Analysis of Dual-Frequency Calibration for Spacecraft VLBI

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In this article a feasibility study is undertaken and a detailed analysis is made of a wide-band very long baseline interferometer (VLBI) for the purpose of ranging and tracking a spacecraft. The system works on two frequencies (S- and X-band). By a new correlation technique, it is shown that it is possible to extract information on the total electron content with a rather high degree of accuracy, an accuracy certainly impossible to achieve with tracking modes currently in use. The total electron content and its time variation are valuable quantities on their own; they give important information on the solar wind. It is also shown that at the same time, potentially, the determination of the declination and right ascension of a spacecraft can be made much more accurately than by existing procedures.

I. Introduction

The ever increasing complexity of future unmanned space missions engenders an ever increasing need for higher accuracy with respect to tracking and ranging a spacecraft. Accordingly, here at JPL, a number of new tracking modes are currently under investigation. Before delving into this, let us first comment on the customary methods employed as of this writing. The range ρ (the distance between the spacecraft and an earthbound observer) is simply measured by measuring the round trip time between the observer and the spacecraft. This is achieved by transmitting a range code (a particular type of modulation of the S-band carrier) and comparing the received range code transmitted back from the spacecraft with the internally generated range code carefully calibrated with an internal clock (usually a Rb-standard, but recently H-masers, which are two orders of magnitude more accurate, are more and more the vogue). This round trip time, when corrected for tropospheric, ionospheric, and other influences on the electromagnetic radiation, is a direct measure of the range. On the other hand, the range rate $\dot{\rho}$ is determined by measuring the doppler shift, in other words by measuring the carrier frequency as a function of time. Again, corrections have to be made for time variations of electron densities, etc. Currently the inherent accuracy of the ranging system[1] is 7 m and the inherent accuracy of the tracking (doppler) system[1] is 1.3 m/12 h (Ref. 1).

Now, the range ρ depends, of course, on the orbital elements of the spacecraft: R the distance from the center of the earth, δ the declination, and α the right ascension, as well as on the station location in a certain way (for details see below). The problem then is to extract information about R, δ, and α from the measured values of ρ and $\dot{\rho}$ (the observables). An error analysis has shown two things (Ref. 2): (1) an accurate determination of the declination near $\delta \approx 0$ is very difficult, and (2) the process noise

[1]This does not take charged-particle or tropospheric calibrations into account.

(random forces due to jet leakage, solar pressure, etc.) is most severely affecting R and this error propagates "downstream" and degrades the determination of δ. If the geocentric range R could somehow be removed, a determination of δ, particularly at low δ, would principally be much more accurate. This can indeed be achieved by tracking and ranging from *two different* stations widely separated and *differencing* their respective topocentric ranges, for then R, being the same for both stations, cancels and we are left with a tidy expression containing only α, δ, and the station locations.

The differencing just described can be realized by a number of possibilities. Here we list the major ones now under active investigation:

(1) *Alternate ranging.* In this mode the two stations range separately and alternately, each for about 30 min during the overlap time, i.e., the time both stations can see the spacecraft simultaneously. The individual measurements are subsequently differenced. A simultaneous ranging is not possible since the two existing ranging machines (τ and μ) are quite different (different range codes, etc.).

(2) *Simultaneous tracking (2–3 way).* Here a signal is transmitted from one station and received at both stations. The received range and doppler cycles at the station which does not transmit are referenced to a synchronized clock. The independent measurements are subsequently differenced and yield the differenced range and range rate.

(3) *Coherent switching between the spacecraft and a radio star.* This is a true very long baseline interferometric (VLBI) mode. The idea is that two stations are receiving simultaneously a noise signal from a radio star and then switching the antennas so that the spacecraft's signal can be received alternately. The interference fringes in both cases give the relative position of the spacecraft against a background of radio stars, and if in turn their positions are known relative to the sky, the angular parameters of the spacecraft are determined.

(4) *Spacecraft VLBI.* In this mode the two stations listen simultaneously to a signal emanating from the spacecraft. This is the mode we are going to analyze in detail in the following.

All modes described here, as all tracking and ranging methods, have to be calibrated for charged particles and the troposphere. Because of the dispersion of the dielectric constant, as far as the interaction of the electromagnetic waves with the plasma is concerned, transmission and (or) reception of signals at two different frequencies will determine the total electron content (see *Section III*) and therefore calibrate for charged particles. The method we are going to analyze subsequently will determine *separately* the differenced range, the differenced range rate, and the total electron content within the ray path.

In *Section II* a VLBI correlation analysis is presented. In *Section III* this analysis is extended to determine the charged-particle effects and a correlation analysis is proposed and analyzed, which allows for a separate determination of the total electron content.

With the notation of Fig. 1 one readily establishes that the vectors from the center of the earth to station 1 or 2 on the surface of the earth are given by

$$\mathbf{r}_1 = r_s^{(1)} (\cos \Omega t\, \mathbf{e}_x + \sin \Omega t\, \mathbf{e}_y) + r_s^{(1)} \tan \epsilon_1\, \mathbf{e}_z \tag{1}$$

and

$$\mathbf{r}_2 = r_s^{(2)} (\cos (\Omega t + L)\, \mathbf{e}_x + \sin (\Omega t + L)\, \mathbf{e}_y) + r_s^{(2)} \tan \epsilon_2\, \mathbf{e}_z \tag{2}$$

where Ω is the angular speed of the earth, L the difference in longitude of the two antennas, and ϵ_1, ϵ_2 their respective latitudes. $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$, finally, are unit vectors conveniently placed in such a way that the longitude of station 1 coincides with the direction of the x axis, which we take to be the vernal equinox, again for convenience. The angle α of Fig. 1 is then the right ascension of the spacecraft. If these conveniences do not apply, an appropriate time translation has to be performed, a consideration which is immaterial for this analysis.

