher, the 10% possibility of a total failure is, to say the least, disquieting, and almost surely unacceptable; even 1% is probably unacceptable.

V. The Prudent Strategy (α-strategy)

In the above examples we have seen two extremes of system design. The question that remains is whether it is possible to eliminate the risk of the gambler as well as the regret of the faint-hearted conservative. It turns out that a trade-off is possible. If one is willing to accept a guaranteed rate less than 10 bits/s it becomes possible to receive at a greater rate when the capacity is 100.

One method for achieving this intermediate state is to time share the rates. The transmitter is designed to transmit at 10 bits/s for a fraction α of the time and at 100 bits/s for $\bar{\alpha} = 1 - \alpha$ of the time. Another way is to allocate the total power P in the above proportions to two subcarriers, one modulated by high rate data using $\bar{\alpha}$ of the power, the other by low rate data using α of the power. Now, the received data rate is 10α if the capacity is 10 but it is $10\alpha + 100\bar{\alpha}$ if capacity is 100. The expected rate is, therefore,

$$\begin{aligned} E[R] &= (10\alpha)\lambda + (10\alpha + 100\bar{\alpha})\bar{\lambda} \\ &= 10\alpha + 100\bar{\alpha}\bar{\lambda} \end{aligned}$$

and the minimum rate is guaranteed to be

$$R_{\min} = 10\alpha$$

Continuing the example with $\lambda = 0.1$, if we can accept an $R_{\min} = 9.0$ we will obtain $E[R] = 18$; i.e., an 80% expected improvement for a 10% risk with a 90% chance of 19 bits/s.

In general, letting C_1 and C_2 designate the low and high capacity values, we have the expected rate vs the guaranteed minimum rate

$$\begin{aligned} E[R] &= \alpha C_1 + \bar{\alpha}\bar{\lambda} C_2 \\ R_{\min} &= \alpha C_1 \end{aligned} \qquad (1)$$

Equation (1) effectively summarizes our three strategies and is plotted in Fig. 1, that of the conservative corresponds to $\alpha = 1$, hence $E[R] = R_{\min}$, while $\alpha = 0$ yields the gamblers approach $E[R] = \bar{\lambda} C_2$, $R_{\min} = 0$.

VI. Extension to Multiple Rates

We shall now find that the α-strategy of transmitting at two rates is optimal even when the number of possible rates is greater than two, provided we restrict ourselves to time-sharing and only constrain the minimum rate. If we restrain rates, also if intermediate conditions occur, then we get a linear programming problem.

Consider a channel with capacities $C_1 \leqq C_2 \leqq \cdots \leqq C_n$ occurring with probability $P(C_k) = \lambda_k$. If we allocate the transmission rates in fractions α_k per C_k the expected rate, for a fixed α_1, is

$$\begin{aligned} E[R] &= \lambda_1 \alpha_1 C_1 + \lambda_2 (\alpha_1 C_1 + \alpha_2 C_2) \\ &\quad + \cdots + \lambda_n (\alpha_1 C_1 + \alpha_2 C_2 + \cdots + \alpha_n C_n) \\ &= \sum_{k=1}^{n} \lambda_k \sum_{i=1}^{k} \alpha_i C_i \\ &= \sum_{k=1}^{n} \alpha_k C_k \sum_{i=k}^{n} \lambda_i \\ &= \alpha_1 C_1 + \sum_{k=2}^{n} \alpha_k C_k \left[\sum_{i=k}^{n} \lambda_i \right] \\ &\leqq \alpha_1 C_1 + (1 - \alpha_1) \max_{k>1} \left[C_k \sum_{i=k}^{n} \lambda_i \right] \end{aligned}$$

The maximum is achieved by allocating the fraction $(1-\alpha_1)$ for transmission at the rate $C_{\text{opt}} = C_m$ such that $C_m(\lambda_m + \lambda_{m+1} + \cdots + \lambda_n) \geqq C_k(\lambda_k + \lambda_{k+1} + \cdots + \lambda_n)$ for all $k > 1$. The fraction α_1 is, as before, determined solely by the minimum rate $R_{\min} = \alpha_1 C_1$.

