Tropospheric Range Effect Due to Simulated Inhomogeneities by Ray Tracing

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A simple ray trace method is developed to study the effect in range correction of a radio wave passing through the troposphere with inhomogeneities. Inhomogeneities were simulated from previous observations. The uncertainties in range correction due to the unmodeled horizontal gradient in refractivity are mostly less than 1% for elevation higher than 5 deg. The average uncertainty due to local inhomogeneities, based on nine simulated cases, is below 1-m in one-way range correction for elevation angles greater than 5 deg.

I. Introduction

The purpose of the article is to investigate the influence of inhomogeneities in the troposphere based on the exact solutions from a two dimensional ray trace technique. Eighteen different cases of inhomogeneities were simulated, based on previous observations. A double-precision computer program called INHOMO was designed for numerical integration and analysis.

In the past, methods for reducing the troposphere-induced errors on interplanetary spacecraft orbit determination employed an exponential refractivity model which is independent of time. It was found that this model is not an adequate approximation to the true physical situation. First, the seasonal fluctuations in the refractivity profiles can cause a maximum of 10% deviation in zenith range effect.[1] Secondly, refractivity is not an exponential function of altitude in the first 12 km, where most of the refraction takes place.[2]

Currently an improved model, which uses Berman's (Ref. 1) time-dependent parameters for zenith range correction and maps it down to lower elevation angle by a two-stage quartic-exponential refractivity profile, is being introduced for tropospheric calibration. This model has reduced the errors in range correction due to seasonal fluctuations in the troposphere by 60%. However, it

[1]"The Repetition of Seasonal Variations in the Tropospheric Zenith Range Effect, by K. L. Thuleen and V. J. Ondrasik in this issue.

[2]New Tropospheric Range Corrections With Seasonal Adjustment," by C. C. Chao in this issue.

neglects the possible inhomogeneities (including asymmetry) in the troposphere that may contribute significant effect to range and range rate.

Bean and Dutton (Ref. 2) studied the effect of atmospheric horizontal inhomogeneity upon ray tracing. They computed only the ray-bending effect on elevation angles for two particular observed situations, and reached the tentative conclusion that the effect of horizontal inhomogeneity is normally small, since ducting, which causes inhomogeneity, will occur less than 15% of the time. For high-precision tracking techniques, i.e., very long baseline interferometry (VLBI) and laser ranging, which require tracking at very low elevation angles, such effect should be more carefully investigated. O. H. von Roos[1] has recently developed a first-order perturbation theory to examine the inhomogeneity influence on range.

He obtained a single integral, although it has to be integrated numerically, which gives the additional range effect due to inhomogeneities in good agreement (at 10-deg elevation angle) with the exact solution of this article. After further verification with the exact solution at various elevation angles his method may become a useful tool in calibrating the additional range effect from observed inhomogeneities.

The ray trace method introduced in this article can also be applied for the study of inhomogeneities in the ionosphere.

II. Ray Trace in an Inhomogeneous Atmosphere

The travel time of a radio wave between two fixed points in a medium can be written as

$$T = \int_1^2 \frac{dS}{V} \tag{1}$$

or

$$T = \frac{1}{c}\int_1^2 n\, dS \tag{2}$$

where

V is the speed of light in medium

c is the speed of light in vacuum

$n = c/V$ index of refraction ($n \geq 1$)

[1]"Tropospheric and Ionospheric Range Corrections for an Arbitrary Inhomogeneous Atmosphere (First-Order Theory)," by O. H. von Roos in this issue.

According to Fermat's Principle, the raypath takes the shortest time between two given points in a medium. Therefore, our problem is to find the path OB in Fig. 1 for which the travel time is a minimum. The infinitesimal change in distance, ds, is

$$dS = \sqrt{1 + r^2\left(\frac{d\phi}{dr}\right)^2}\, dr$$

Thus, we can rewrite the above equation as

$$\rho = cT = \int_{R_0}^{R_b} n \sqrt{1 + r^2\left(\frac{d\phi}{dr}\right)^2}\, dr \tag{3}$$

where

ρ = traveled distance between O and B

$\phi = \phi(r)$ along the raypath

and for a nonsymmetric atmosphere

$n = n(\phi, r)$

Since r is the independent variable, the integrand in Eq. (3) can be defined as the Langrangian of our problem.

$$L = L\left(\phi(r), \frac{d\phi}{dr}, r\right) = n\sqrt{1 + r^2\left(\frac{d\phi}{dr}\right)^2} \tag{4}$$