From Fig. 1 it is clear that

$$\mathbf{r}_1 + \boldsymbol{\rho}_1 = \mathbf{r}_2 + \boldsymbol{\rho}_2 = \mathbf{R} \tag{3}$$

where $\mathbf{R}$ is the geocentric range and $\boldsymbol{\rho}$ the topocentric range of the spacecraft. Representing $\mathbf{R}$ by

$$\mathbf{R} = R\, \{\sin \delta \mathbf{e}_z + \cos \delta\, (\cos \alpha \mathbf{e}_x + \sin \alpha \mathbf{e}_y)\} \tag{4}$$

where δ is the declination and α the right ascension of the spacecraft, and assuming $R >> r_s^{(1)} \approx r_s^{(2)}$, we obtain

$$|\boldsymbol{\rho}_1| = R - r_s^{(1)} (\cos \delta \cos (\Omega t - \alpha) + \sin \delta \tan \epsilon_1) \tag{5a}$$

and

$$|\boldsymbol{\rho}_2| = R - r_s^{(2)} (\cos \delta \cos (\Omega t + L - \alpha) + \sin \delta \tan \epsilon_2) \tag{5b}$$

One readily establishes that the time delay between the reception of a signal at station 1 and the reception of the same signal at station 2 is given by

$$T_g = \frac{1}{c}[|\rho_1| - |\rho_2|] = \frac{1}{c}\{D\cos\delta\cos(\Omega t - B)$$

$$- (r_s^{(1)}\tan\epsilon_1 - r_s^{(2)}\tan\epsilon_2)\sin\delta\} \tag{6a}$$

$$\dot{T}_g = \frac{D}{c}[(\Omega - \dot{\alpha})\cos\delta\sin(\Omega t - B) + \dot{\delta}\sin\delta\cos(\Omega t - B)]$$

$$- \frac{\dot{\delta}}{c}(r_s^{(1)}\tan\epsilon_1 - r_s^{(2)}\tan\epsilon_2)\cos\delta \tag{6b}$$

with

$$D = [(r_s^{(1)})^2 + (r_s^{(2)})^2 - 2r_s^{(1)}r_s^{(2)}\cos L]^{1/2} \tag{7}$$

$$\tan B = \frac{r_s^{(1)}\sin\alpha + r_s^{(2)}\sin(L - \alpha)}{r_s^{(1)}\cos\alpha - r_s^{(2)}\cos(L - \alpha)} \tag{8}$$

α and δ in Eq. (6) are the time-dependent quantities of greatest interest and can be determined more accurately with a VLBI technique than with existing methods, as will be shown on the following pages.

II. Development of the Analysis of the VLBI Technique

Since we are interested in ranging measurements, we are dealing with a *modulated* (wide-band) signal rather than a monochromatic (narrow-band) signal as in doppler.[2] If $F(\omega)$ is the Fourier spectrum of the signal we may represent the voltages at the output of the two RF amplifiers of the two stations relative to each other by

$$V_1 = \int d\omega\, F(\omega)\cos\omega t \tag{9a}$$

$$V_2 = \int d\omega\, F(\omega)\cos[\omega(t + T_g)] \tag{9b}$$

where T_g is given by Eq. (6) and the time t must be synchronous for the two stations. Since the frequency ω is usually very high (the modulation F is centered at the carrier frequency $\omega_0 \approx 10^{10}\,\mathrm{sec}^{-1}$), the signals (9a) and (9b) must be heterodyned to obtain a usable intermediate frequency. Let the local oscillators have frequencies ω_1 and $\omega_2 \approx \omega_1$. After mixing and filtering out the high-frequency component at each site, there results a signal which we may express as

$$V_1^{(if)} \sim \int d\omega\, F(\omega)\cos[(\omega_1 - \omega)t + \phi_1(t)] \tag{10a}$$

$$V_2^{(if)} \sim \int d\omega\, F(\omega)\cos[(\omega_2 - \omega)t + \omega T_g + \phi_2(t)] \tag{10b}$$

The phase errors ϕ_1 and ϕ_2 are introduced by the time standards and the electronic equipment. Both $\omega_1 - \omega$ and $\omega_2 - \omega$ are sufficiently slowly varying so that $V_1^{(if)}$ and $V_2^{(if)}$ can be recorded with ease (on tape for instance).

After recording the data (Eqs. 10a and 10b), which introduces additional phase errors ϕ_1^r and ϕ_2^r that we lump together with the previous phase errors, we obtain the cross correlation $C(\tau)$:

$$C(\tau) = \frac{1}{2\delta}\int_{-\delta}^{\delta} dt\, V_1^{(if)}(t)\, V_2^{(if)}(t + \tau)$$

$$= \frac{1}{2\delta}\int_{-\delta}^{\delta} dt \int d\omega\, d\omega'\, F(\omega)\, F(\omega')$$

$$\times \cos[(\omega_1 - \omega)t + \phi_1(t)]$$

$$\times \cos[(\omega_2 - \omega')(t + \tau) + \omega' T_g(t + \tau) + \phi_2(t + \tau)] \tag{11}$$

For the noise pulse spectrum $F(\omega)$, we take the following as a simple analytical model which is, however, general enough to be quite adequate:

$$F(\omega) = \frac{1}{\sqrt{\pi}\,\omega_b}\sum_{i=-\infty}^{+\infty} R_i\cos[(\omega - \omega_0)T_i]\exp\left(-\frac{(\omega - \omega_0)^2}{\omega_b^2}\right) \tag{12}$$

Here R_i and T_i are random variables. R_i is the amplitude with zero mean and T_i the inverse repetition rate or the time elapsed between two pulses, itself a random variable with (possibly) a Poisson distribution. We will see presently that the underlying probability distribution for both R_i and T_i need not be known.[3] ω_0 is the carrier frequency and ω_b is essentially the bandwidth. For mathematical convenience we choose a Gaussian pulse shape. From Eqs. (12) and (10) we obtain in the time domain:

[2]The first part of the following analysis is similar to one given by P. S. Callahan (Ref. 3). Even so, this analysis is far more detailed. On the other hand, the only other paper in the open literature addressing itself to the same subject is by G. W. Swenson and N. C. Mathur (Ref. 4). That paper, however, does not address itself to a charged-particle calibration, which is the central theme of this article.

