An Inventory Policy for the Deep Space Network

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This article describes a proposed inventory and procurement policy for optimal procedures for ordering and allocating for the Network Supply Depot (NSD). The policy defined differs from conventional inventory stockage and resupply systems in that it takes into consideration the inventory status not only at NSD but also at each of the Complex Supply Facilities.

I. Basic Features of the Proposed Policy

For each item the Network Supply Depot (NSD) supplies to the Complex Supply Facilities (CSFs), there are three fundamental questions that the policy must answer:

(1) When should NSD reorder the items?

(2) How much should be ordered?

(3) How should the NSD supply be allocated among the CSFs?

(1) Any policy, such as the present system, in which NSD reorders whenever its own inventory reaches a prescribed minimum is wasteful, incurring excessive inventory costs. This is because NSD's inventory may reach its minimum at a time when the levels of inventory on hand at the CSFs are large. An efficient cost-minimizing policy must base reorder decisions upon the state of the entire system, i.e., the levels of inventory on hand at the CSFs (continuously monitored by the Integrated Logistics System) as well as the NSD level. The proposed policy establishes a safe minimum inventory level for each CSF and requires NSD to reorder whenever one or more CSFs reach the prescribed minimum.

(2) The conventional economic lot size model (Refs. 1 and 2) has been modified to take into account the effects of ordering simultaneously for all CSFs. An equation is set up which expresses for each item the average cost per year incurred in operating the inventory system. The optimal mean time between orders can be calculated, and from this the order size is determined, using estimated mean demand levels for the CSFs.

(3) A mathematical analysis based upon the average cost equation reveals that optimality is achieved by distributing the NSD supply to the CSFs in such a way

as to minimize the average total inventory in the system at reorder time. Since each CSF has at least its prescribed minimum inventory level, this amounts to minimizing on the average the sum of the "excess" inventories (over minimum levels) of the CSFs. The "ideal" situation is one where all the CSFs reach minimum at the same time. However, this ideal is seldom attained because the model assumes the CSF inventories are depleted by orders arriving at random. The next section deals with minimization of excess inventories.

II. Allocation of NSD Supply Among CSFs

It is conceivable that NSD could distribute items to the CSFs one at a time as the CSFs' inventories are depleted, thereby keeping each CSF from accumulating any excess inventory. The time required for shipping and the cost of handling these shipments render such an approach obviously impractical. Two practical approaches are considered. A *no-resupply policy* (NR-policy) and a *one-stage resupply policy* (R-policy) under which NSD distributes a resupply inventory R among the CSFs the first time one of them reaches its minimum.

Under the NR-policy, an optimal allocation for a given order size can be computed by means of an algorithm based only on estimated ratios of mean demand between CSFs. These estimates can be based on demand experienced during previous order cycles. Tables listing these optimal allocations are easily constructed for a variety of demand ratios.

An advantage of the NR-policy is that it permits NSD to process an incoming order and send shipments to the CSFs without maintaining an inventory of its own. However, the excess inventory at reorder time is further reduced by the R-policy, a fact which may justify the cost of additional shipping and handling, particularly for high-cost items. The following example illustrates the comparison of excess inventories between the two policies.

Starting with an initial inventory 142 larger than the sum of the CSF minimum levels, the NR-policy optimum allocation based upon six CSFs with demand ratios 7:8:9:10:11:12 is 19, 21, 23, 25, 26, 28, respectively. The resulting excess inventory averages 34.8. Using the R-policy with $R = 10$, the excess inventory averages 19.5, a reduction of 44%.

III. Setting Minimum Levels

If the demand distribution for a CSF is known, then for each possible value of the minimum stockage level s, one can compute the expected shortage (i.e., demands not satisfied) during the lead period required for resupply. One also computes the average level of inventory resulting from the choice of s. The standard inventory model based upon known demand distribution (Ref. 2) then proceeds by choosing s so as to minimize a performance criterion of the form

$$cs + \mathcal{E}(s) \tag{1}$$

where

$\mathcal{E}(s)$ – expected shortage per lead period

and c depends upon the frequency of lead periods and the relative importance of shortages.

Of course, the true distribution of demand for an individual CSF is not known and must be estimated. A natural approach is first to estimate this distribution and then, using the estimated distribution, choose the optimal s by the standard methods. This approach is unsatisfactory because it fails to weigh the consequences of errors of estimation. To illustrate the nature of the difficulty, let us suppose that the demand is subject to a Poisson distribution with mean demand (per lead period) M, assumed unknown. The usual estimate of M is D/T, where D is the observed demand over a period of time equal to T lead periods. If $M = 10$ and $T = 3$, then the probability that the estimate $D/T = M$ is 0.073, and the probabilities of other values are shown by the heights of the bars in Fig. 1. To each value of D there corresponds a value of $s = s(D/T)$ (chosen optimally for mean demand D/T). As a consequence of using $s(D/T)$ rather than $s(M)$ (the optimal s-value for the true mean demand, M), one incurs a change in expected shortage,