The Hamiltonian can then be obtained from the definition

$$H = H(\phi(r), p(r), r) = p\frac{d\phi}{dr} - L \tag{5}$$

where p is the momentum and can be given as

$$p = \frac{\partial L}{\partial \phi_1} = \frac{n\, r^2\, \phi_1}{\sqrt{1 + r^2\, \phi_1^2}} \tag{6}$$

where $\phi_1 = d\phi/dr$, and it is expressed in terms of p as

$$\phi_1 = \frac{p}{r\sqrt{n^2r^2 - p^2}} \tag{7}$$

Upon substitution, we then easily obtain the Hamiltonian in the following simple form:

$$H = -\frac{\sqrt{n^2r^2 - p^2}}{r} \tag{8}$$

Therefore, the raypath of a radio wave passing through an inhomogeneous medium can be obtained by integrating the following equations:

$$\frac{d\phi}{dr} = \frac{\partial H}{\partial p} \tag{9}$$

$$\frac{dp}{dr} = -\frac{\partial H}{\partial \phi} \tag{10}$$

or

$$\frac{d\phi}{dr} = \frac{p}{r\sqrt{n^2r^2 - p^2}} \tag{11}$$

$$\frac{dp}{dr} = \frac{nr\dfrac{\partial n}{\partial \phi}}{\sqrt{n^2r^2 - p^2}} \tag{12}$$

For a spherically symmetric atmosphere, $\partial n/\partial \phi$ vanishes and p is a constant. Then Eq. (11) will lead to the familiar integral of ray trace.

$$\phi = \int_{R_0}^{r} \frac{c_0}{r\sqrt{n^2r^2 - c_0^2}}\, dr \tag{13}$$

where

$$c_0 = r_0 n_0 \cos \gamma_0$$

The above two-dimensional approach assumes that the perturbed raypath due to inhomogeneities is always remaining in the same vertical plane. The bending in the azimuth direction has been found insignificantly small from three-dimensional ray trace results. The horizontal gradients in refractivity due to tropospheric inhomogeneities are much smaller than the vertical gradients, which causes sizable bending effect at very low elevation angles.

III. Range Effect Due to Simulated Inhomogeneities

The inhomogeneities in troposphere are defined here as any departure from spherical symmetry and local irregularities in refractivity. Bean and Dutton made several nation-wide surveys of the variations in refractivity. Figure 2 shows the mean values of sea level refractivity N_0 in the August months over the U.S. A horizontal gradient of about 30 N units per 500 km can be found in California. When a bad storm is passing through, the horizontal gradient increases to 50 N units per 500 km, as shown in Figs. 3 and 4. A sample of the corresponding variation in refractivity in a space cross-section in N units is shown in Fig. 5. The local irregularities in refractivity were also shown by the observed results made along a straight path which began at Cape Kennedy in 1957 by Bean and Dutton (Fig. 6).

Based on the above observed results originating from Ref. 4, an analytic expression of the variation of refractivity N was developed for the two-dimensional ray trace computation. Figure 5 seems to indicate that the variations are sinusoidal in nature; thus, we can approximately simulate the inhomogeneities by the following analytic form:

$$N(r,\phi) = N_0\left(1 - \frac{r - r_0}{H}\right)^4 + \Delta N_{\text{inhomo}}(r,\phi)$$

where

$$\Delta N_{\text{inhomo}} = \Delta N_0 \left(1 - \frac{r - r_0}{H_1}\right)^4 \left[A\phi + B \sin\left(\frac{\pi}{2}\frac{\phi}{W}\right) + c \sin\left(\frac{\pi}{2}\frac{r - r_0}{L}\right)\right]$$

and

ΔN_{inhomo} = change of refractivity due to inhomogeneity

ΔN_0 = change of surface refractivity due to inhomogeneities

ϕ = raypath angle, rad

H_1 = the light of the layer in which inhomogeneities takes place

A = parameter of the constant gradient along ϕ direction

B,C = parameters of the amplitude of sine function along ϕ and r direction, respectively

W,L = one fourth of the wavelength of the two sine functions along ϕ and r direction

The values of those constants in the above equation are estimated based on the observed results shown in Figs. 2 to 6. Samples of 18 different cases (Table 1) were computed by the two-dimensional ray trace program. The first nine cases simulate the horizontal gradient, and the second nine cases (from 10 to 18 in Table 1) approximate the local inhomogeneities as shown in Fig. 6. The additional influences on range correction of these 18 cases are tabulated in Table 2.