[3]The bandwidth ω_b could also be made a random variable erratically changing from noise pulse to noise pulse. The outcome of the analysis would be exactly the same.

$$V_1^{(if)}(t) \sim \frac{1}{2}\sum_i R_i \cos[(\omega_1 - \omega_0)t + \phi_1(t)]$$
$$\times \left\{ \exp\left[-\frac{\omega_b^2}{4}(t - T_i)^2\right] + \exp\left[-\frac{\omega_b^2}{4}(t + T_i)^2\right] \right\} \tag{13a}$$

$$V_2^{(if)}(t) \sim \frac{1}{2}\sum_i R_i \cos[(\omega_2 - \omega_0)t + \phi_2(t) + \omega_0 T(t)]$$
$$\times \left\{ \exp\left[-\frac{\omega_b^2}{4}(t - T_g - T_i)^2\right] + \exp\left[-\frac{\omega_b^2}{4}(t - T_g + T_i)^2\right] \right\} \tag{13b}$$

To evaluate the cross correlation we have to multiply the two voltages (13a) and (13b) together and integrate over time. Assuming that the integration time is only a few minutes and that the delay time T and the phases ϕ are slowly varying functions of time, we put

$$\phi_1 = \phi_{10} + \dot{\phi}_{10}t; \qquad \phi_2 = \phi_{20} + \dot{\phi}_{20}t \tag{14}$$

and

$$T_g = T_0 + \dot{T}_0 t \tag{15a}$$

as well as

$$\tau = \tau_0 + \dot{\tau}_0 t \tag{15b}$$

We also filter out the high-frequency component of the product (13a) times (13b). Because of the sharpness of the peaks in amplitude, the integration can safely be extended to infinity. Also, since we are only interested in positive times (the experiment starts at $t = 0$), the second terms in the braces of Eq. (13) may be discarded. Taking advantage of the fact that the R_j are random with zero mean so that the expectation $E(R_i R_j) = \delta_{ij} R_j^2$, we obtain for the cross correlation the following expression:

$$C(\tau) = \frac{\sqrt{\pi}}{8\omega_b \delta \sqrt{\beta}} \sum_j R_j^2 \cos\left(\dot{\alpha}\sqrt{\frac{B_j}{\beta}} + \gamma\right)$$
$$\times \exp\left\{-\frac{\alpha^2}{\omega_b^2 \beta} - \frac{\omega_b^2}{4}(A_j - B_j)\right\} \tag{16}$$

where

$$\dot{\alpha} = \omega_2 - \omega_1 + \dot{\phi}_{20} - \dot{\phi}_{10} + \omega_0 \dot{T}_0 + (\omega_2 - \omega_1)\dot{\tau}_0 \tag{17a}$$

$$\beta = 1 + (1 + \dot{\tau}_0 - \dot{T}_0)^2 \tag{17b}$$

$$\gamma = \phi_{20} - \phi_{10} + \dot{\phi}_{20}\tau_0 + \omega_0 T_0 + \omega_0 \dot{T}_0 \tau_0 + (\omega_2 - \omega_0)\tau_0 \tag{17c}$$

$$A_j = T_j^2 + (\tau_0 - T_j - T_0 - \dot{T}_0\tau_0)^2 \tag{17d}$$

$$B_j = \beta^{-1}[(1 + \dot{\tau}_0 - \dot{T}_0)(\tau_0 - T_j - T_0 - \dot{T}_0\tau_0) - T_j]^2 \tag{17e}$$

and 2δ is the integration time normalizing the summation over j. If we neglect $\dot{T}_0$ for a moment and set $\dot{\tau} = 0$ (constant time offset) and $\dot{\alpha} \approx 0$, we obtain from Eq. (16):

$$C(\tau) = \sqrt{\frac{\pi}{2}}\frac{1}{8}\cos\gamma \exp\left(-\frac{\omega_b^2}{8}(\tau - T_0)^2\right)\frac{1}{\omega_b \delta}\sum R_j^2 \tag{18}$$

The cross correlation $C(\tau)$ has a sharp maximum at $\tau = T_0$ and can, therefore, be measured accurately.[4] However, $\dot{T}_0$ is not zero. Disregarding the motion of the spacecraft relative to the earth, the order of magnitude of $\dot{T}_0$ is approximately given by $\dot{T}_0 \approx \Omega D/c$, where Ω is the angular velocity of the earth and D is the baseline of the VLBI. For $D = 7000$ km and $\Omega = 7.3\ 10^{-5}$, $\dot{T}_0 \approx 10^{-6} << 1$. In this case a *constant* offset time τ_0 would not give any information about T_0 in contradistinction to the case considered in Eq. (18). The offset time τ must be changed with time at the same rate as T_0 changes with time. Since $\dot{T}_0 << 0$, it suffices to consider only linear terms in $\dot{T}_0$ (or $\dot{\tau}_0$).

Care must be taken here if large integration times are contemplated. For we have, conservatively speaking,

$$T \approx T_0 + \frac{\Omega}{c}Dt + \frac{1}{2}\frac{\Omega^2}{c}Dt^2 \cdots$$

In the following analysis we only keep the linear term. But if integration times are of the order of 10 min, the term quadratic in time will give a contribution of $2 \cdot 10^{-5}$ at the end of the integration interval. Reducing the integration time to 1 min reduces the quadratic correction to $2 \cdot 10^{-7}$, which is acceptable since the range is then accurate to the 30-cm level. On the other hand, a shorter integration time is quite adequate in the case of spacecraft VLBI for the simple reason that the spacecraft puts out a larger flux than the faint and distant radio stars. For instance the proposed (proposed for the grand tour) spacecraft transponder of 10 W at S-band yields 60 flux units under the assumption that the noise fills the whole 1-MHz band (1 flux unit = 10^{-26} W/m²-Hz), whereas the 4-W X-band

[4]The accuracy of the measurement depends strongly, of course, on the bandwidth ω_b and the possibility of digitizing the information at a high bit rate.

transmitter yields approximately 1000 flux units at a distance of 1 AU. Compared to that a distant radio source emits about 1 flux unit. An integration time of 1 min is sufficient and the linear approximation is valid.