VII. Equivalent and Mixed Strategies

Note that the strategy may not be unique because the function

$$C_k \sum_{i=k}^{n} \lambda_i$$

may achieve the same maximum for two or more (say M) values of k. In this case there will be an infinite set of equivalent strategies to achieve the maximum expected rate, of which at least M are two-rate strategies. For example, suppose that

$$C_2(\lambda_2 + \lambda_3 + \cdots + \lambda_n) = C_m(\lambda_m + \lambda_{m+1} + \cdots + \lambda_n) > C_k(\lambda_k + \cdots + \lambda_n)$$

for all $k > 1$, $k \neq 2, m$; then

$$\max E[R] = \alpha_1 C_1 + (1-\alpha_1) C_2 \sum_{k=2}^{n} \lambda_k$$

$$= \alpha_1 C_1 + (1-\alpha_1) C_m \sum_{k=m}^{n} \lambda_k$$

$$= \alpha_1 C_1 + (1-\alpha_1)\left[\beta C_2 \sum_{k=2}^{m} \lambda_k + (1-\beta) C_m \sum_{k=m}^{n} \lambda_k\right]$$

where $0 \leqq \beta \leqq 1$. Therefore, the set of optimal strategies is $\{\alpha_1, \alpha_2 = (1-\alpha)\beta\ \alpha_m = (1-\alpha_1)(1-\beta)\}$ for any β in $[0, 1]$, and there are two strategies which can be realized by transmission at only two rates, viz., C_1 and C_2, or C_1 and C_m, ($\beta = 0$ and $\beta = 1$). The rest require a mixture of the above two and transmission at C_1, C_2 and C_m.

VIII. Higher Order Strategies

When faced with a situation which can be achieved by means of a mixed strategy it is possible to employ the previous ideas to select a unique strategy on the basis of secondary preferences. This is best seen by an example: suppose $C_1 < C_2 < C_3$ and $C_2(\lambda_2 + \lambda_3) = C_3\lambda_3$. For this case we can achieve the maximum expected rate while ensuring a minimum rate by allocating $(1-\beta)$ $(1 - R_{\min}/C_1)$ to C_2 and $\beta(1 - R_{\min}/C_1)$ to C_3. Next, we note, on the one hand, that the *probability of receiving more than the minimum rate* is

$$P_r\{R > R_{\min}\} = (1-\beta)(\lambda_2 + \lambda_3) + \beta\lambda_3$$
$$= \lambda_2 + \lambda_3 - \beta\lambda_2$$
$$\leqq \lambda_2 + \lambda_3,$$

and this probability achieves a maximum for $\beta = 0$. On the other hand, the probability of receiving at the highest rate (should it occur) is now nil. Thus, if we wish to *minimize the probability of missing the high-rate situation* we should consider

$$P_r\{R < C_3\} = 1 - \beta\lambda_3$$
$$\geqq 1 - \lambda_3,$$

which is minimal when $\beta = 1$.

It is now evident that by varying β we can vary the apportionment of medium and high rate possibilities without affecting the maximum expected rate. (It is possible to present arguments for or against any such combination.) The most convincing case against mixed strategies, however, is that they require a greater variety of transmission rates, and thus, higher cost and other unavoidable losses, e.g. crossmodulation for frequency multiplexed channels or frame synchronization for time shared channels.

IX. Channels with a Continuum of Capacities

We conclude this topic by noting that the two-rate strategy is also valid for channels with capacity $C \varepsilon [C_1, \infty]$ and an associated probability distribution $F(C) = Pr\{x < C\}$. In this case

$$E[R] = R_{\min} + (1 - R_{\min}/C_1) \sup_C [C(1 - F(C))]$$

and the value of C which achieves the supremum is the maximum transmission rate for us.

X. Implementation on Mariner Type Telemetry Systems

Mariner-type telemetry systems are particularly suited to the implementation of an α-strategy because they have been designed to transmit at two independent rates simultaneously, provided carrier phase coherence can be maintained. However, of the total power transmitted, some must be dedicated to a carrier phase reference, and some is lost due to crossmodulation between the two channels

(Ref. 1). Since the crossmodulation loss is absent in the case of a single channel, it must be included in the evaluation of a two channel strategy as a reduction of channel capacity.