IV. Results and Discussion

The additional range effects due to the 18 different simulated inhomogeneities were ray traced by a double precision computer program called INHOMO. It employed a fifth-order Runge-Kutta integrator with automatic step size control. The results can be grouped into four types:

I. Horizontal gradient assumed in the entire troposphere ($H_1 = 42.6$ km).

II. Horizontal gradient occurs only in the first 13.6 km from surface ($H_1 = 13.6$ km).

III. Irregularities assumed in the entire troposphere ($H_1 = 42.6$ km).

IV. Irregularities are confined within the first 13.6 km from surface ($H_1 = 13.6$ km).

The results for type I are shown by the first three cases in Table 2. They have higher values than the type II results, as shown by cases 4 to 9 in Table 2. According to the radiosonde balloon data (Ref. 2), most of the fluctuations in refractivity were found in the first 14 km of altitude. Thus the actual influence on range correction due to horizontal gradient is expected to be in the same magnitude of type II results. A normalized range effect due to horizontal gradient is plotted in Fig. 7. The influence due to local irregularities similar to the one shown in Fig. 6 are difficult to simulate with a simple equation. However, the second nine cases in Table 1 will reveal, quantitatively, the effect on range due to local irregularities. As shown in Table 2, the effects on range caused by local irregularities are more significant than that induced by horizontal gradient. A 2 to 12% change in range effect can be found at various elevation angles in type III results (cases 11, 12, 13, 16, 17, 18), and the type IV results have a smaller variation from 1.2% to 6% (cases 10, 14, 15). Similarly, type IV results are expected to better approximate the actual normalized range effect due to local irregularities is plotted in Fig. 8.

From the above results, one conclusion may be reached that the additional range effects due to possible horizontal gradient are mostly insignificant for elevation angle greater than 5 deg ($\Delta\rho < 0.1$ m), the corresponding effects due to local inhomogeneities are sizable—from 5 to 20 deg of elevation ($0.1\,\text{m} < \Delta\rho < 1.0\,\text{m}$). However, according to Bean and Dutton (Ref. 4), the local irregularities usually caused by bad storm and ducting will occur less than 15% of the time. Thus, in case of a bad storm or a noticeable ducting, the tracking data should be deleted or loosely weighed before advanced techniques of tropospheric calibration, such as line-of-sight correction by microwave radiometer, are available.

References

1. Berman, A., "A New Tropospheric Range Refraction Model," in *The Deep Space Network*, Space Programs Summary 37-65, Vol. II, pp. 140–153. Jet Propulsion Laboratory, Pasadena, Calif., August 30, 1970.

2. Bean, B., and Dutton, E., *Radio Meteorology*, Monograph 92, U.S. Department of Commerce, National Bureau of Standards, March 1966.

Table 1. Simulated tropospheric inhomogeneities

Case No.	N_0	ΔN_0	H_1	A	B	C	W, km	L, km
1	350	70	42.6	0	1	0	800	–
2	300	50	42.6	0	1	0	800	–
3	300	50	42.6	0	1	0	1600	–
4	300	50	13.6	6	0	0	–	–
5	300	50	13.6	12	0	0	–	–
6	300	50	13.6	0	1	0	300	–
7	300	50	13.6	0	1	0	1600	–
8	300	50	13.6	0	1	0	800	–
9	350	70	13.6	0	1	0	800	–
10	300	50	13.6	12	0	0.5	–	1.0
11	300	50	42.6	0	1	0.5	500	2.0
12	300	50	42.6	0	1	0.5	300	2.0
13	300	50	42.6	0	1	0.5	100	2.0
14	300	50	13.6	0	1	0.5	500	1.0
15	300	50	13.6	–	1	1.0	100	1.0
16	300	25	42.6	0	1	0.5	500	2.0
17	300	25	42.6	0	1	0.5	300	2.0
18	300	25	42.6	0	1	0.5	100	2.0

Table 2. Additional influence in tropospheric range correction due to simulated inhomogeneities