It turns out that the function $F_j(\tau) = A_j - B_j$ of Eq. (16) has a minimum at

$$\tau = \tau_j = T_0(1 + \dot{T}_0) + T_j(\dot{T}_0 - \dot{\tau}_0) \tag{19}$$

which is independent of j if $\dot{\tau}_0$ is set equal to $\dot{T}_0$. The value of $F_j(\tau)$ at the minimum with this choice for $\dot{\tau}_0$ is now

$$F_j(\tau = T_0[1 + \dot{T}_0]) = 0 \tag{20}$$

again independent of j. The cross correlation (16) will exhibit a sharp maximum at[5] $\tau_0 = T_0(1 + \dot{T}_0)$ provided that the offset time τ is given by

$$\tau = \tau_0 + \dot{T}_0 t = T_0(1 + \dot{T}_0) + \dot{T}_0 t \tag{21}$$

Neglecting $\dot{\alpha}/\omega_b$ we now have for $C(\tau)$:

$$C(\tau) = \sqrt{\frac{\pi}{2}}\frac{1}{8}\exp\left\{-\frac{\omega_b^2}{8}(\tau - \tau_0)^2\right\}$$
$$\times \frac{1}{\omega_b \delta}\sum_j R_j^2 \cos[\gamma + (\omega_1 - \omega_2 + \dot{\phi}_{10} - \dot{\phi}_{20} - \omega_2\dot{T}_0)T_j] \tag{22}$$

where τ_0 is given by Eq. (21). The expression (22) is valid, after neglecting insignificant higher order terms, in the neighborhood of $\tau = \tau_0$ and exhibits again a very sharp maximum at $\tau = \tau_0$ if the cos terms in the series of Eq. (22) are slowly varying throughout the integration time 2δ. But this is assured if

$$2\delta(\omega_1 - \omega_2 + \dot{\phi}_{10} - \dot{\phi}_{20} - \omega_2\dot{T}_0) < \pi \text{ rad} \tag{23}$$

A conservative number for the fringe rate $\dot{T}_0$ is 10^{-6} assuming that $\dot{T}_0$ is solely given by $\dot{T}_0 = \Omega D/c$ with Ω the angular velocity of the earth and D the baseline of 7000 km. Neglecting the phase drifts in Eq. (23) and assuming an integration time of 1 min, the requirement (23) becomes

$$\omega_1 - \omega_2 - 10^{-6}\omega_2 < 10^{-2} \text{ Hz} \tag{24}$$

[5]The reason for the odd expression $\tau_0 = T_0(1 + \dot{T}_0)$ rather than $\tau_0 = T_0$ as in Eq. (11) is "retardation." The signal which reaches antenna 1 at time t does not reach antenna 2 at $t + T(t)$ but at $t + T(t + \tau) = t + \tau$ so that $T(t) + \tau\dot{T}(t) = \tau$ or $\tau = T(1 + \dot{T})$ in first order.

a condition which can easily be met. Since the drift rates $\dot{\phi}_0$ are unpredictable, they should not be allowed to exceed 10% of the 10^{-2} Hz just quoted or translated into frequency stability requirements; the fractional frequency deviation should be better than 5×10^{-14}, a requirement only a hydrogen maser can meet to date.

In principle it is, therefore, possible to measure with a high degree of accuracy both the fringe (the time delay or differenced topocentric range Eq. 6) and the fringe rate (the time rate of change of the time delay) by means of a wide-band VLBI. Obviously the accuracy becomes better the larger the bandwidth ω_b, since the maximum of the cross correlation at the correct offset time (Eq. 21) becomes more pronounced. There is no maximum if τ_0 is not chosen correctly, and there is only a weak maximum if $\dot{\tau}_0$ is chosen incorrectly. By varying τ and $\dot{\tau}$ around their expected values, the absolute maximum of $C(\tau)$, therefore, yields both T_0 and $\dot{T}_0$.

There are, however, two major contributors toward a degradation of the measurements. The first consists of the influence of the charged particles (the solar and ionospheric plasma) and the second consists of the influence of the troposphere. The tropospheric corrections must be made at each station separately, probably in the same manner as in the past. Radiosonde measurements, etc., are necessary for the determination of air density as a function of altitude, water vapor content, and so forth. The range corrections due to the troposphere vary typically from 2 to 100 m, depending on elevation angle, and will not be discussed further here.

III. Charged-Particle Corrections

The charged-particle corrections will keep us occupied for most of the remainder of this article. Taking into account the dispersive dielectric constant of the charged medium, it is easy to see (Ref. 5) that Eqs. (9) have to be replaced by

$$V_1 = \int d\omega\, F(\omega)\cos\left[\omega t - \frac{a}{\omega}I_1(t)\right] \tag{25a}$$

$$V_2 = \int d\omega\, F(\omega)\cos\left[\omega(t + T_g) - \frac{a}{\omega}I_2(t)\right] \tag{25b}$$

where $a = 2\pi e^2/mc$ and

$$I_j = \int_{\text{raypath}} dS\, N_j(S, t) \tag{26}$$

is the total electron content in the raypath between the spacecraft and station j ($j = 1, 2$). Following now exactly in the footsteps leading from Eq. (9) to Eq. (13), we obtain for the intermediate frequencies[6]

$$V_1^{(if)}(t) \sim \frac{1}{2}\sum_i R_i \cos\left[(\omega_1 - \omega_0)\,t + \phi_1(t) + \frac{a}{\omega_0} I_1(t)\right] \times \exp\left\{-\frac{\omega_0^2}{4}\left(t - \frac{a}{\omega_0^2} I_1 - T_i\right)^2\right\} \tag{27a}$$

$$V_2^{(if)}(t) \sim \frac{1}{2}\sum_i R_i \cos\left[(\omega_2 - \omega_0)\,t + \phi_2(t) + \frac{a}{\omega_0} I_2(t) + \omega_0 T_g(t)\right] \exp\left\{-\frac{\omega_0^2}{4}\left(t - T_g - \frac{a}{\omega_0^2} I_2 - T_i\right)^2\right\} \tag{27b}$$