$$E = \mathcal{E}\left(s\left(\frac{D}{T}\right)\right) - \mathcal{E}(s(M)),$$

and a (possible) savings in the inventory term in expression (1),

$$I = c \cdot \left(s(M) - s\left(\frac{D}{T}\right)\right),$$

which are plotted in Fig. 1 for $c = 0.02$. The term *regret* will be used to refer to

$$E - I = \left[c \cdot s\left(\frac{D}{T}\right) + \mathcal{E}\left(s\left(\frac{D}{T}\right)\right)\right] - [c \cdot s(M) + \mathcal{E}(s(M))]$$

which is the increase in expression (1) resulting from using $s = s(D/T)$ rather than the $s = s(M)$ (recall that since M is the actual mean demand, $s = s(M)$ minimizes expression (1)). The effect of errors of estimation (deviations of D/T from M) are apparent from Fig. 1. While the regret increases as D/T moves away from M in either direction, note that the regret rises much more sharply for D/T less than M, i.e., underestimating M is much more costly than overestimating M. The explanation of this profound asymmetry is that the term $c \cdot s$ grows linearly with s, while $\mathcal{E}(s)$ is nonlinear, being very small for s comfortably larger than M (when the probability of incurring any shortage is small), but rising steeply as s decreases to M and below (where the expected shortage is roughly $M - s$).

Obviously, what would be desirable is a procedure for setting s (based on D and T) that minimizes the average regret. The problem is complicated because the average regret of a given procedure depends on the true M and no procedure minimizes it for all M. Application of statistical decision theory leads to a class of readily computable procedures which have certain optimal properties. Selection of procedures within this class is greatly facilitated by a specially developed computer program which evaluates and plots average regret as a function of M. One can thus select a procedure whose performance is reasonably good for a broad range of values of M and is particularly strong for M in a smaller range where the mean demand is thought to lie. Preliminary investigations have led to some useful formulas for choosing s.

Several refinements of this basic approach have already been made and incorporated into the general policy and associated computer programs. One of these concerns the modifications required to deal with items which are dispensed by a CSF in different quantities, e.g., one or two items, a dozen, or two dozen. Another concerns the problem of sporadic demand. Any procedure for setting s based on observed demand over time T may encounter, particularly if T is small, the difficulty that only small-size orders were observed during time T but large orders may occur occasionally. This calls for a substantially larger s to keep the expected shortage under control. A flexible and efficient way of setting s in such cases has been developed, and its performance studied using the computer evaluation program referred to above.

References

1. *The Economic Order Quantity Principle & Applications.* GSA Handbook, FMPR 101-27.1, May 1966.

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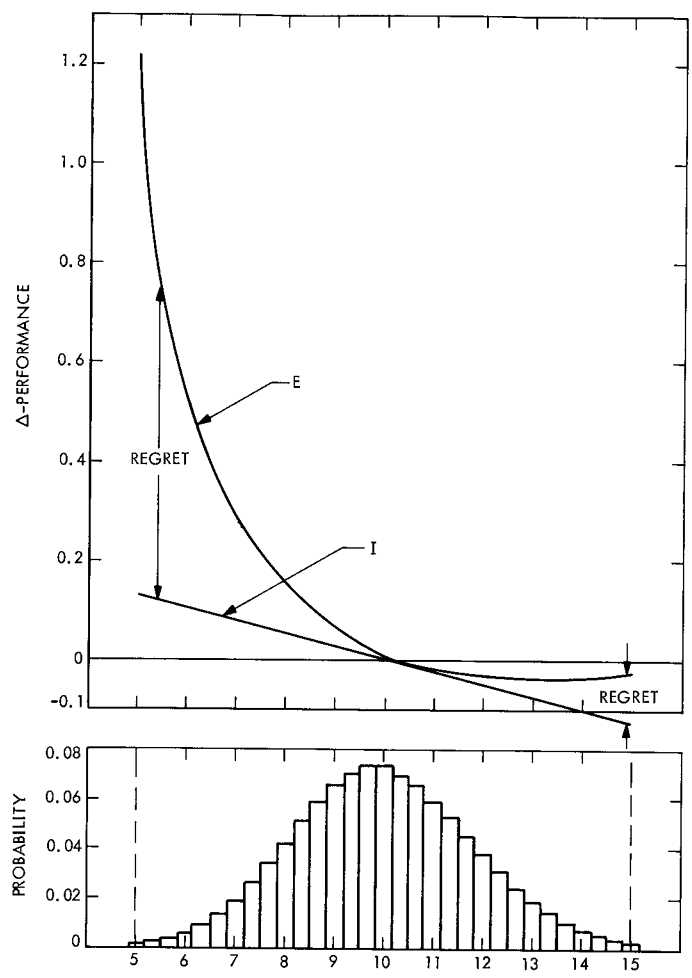

Fig. 1. Effect of the error in estimating demand