Case No.	Percent of total range correction					
	(70)[a] At 1 deg	(36) At 3 deg	(24) At 5 deg	(12.7) At 10 deg	(8.7) At 15 deg	(6.5) At 20 deg
1	6.15	4.67	3.35	1.96	1.36	1.00
2	4.28	3.31	2.4	1.41	0.94	0.72
3	2.23	1.68	1.23	0.70	0.47	0.36
4	0.47	0.24	0.15	0.08	0.05	0.04
5	0.93	0.48	0.31	0.15	0.10	0.08
6	2.38	1.33	0.84	0.43	0.28	0.21
7	0.47	0.25	0.16	0.08	0.05	0.04
8	0.96	0.50	0.32	0.16	0.10	0.08
9	1.36	0.70	0.44	0.22	0.15	0.11
10	2.42	1.71	1.43	1.23	1.15	1.15
11	8.87	7.43	6.02	4.33	3.58	3.22
12	11.30	10.20	8.30	5.79	4.56	4.00
13	7.17	11.10	12.58	11.11	8.95	7.54
14	2.88	2.00	1.63	1.34	1.23	1.20
15	6.92	5.61	4.51	3.41	2.94	2.77
16	4.66	3.79	3.04	2.18	1.79	1.62
17	5.86	5.16	4.17	2.89	2.30	2.0
18	3.63	5.55	6.31	5.59	4.47	3.76

[a]Values in parenthesis give the total range correction in meters.