and we see that the pulses are delayed by the interaction with the plasma (group velocity). In Eq. (27) we only took those terms into account for which the arrival time lies within the time interval $(0, 2\delta)$ in which the observations are actually made. Just as in Eqs. (14) and (15), we put:

$$I_1(t) = I_{10} + \dot{I}_{10} t \tag{28}$$

and similarly for I_2. Expansion (28) is again valid to a sufficient accuracy within the integration time 2δ, as it was the case with Eqs. (14) and (15). The analysis leading from Eqs. (13) to Eq. (16) can now be performed in complete analogy starting with Eqs. (27). After some tedious algebra, one is led to Eq. (16) again. Only the meaning of the parameters $\dot{\alpha}$, β, γ, A_j and B_j has changed. The result is the following: taking the charged-particle effects explicitly into account, the cross correlation is again given by Eq. (16), but the parameters (Eqs. 17) have to be replaced by

$$\dot{\alpha} \to \dot{\alpha}' = \omega_2 - \omega_1 + \dot{\phi}_{20} - \dot{\phi}_{10} + \omega_0 \dot{T}_0 + (\omega_2 - \omega_0)\,\dot{\tau}_0 + \frac{a}{\omega_0}(\dot{I}_{20} - \dot{I}_{10}) + \frac{a}{\omega_0^2}(\omega_2 - \omega_1)\,\dot{I}_{10} \tag{29a}$$

$$\beta \to \beta' = 1 + \left[1 + \dot{\tau}_0 - \dot{T}_0 + \frac{a}{\omega_0^2}(\dot{I}_{10} - \dot{I}_{20})\right]^2 \tag{29b}$$

$$\gamma \to \gamma' = \phi_{20} - \phi_{10} + \dot{\phi}_{20}\tau_0 + \omega_0 T_0 + \omega_0 \dot{T}_0 \tau_0 + (\omega_2 - \omega_1)\,\tau_0 + \frac{a}{\omega_0^2}(\omega_2 - \omega_1)\, I_{10} + \frac{a}{\omega_0}(I_{20} - I_{10}) + \frac{a}{\omega_0}\dot{I}_{20}\tau_0 \tag{29c}$$

$$A_j \to A_j' = T_j^2 + \left[\tau_0 - T_j - T_0 - \dot{T}_0\tau_0 - \frac{a}{\omega_0^2}\dot{I}_{20}\tau_0 + \frac{a}{\omega_0^2}(I_{10} - I_{20})\right]^2 \tag{29d}$$

$$B_j \to B_j' = (\beta')^{-1}\left\{\left[1 + \dot{\tau}_0 - \dot{T}_0 + \frac{a}{\omega_0^2}(\dot{I}_{10} - \dot{I}_{20})\right]\left[\tau_0 - T_j - T_0 - \dot{T}_0\tau_0 + \frac{a}{\omega_0^2}(I_{10} - I_{20}) - \frac{a}{\omega_0^2}\dot{I}_{20}\tau_0\right] - T_j\right\}^2 \tag{29e}$$

Terms containing the time derivative of the electron content $\dot{I}$ cannot be neglected in the expressions above. Typical time variations $\dot{I}$ are of the order of 10^{11} cm^{-2} s^{-1} (Ref. 6) leading to a value for an $\dot{I}/\omega_0^2$ of about 10^{-10}. Now, $\dot{T}_0$ is of the order of 10^{-6}, and it is desired that it be known to an accuracy of one part in 10^5, corresponding to an accuracy of 1 mm/s for the differenced range rate. Time variations of the intervening plasma are likely to be an order of magnitude larger than the desired accuracy. To proceed with the analysis we determine the minimum of $F_j' = A_j' - B_j'$ with respect to τ_0, which yields the maximum of the cross correlation according to Eq. (16). It is given by

$$\tau = \tau_0 = T_0\left(1 + \dot{T}_0 + \frac{a}{\omega_0^2}\dot{I}_{20}\right) + T_j\left[\dot{T}_0 - \dot{\tau}_0 - \frac{a}{\omega_0^2}(\dot{I}_{10} - \dot{I}_{20})\right] - \frac{a}{\omega_0^2}(I_{10} - I_{20}) \tag{30}$$

and is independent of T_j if

$$\dot{\tau}_0 = \dot{T}_0 - \frac{a}{\omega_0^2}(\dot{I}_{10} - \dot{I}_{20}) \tag{31}$$

With this choice of $\dot{\tau}_0$ the cross correlation is given by

$$C(\tau) = \sqrt{\frac{\pi}{2}\frac{1}{\delta}}\exp\left\{-\frac{\omega_0^2}{8}(\tau - \tau_0)^2\right\} \times \frac{1}{\omega_0\delta}\sum_j R_j^2 \cos\left[\gamma' - T_j\dot{\alpha}'\right] \tag{32}$$

in the neighborhood of the maximum. τ_0 is now given by Eq. (30), γ' and $\dot{\alpha}'$ are given by Eqs. (29a) and (29c). Again we have a very sharp maximum if Eqs. (30) and (31) hold. Also, in order that no cancellation occurs in the sum over j we must have

[6]Taking the dispersion into account as in Ref. 5, p. 408.

$$2\delta\left[\omega_1 - \omega_2 + \dot{\phi}_{10} - \dot{\phi}_{20} - \omega_2 \dot{T}_0 + \frac{a}{\omega_0^2}(\omega_1 \dot{I}_{10} - \omega_2 \dot{I}_{20})\right] < \pi \tag{33}$$

This condition on the beat frequencies is essentially the same as Eq. (23) because of the smallness of the plasma correction terms.