The total power is

$$P_T = P_a + P_b + P_c + P_{cm}$$

where P_a, P_b are the power allocations for the two channels, P_c is the carrier reference power and P_{cm} is the crossmodulation power. The crossmodulation power depends on the modulation scheme as well as upon P_a, P_b and P_c as follows:

$$P_{cm} = \begin{cases} \dfrac{\min(P_a, P_b)\, P_c}{\max(P_a, P_b)} & \text{for Interplex modulation (Ref. 1)} \\ P_a P_b / P_c & \text{for conventional PSK/PCM/PM} \end{cases}$$

Since $P_a P_b = \min(P_a P_b) \max(P_a P_b)$ we have, from above, that Interplex is preferable when $P_c \leqq \max(P_a P_b)$, otherwise the conventional modulation scheme is better.

Now we shall be concerned with the quantity

$$P = P_a + P_b + P_{cm}$$

which represents the maximum amount of power the system can transmit as data, as, for instance, in the case of a single channel where P_{cm} and either P_a or P_b is zero. The practical capacity of the channel is $C = (P/N) \times \text{const}$ where N is the spectral density of additive Gaussian white noise and the constant is about 0.4 for practical encoding/decoding schemes. We are supposing that for one reason or another this channel capacity is either C_1 with probability λ or $C_2 > C_1$ with probability $1 - \lambda = \bar{\lambda}$. This may be due to changes in the strength of the noise (or lack of prior knowledge thereof), or fading of signal strength due to mispointing of the transmitting antenna, caused by meteor impact, or, when entering a planetary atmosphere slowing due to gusts.

The power allocation called forth by the α-strategy is

$$P_a = \alpha (P - P_{cm}) = \alpha P (1 - P_{cm}/P)$$

$$P_b = 1 - \alpha (P - P_{cm}) = \bar{\alpha} P (1 - P_{cm}/P)$$

Therefore, in the case of Interplex modulation,

$$\frac{P_{cm}}{P} = \Delta \frac{\min(\alpha, \bar{\alpha})}{\max(\alpha, \bar{\alpha})}$$

where

$$\Delta = P_c/P,$$

and, in the case of conventional PSK/PM

$$\frac{P_{cm}}{P} = \begin{cases} (2\sqrt{\alpha\bar{\alpha}} - \Delta)/(1 + 2\sqrt{\alpha\bar{\alpha}}) & \text{if } \sqrt{\alpha\bar{\alpha}} \geqq \Delta \\ 1 + \dfrac{\Delta}{2\alpha\bar{\alpha}} - \sqrt{\dfrac{\Delta}{\alpha\bar{\alpha}} + \left(\dfrac{\Delta}{2\alpha\bar{\alpha}}\right)^2} & \text{otherwise} \end{cases}$$

The above expressions for P_{cm}/P must be included in plotting the "operating characteristics" ($E[R]$ vs $R_{\min}$) of the α-strategy, since

$$E[R] = (\alpha C_1 + \bar{\lambda}\bar{\alpha} C_2)(1 - P_{cm}/P)$$

and

$$R_{\min} = \alpha C_1 (1 - P_{cm}/P)$$

Figure 2 is a sample plot of the operating characteristics for the case $C_1 = 10$ bits/s, $C_2 = 100$ bits/s and $\lambda = 0.1$.

It is important to stress that the operating characteristics of Fig. 2 compare only the efficiency of systems with the same net power $P = P_T - P_C$. This is done because the carrier reference power has, until this writing, been set by doppler navigation considerations beyond the control of the telemetry systems engineer. Keeping this in mind it is evident that the overall system efficiency $(P_a + P_b)/P_T$ decreases as Δ increases in both single channel and double channel systems.

Under most normal conditions, however, Δ is small ($\Delta < 0.2$) and Interplex modulation becomes almost as good as ideal time share and may even be better after provisions for timing sync in time share are taken into account.