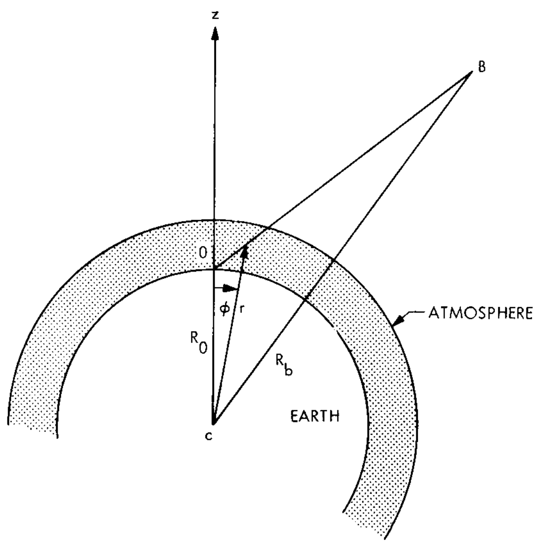


Fig. 1. Geometry of the radio wave refraction

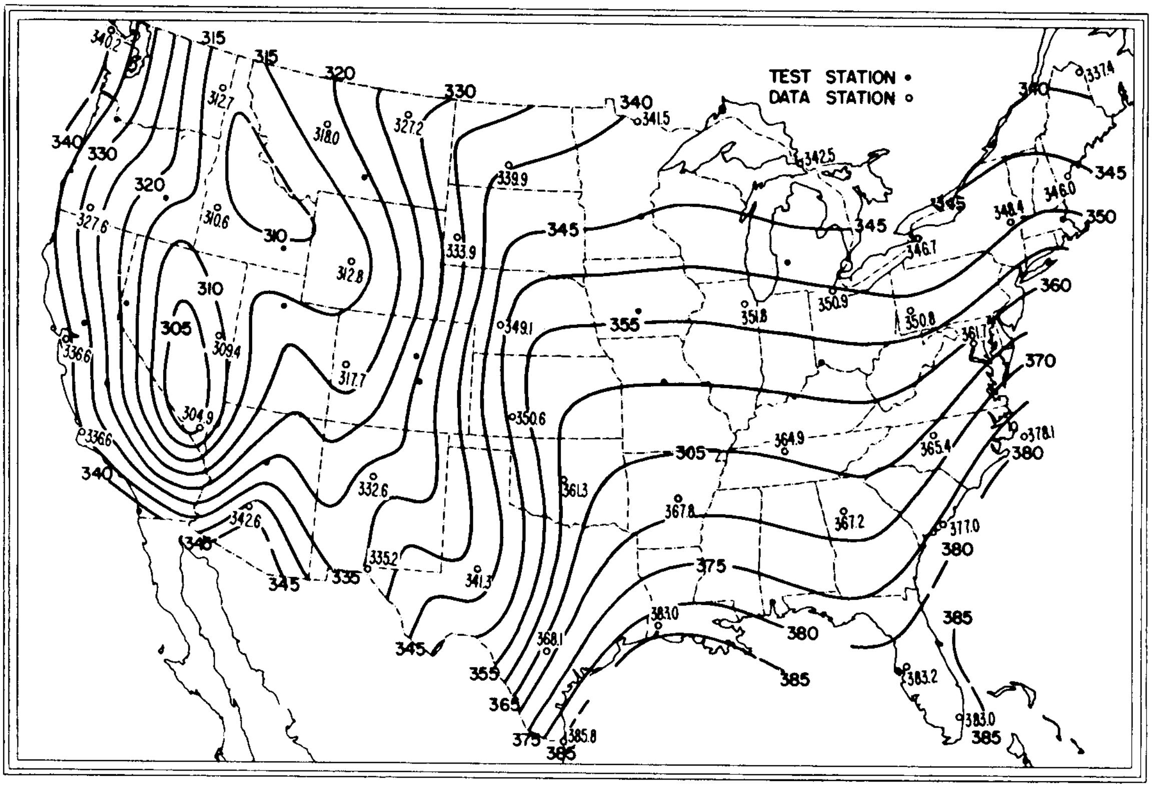


Fig. 2. Test chart of mean N_0 on August at 02:00 local time (taken from Ref. 2)

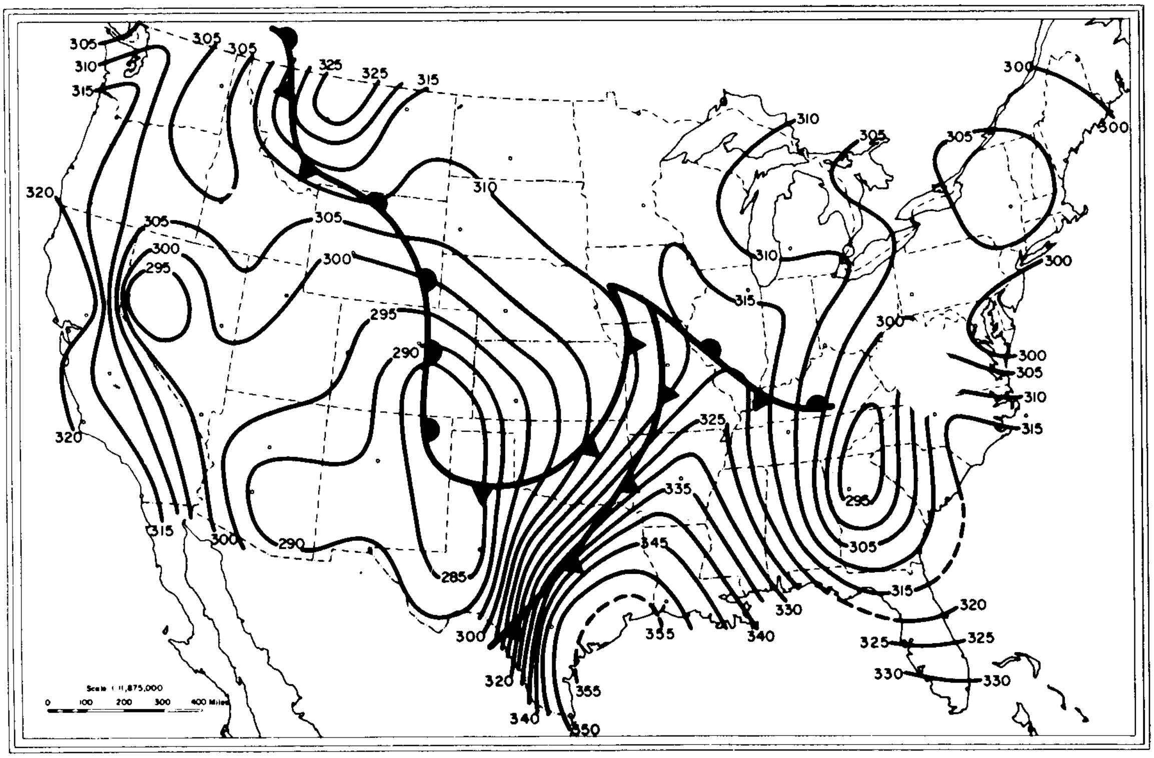


Fig. 3. N_0 chart for storm system 1330E, Feb. 19, 1952 (Ref. 2)