It is clear from the foregoing that it has been achieved to measure accurately the quantities:

$$\tau_0 = T_0\left(1 + \dot{T}_0 + \frac{a}{\omega_0^2}\dot{I}_{20}\right) - \frac{a}{\omega_0^2}(I_{10} - I_{20}) \tag{34a}$$

and

$$\dot{\tau}_0 = \dot{T}_0 - \frac{a}{\omega_0^2}(\dot{I}_{10} - \dot{I}_{20}) \tag{34b}$$

and it is necessary to correct for the corrupting charged-particle effect. This can be done only with additional measurements. The well-known Differenced Range Versus Integrated Doppler (DRVID) method cannot be employed, since it measures only the time derivative of the total electron content and not its absolute value. There is another reason why DRVID is a poor choice for correcting the VLBI measurements contemplated here. DRVID measures the time variations of the plasma between the spacecraft and the ground station on both the uplink and the downlink. It does not differentiate between the two links. Therefore ½ $\dot{I}$ as determined by DRVID is not necessarily the same as the quantity $\dot{I}_1$ or $\dot{I}_2$ of Eq. (27) (Ref. 7). Therefore, an independent determination of the total electron content for each raypath has to be made using two different frequencies (dual-frequency method).

At this point there are essentially two avenues at our disposal, remembering that we have a dual-frequency system available. The first thing to do if we want to calibrate for charged-particle effects is to simply measure the quantities Eqs. (30) and (31) at a different carrier frequency ω_0'. Then we have, exactly as Eqs. (30) and (31),

$$\tau_0' = T_0\left(1 + \dot{T}_0 + \frac{a}{(\omega_0')^2}\dot{I}_{20}\right) - \frac{a}{(\omega_0')^2}(I_{10} - I_{20}) \tag{30a}$$

and again

$$\dot{\tau}_0 = \dot{T}_0 - \frac{a}{(\omega_0')^2}(\dot{I}_{10} - \dot{I}_{20}) \tag{31a}$$

Then subtracting Eq. (30a) multiplied by $(\omega_0')^2/\omega_0^2$ from Eq. (30), we have

$$\tau_0 - \frac{(\omega_0')^2}{\omega_0^2}\tau_0' = T_0(1 + \dot{T}_0)\left(1 - \frac{(\omega_0')^2}{\omega_0^2}\right) \tag{31b}$$

and all charged-particle ambiguities are removed. However, the total electron content $I(t)$ is in itself a valuable quantity to know since it gives information on the solar wind. We like to propose, therefore, a method in the following which gives just that information without being unduly expensive and without impairment of the range and range rate accuracy.

Suppose there are two phase-locked wide-band transmitters on board the spacecraft operating at frequencies ω_0 and ω_0' with bandwidths ω_b and ω_b', respectively. A ground station receives simultaneously a noise voltage with its receivers, which may be expressed by

$$V_1 = \int d\omega\, F(\omega)\cos\left[\omega t - \frac{a}{\omega} I(t)\right] \tag{35a}$$

and

$$V_2 = \int d\omega\, F'(\omega)\cos\left[\omega t - \frac{a}{\omega} I(t) + \phi\right] \tag{35b}$$

The indices 1 and 2 now refer to the two transmitters. ϕ is a *constant* phase, since the transmitters are phase locked. The Fourier spectrum $F(\omega)$ is given by Eq. (12) and $F'(\omega)$ is[7]:

$$F'(\omega) = \frac{1}{\sqrt{\pi\omega_b'}}\sum_j R_j \cos[(\omega - \omega_0')T_j]\exp\left[-\frac{(\omega - \omega_0')^2}{(\omega_b')^2}\right] \tag{36}$$

The subsequent analysis is quite similar to the previous calculations so we shall be brief in the following. The intermediate frequencies are given by

[7]We assume coherent noise, of course, so that the signals from both transmitters stem from the same noise source.

$$V_1^{(if)}(t) \sim \frac{1}{2}\sum R_i \cos\left[(\omega_1 - \omega_0)t + \phi_1(t) + \frac{a}{\omega_0} I(t)\right]\exp\left[-\frac{\omega_b^2}{4}\left(t - \frac{a}{\omega_0^2} I - T_i\right)^2\right] \tag{37a}$$

$$V_2^{(if)}(t) \sim \frac{1}{2}\sum R_i \cos\left[(\omega_2 - \omega_0')t + \phi_2(t) + \frac{a}{\omega_0'} I(t) + \phi\right]\exp\left[-\frac{(\omega_b')^2}{4}\left(t - \frac{a}{(\omega_0')^2} I - T_i\right)^2\right] \tag{37b}$$

ω_1 and ω_2, etc., are, of course, not the same as in previous calculations since 1 and 2 refer to the two different channels of the same station rather than to the two stations. Again we use the linear approximation for $\phi(t)$ and $I(t)$ which is quite sufficient for the integration times involved (1 min). After some algebra analogous to previous calculations we obtain:

$$C_I(\tau) = \frac{\sqrt{\pi}}{8\omega_b\,\delta\sqrt{\epsilon}} \sum_j R_j^2 \cos\left(\dot{\xi}\sqrt{\frac{E_j}{\epsilon}} + \eta\right) \exp\left\{-\frac{\xi^2}{\omega_b^2\epsilon} - \frac{\omega_b^2}{4}(D_j - E_j)\right\} \tag{38}$$

where

$$\epsilon = 1 + \left(\frac{\omega_b'}{\omega_b}\right)^2 \left[1 + \left(\frac{1}{\omega_0^2} - \frac{1}{(\omega_0')^2}\right) a\dot{I}_0 + \dot{\tau}_0\right]^2 \tag{39a}$$

$$\dot{\xi} = \omega_2 - \omega_1 + \omega_0 - \omega_0' + \dot{\phi}_{20} - \dot{\phi}_{10} + (\omega_2 - \omega_0')\,\dot{\tau}_0 + (\omega_2 - \omega_1 - \omega_0')\frac{a}{\omega_0^2}\dot{I}_0 + \frac{a}{\omega_0'}\dot{I}_0 \tag{39b}$$

$$\eta = \phi_{20} - \phi_{10} + \phi + \left(\omega_2 - \omega_0' + \dot{\phi}_{20} + \frac{a}{\omega_0'}\dot{I}_0\right)\tau_0 + (\omega_2 - \omega_1 - \omega_0')\frac{a}{\omega_0^2}\dot{I}_0 + \frac{a}{\omega_0'}\dot{I}_0 \tag{39c}$$