We conclude this section with an example based on Fig. 2, with $\Delta = 0.1$, which shows that an α-strategy using two-channel Interplex increases the expected data rate from 10 bits/s of the conservative design ($\max R_{\min}$) to 23 bits/s if we can accept a minimum guarantee of 8 bits/s. Moreover, the probability of achieving this 3 + dB gain is 90%. Had C_2/C_1 been equal to 100 the gain would have been in excess of 12 dB.

XI. Conclusion

This article presented a practical approach toward improving the cost effectiveness of deep space telemetry systems for missions in which the transmission rate cannot be ascertained in advance, or altered after launch. The key idea is to back off from the maximum data rate in the worst case in order to have some facility for high rate transmission in case conditions are better.

We conclude by noting that the approach presented here appears to be applicable to situations where the state of the channel may require different signaling methods, i.e., in one state, non-coherent communications may be required while in other states coherent communication, with or without coding, may be possible; or, non-coherent transmission may be required in all cases. The latter could occur if carrier frequency and/or phase synchronism cannot be maintained due to rapidly varying doppler, as when approaching a planet.

Another possibly fruitful endeavor is the development of signaling schemes that can be decoded not only at high rates when conditions are favorable, but, also at low rates to recover part of the transmitted data when conditions are adverse.

Reference

1. Butman, S., and Timor, U., "Interplex Modulation," in *Quarterly Technical Review*, Vol. I, pp. 97–105. Jet Propulsion Laboratory, Pasadena, Calif., January 1971.

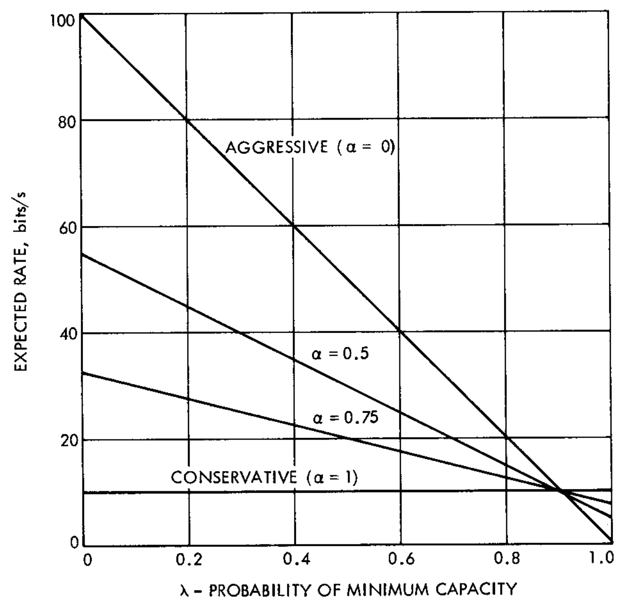

Fig. 1. Illustration of α-strategy

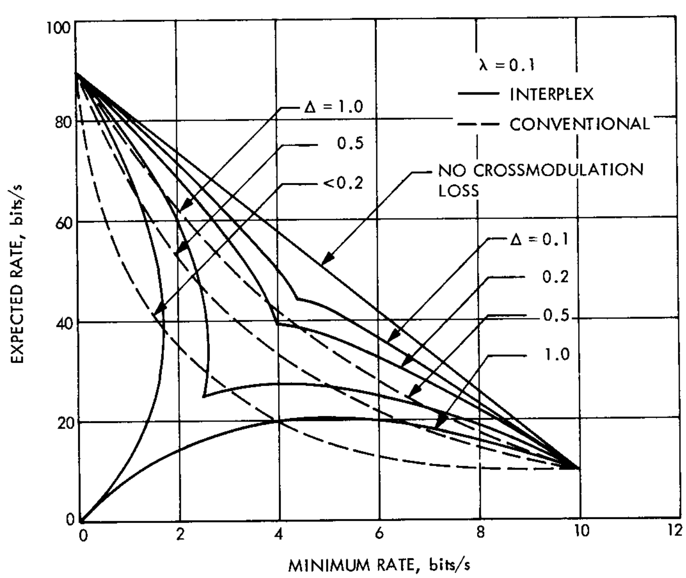

Fig. 2. Operating characteristics of *Mariner* telemetry using α-strategy