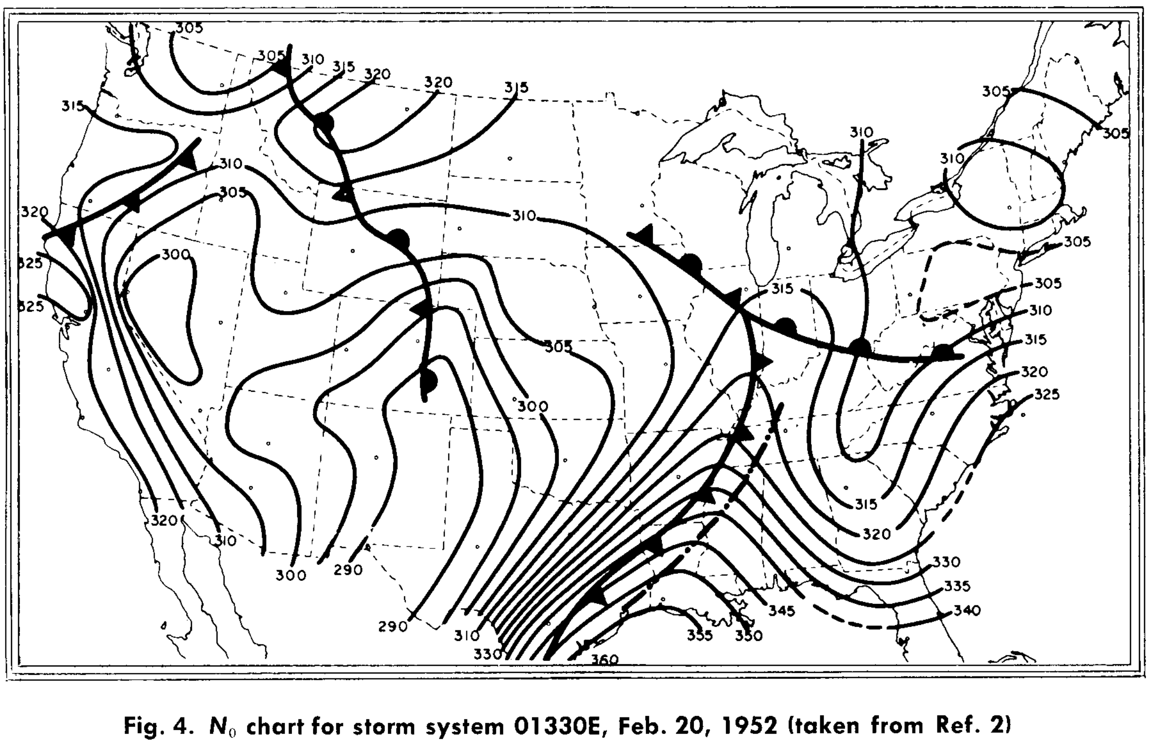


Fig. 4. N_0 chart for storm system 01330E, Feb. 20, 1952 (taken from Ref. 2)