$$D_j = T_j^2 + \left(\frac{\omega_b'}{\omega_b}\right)^2 \left[\tau_0\left(1 - \frac{a}{(\omega_0')^2}\dot{I}_0\right) + \left(\frac{a}{\omega_0^2} - \frac{a}{(\omega_0')^2}\right) I_0 - T_j\right]^2 \tag{39d}$$

$$E_j = \epsilon^{-1}\left\{-T_j + \left(\frac{\omega_b'}{\omega_b}\right)^2 \left[1 + \dot{\tau}_0 + a\dot{I}_0\left(\frac{1}{\omega_0^2} - \frac{1}{(\omega_0')^2}\right)\right]\left[\tau_0 + aI_0\left(\frac{1}{\omega_0^2} - \frac{1}{(\omega_0')^2}\right) - \frac{a}{(\omega_0')^2}\dot{I}_0\tau_0 - T_j\right]\right\}^2 \tag{39e}$$

The correlation (38) depends critically on the bandwidth differences of the two channels ω_b and ω_b'. If $\omega_b = \omega_b'$, C_I exhibits a pronounced maximum if the offset time τ is chosen to be:

$$\tau_0 = -aI_0\left(\frac{1}{\omega_0^2} - \frac{1}{(\omega_0')^2}\right) \tag{40a}$$

and

$$\dot{\tau}_0 = -a\dot{I}_0\left(\frac{1}{\omega_0^2} - \frac{1}{(\omega_0')^2}\right) \tag{40b}$$

and the electron content I may be measured accurately. If, however, the bandwidths differ, no such claim can be made; in that case the maximum of each individual term in the series (38) depends on T_j so that no common τ exists which maximizes all terms of the series (38) simultaneously. It follows also from Eq. (38) that the frequencies have to be chosen such that the product:

$$2\delta\left[\omega_2 - \omega_1 + \omega_0 - \omega_0' + \dot{\phi}_{20} - \dot{\phi}_{10} + a\dot{I}_0\left(\frac{\omega_2}{(\omega_0')^2} - \frac{\omega_1}{\omega_0^2}\right)\right] < \pi \tag{41}$$

a condition which can easily be met.

A possible implementation of this calibration technique is shown in Fig. 2.

Two problems finally remain before the system described here can be employed within the accuracy demanded, i.e., better than 1 m in range and 0.1 mm/s in range rate.

The first problem is the necessity to calibrate for tropospheric effects. This must, of course, be done at both stations independently since the atmospheric properties are expected to be quite different at the widely separated antenna sites. How the calibration is effectuated is explained in detail elsewhere (Ref. 8).[8] Range corrections are a sensitive function of the elevation angle and are typically of the order of 2 m (at zenith) to 100 m (at 0-deg elevation angle). The accuracy of this calibration is about 5% or 10 cm at zenith, which is quite adequate. However, it appears to be 7% at very low elevation angles, for instance,

[8]See also "New Tropospheric Range Corrections With Seasonal Adjustment," by C. C. Chao in this issue.

corresponding to about 4 m at 10 deg (quite inadequate), so that for elevation angles less than 10 deg the errors become intolerably large, and the data must be discarded if we wish to stay within the accuracy of better than 1 m.

The second problem is that of time synchronization. It is clear that if there is a time difference between the times measured independently at the two stations it will show up as a range error. Assuming the clocks have been synchronized once, how long will they stay this way? Now, it turns out that the very stable time standards which must be used in the VLBI system are hydrogen masers. They have an (unpredictable) drift of equivalent range of about 30 cm in 24 h. Assuming that the drift rates are solely due to the hydrogen masers (a rather bold assumption), it is then necessary to resynchronize again within three days to keep the range error below the 1-m level. We like to propose here a rather accurate method of synchronization which uses laser ranging.

It is known that the accuracy of a laser ranging system presently being used is 30 cm. It is also known that within the present state of the art the accuracy can be improved to the 3-cm level[9] without the need of any breakthrough. Suppose, then, that a VLBI is pointed at a synchronous earth satellite a few earth radii away and that at the same time two lasers, one at each site, are ranging the same satellite.[10] Then we would obtain from the lasers the information

$$cT_g^{(L)} = \rho_1 - \rho_2 \pm \sqrt{2} \cdot 30\,\text{cm (or better)} \tag{42}$$

and from the VLBI:

$$cT_g = c\,(T_g^{(L)} + \Delta_S) \tag{43}$$

where Δ_S is the error in clock synchronization which may be obtained from the two independent measurements (42) and (43) and corrected for within the accuracy of the laser ranging. Of course, Eq. (43) represents only an idealization in as much as the previously discussed error sources are not mentioned explicitly. They have to be accounted for as discussed above. Another trivial fact that needs mention in passing is that there is a difference in location between the satellite's antenna and corner reflectors as well as between the lasers and VLBI antennas that has to be accounted for.

[9]Private communication with R. Jaffe.

[10]The satellite must be equipped with a dual-frequency transmitter.

IV. Summary

On the preceding pages the feasibility and the potential enhancement of accuracy over the currently used ranging systems as described in the introduction have been shown. It has also been shown how to measure the total electron content separately within the raypath, a quantity which is of great interest to solar plasma physicists. The limitations in accuracy of the "coherent noise" system are due to *four* error sources:

(1) Bandwidth. A small bandwidth (as seen from Eq. 18) is quite detrimental. But, because of the exponential nature of the maximum of the correlation function, the bandwidth limitation is not critical. A 1-MHz bandwidth will yield a value of T_g good to 1 part in 10^7, approximately.

(2) Coherence between the X- and S-band transmitters. To take out the plasma effects, or in other words, to calibrate for the charged particles, the two transmitters on the spacecraft have to have a certain amount of coherence. First of all, they must be *phase*-locked; secondly, their bandwidths must be *identical.* The reason for these requirements is easily seen from Eqs. (39). A phase drift of the two receivers with respect to each other and a mismatch of the bandwidths would result in an unacceptable charged-particle calibration. However, we have to remember that the aforementioned calibration does not have to be performed to the same degree of accuracy as the differenced range measurements (the ubiquitous T_g of Eq. 6a). The correlation analysis may still be performed with the 1-MHz bandwidth originally assumed, the error in the charged-particle calibration amounting to no more than 1%.