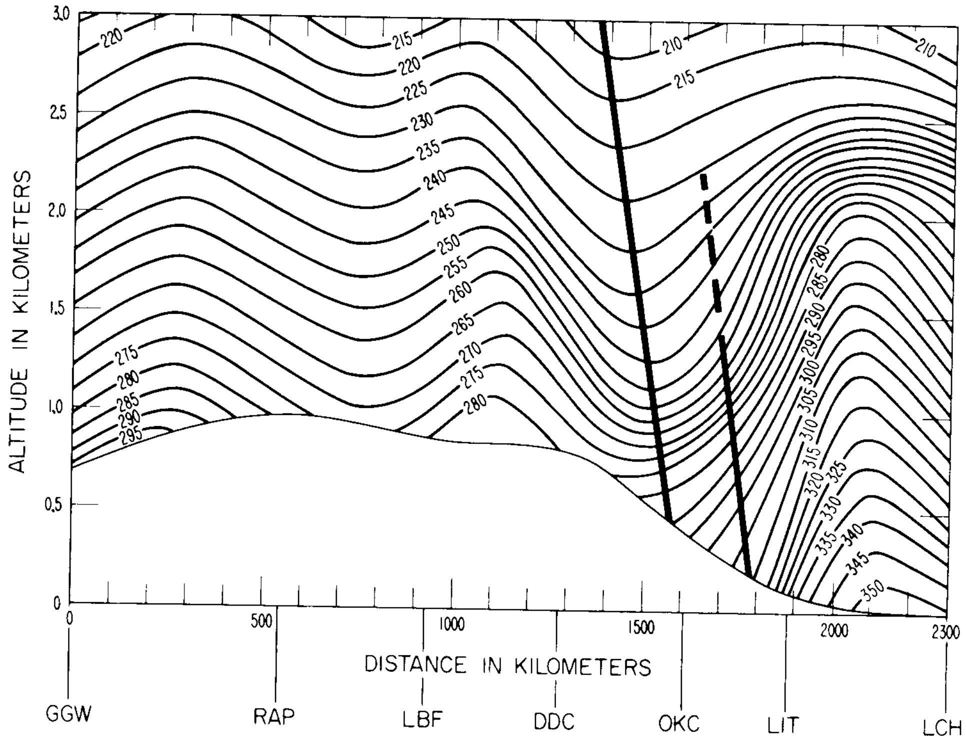
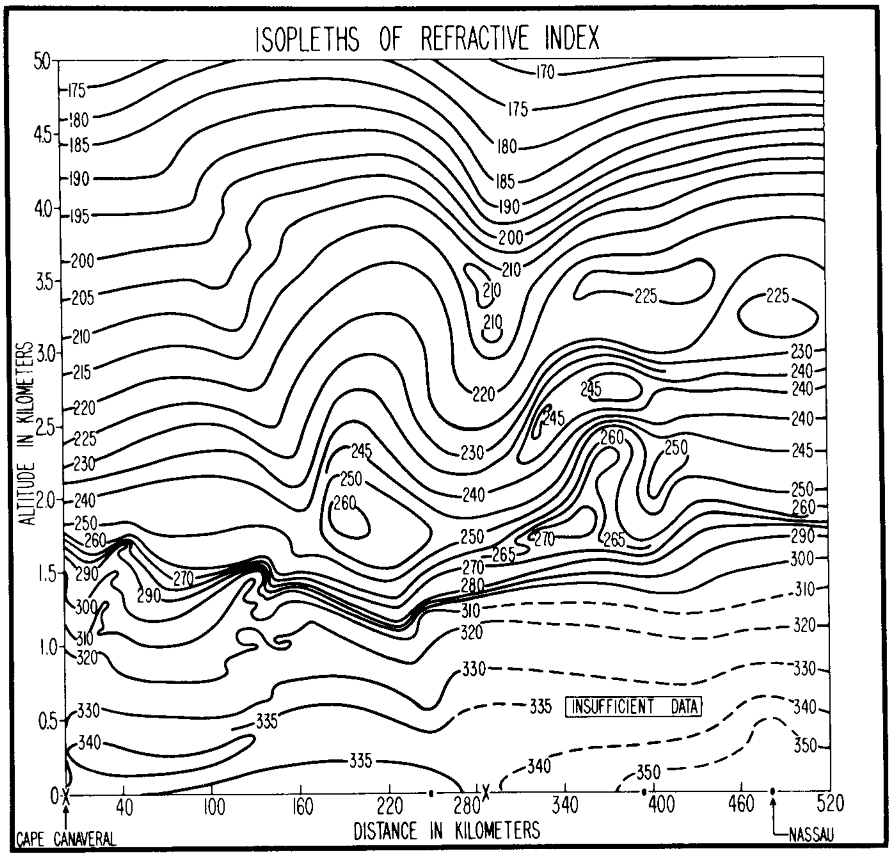


Fig. 5. Space cross section in *N* units, 15:00Z on Feb. 19, 1952 (taken from Ref. 2)



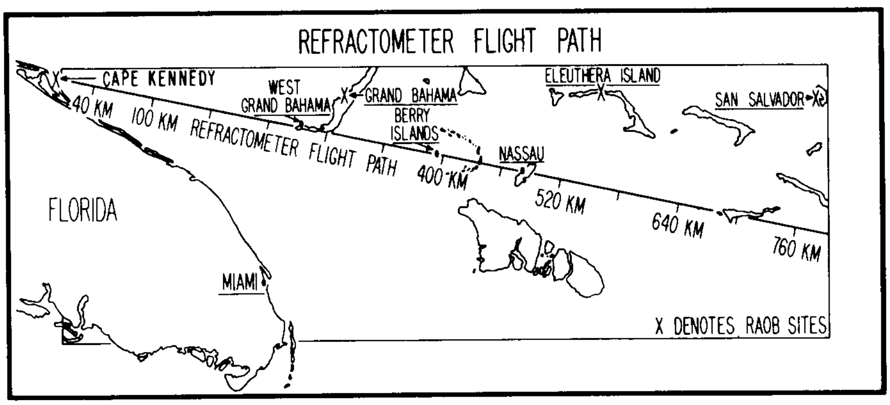


Fig. 6. Isopleths of refractive index and map of refractometer flight path for May 7, 1957, Cape Kennedy to Nassau (taken from Ref. 2)

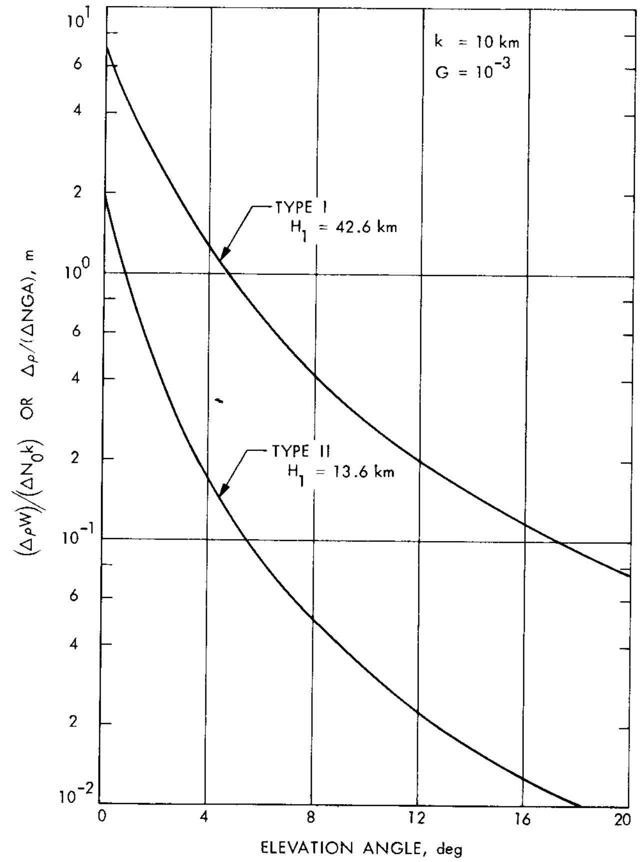


Fig. 7. Normalized range effect due to horizontal gradient effect

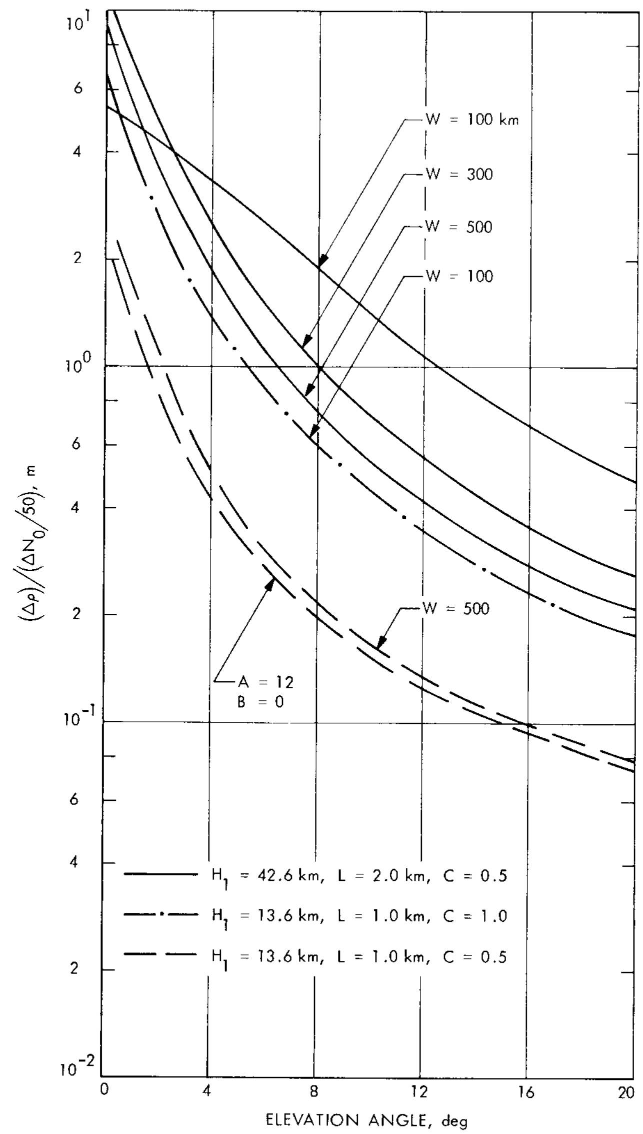


Fig. 8. Normalized range effect due to local inhomogeneities