(3) Tropospheric influences. They have been mentioned on the preceding pages and do not need any amplification, except that one should always be aware of their degrading influence on range accuracy.

(4) Synchronization. Synchronization is essential for this system to work. Without it, all advantages of the spacecraft VLBI will be utterly lost. It is therefore vital to implement a viable system allowing for the best accuracy available. The laser system proposed in this article seems to be more accurate in this context than any other known to the author at present.

References

1. *DSN Flight Project Interface Design Handbook,* 810-5, Rev. B, Aug. 1, 1971 (JPL internal document).

2. Ondrasik, V. J., and Rourke, K. H., "Applications of Quasi-VLBI Tracking Data Types to the Zero Declination and Process Noise Problems," presented at the AAS/AIAA Astrodynamics Specialists Conference 1971, Fort Lauderdale, Florida, August 17–19, 1971.

3. Callahan, P. S., *The VLBI Expected Correlation Over a Finite Bandpass,* Technical Memorandum 391-87, Apr. 8, 1970 (JPL internal document).

4. Swenson, G. W., and Mathur, N. P., "The Interferometer in Radio Astronomy," *Proc. IEEE,* p. 2114, Dec. 1968.

5. Ginzburg, V. L., *Propagation of Electromagnetic Waves in Plasma.* Gordon and Breach, Science Publishers, Inc., New York, 1961.

6. von Roos, O. H., "Second Order Charged Particle Effects on Electromagnetic Waves in the Interplanetary Medium," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. II. Jet Propulsion Laboratory, Pasadena, Calif., April 15, 1971.

7. von Roos, O. H., "Analysis of the DRVID and Dual Frequency Tracking Methods in the Presence of a Time-Varying Interplanetary Plasma," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. III, pp. 71–76. Jet Propulsion Laboratory, Pasadena, Calif., June 15, 1971.

8. Miller, L. F., et al., "A Cursory Examination of the Sensitivity of the Tropospheric Range and Doppler Effects to the Shape of the Refractivity Profile," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. I, pp. 22–30. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1971.

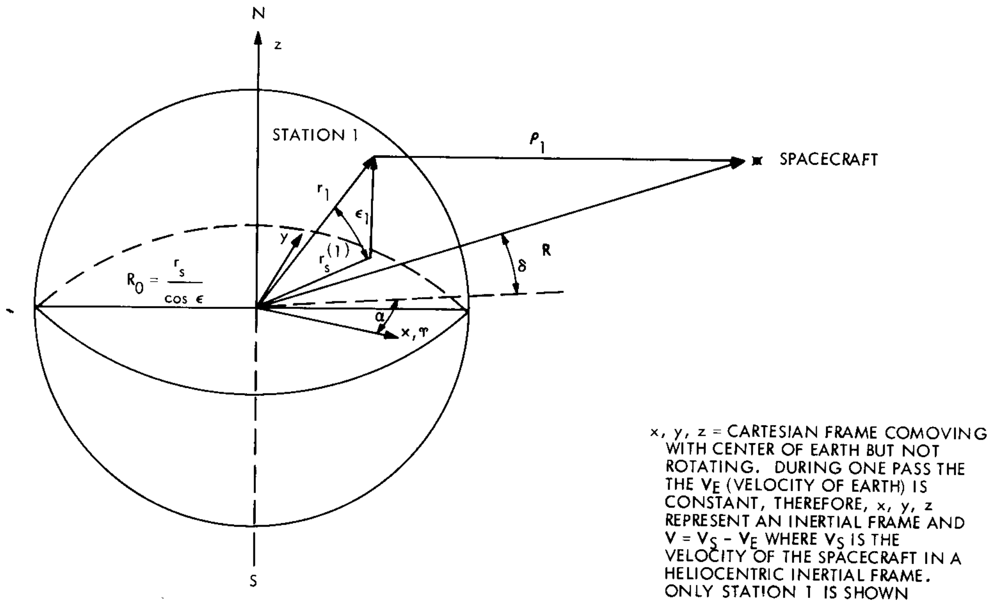

Fig. 1. Ranging geometry

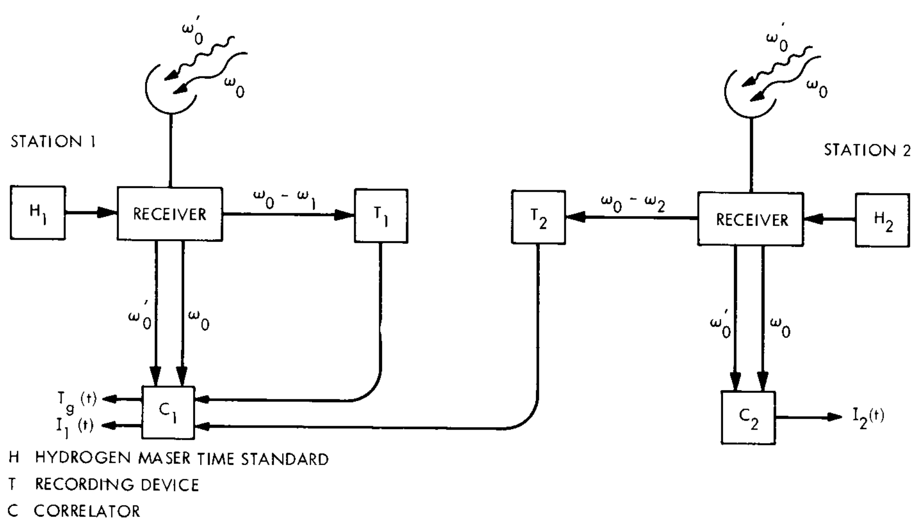

Fig. 2. Interferometer schematic (note the possible advantage for at least two correlators for independent charged-particle calibration)