Automatic Angle Tracking: Angle Error Analysis and Tests

R. D. Rey
DSIF Operations Section

Tests are being developed to measure the angle errors of the 26-meter-diameter antenna stations. Analysis is performed in order to define performance requirements. Receiver degradation due to the mean and variance of the angle error is determined using an approximation of the antenna gain pattern. The equation for the angle error variance is determined. Measured data were compared with the theoretical results and were found to agree well.

I. Introduction

A part of the angle tracking analysis and test development effort (Ref. 1) is the angle variance tests which are being developed. The tests are to be used to test the automatic tracking capability which exists at both the standard and integrated 26-meter-diameter antenna stations.

In order to define ideal performance requirements for the tests, an analysis is performed. The analysis is broken into two parts. The first part is an estimation of receiver degradation due to the mean angle error and angle error variance. The purpose of this analysis is to relate angle error to the performance of the receiver. The second part of the analysis determines the angle error variance versus signal level. The curves resulting from this analysis will be used in test procedures as performance criteria.

II. Receiver Degradation Due to Angle Error

The variance of the antenna about a mean angle will result in degradation of receiver efficiency. This section will relate this degradation to boresight angle error and to the variance of the angle error. The power into the receiver will be modified by the average gain of the antenna and the variance of this gain. These two values can be used to define receiver degradation. A special case will be calculated by setting the mean angle error (boresight error) equal to zero.

The antenna gain has been determined to be

$$|P(\theta)|_{\mathrm{dB}} = 20\log_{10}\left(\frac{\sin 8.43\theta}{8.43\theta}\right) + 53.3$$

for θ in radians and $|\theta| \leq 0.2$ degrees by matching a $\sin x/x$ pattern to the 3-dB points of a 26-meter-diameter antenna (JPL Spec DOW-1389-DTLA).

Since only the degradation due to angle error is to be determined, it is not necessary to deal with the effective gain of the antenna. The antenna pattern can be normalized with respect to the isotropic gain so that the pattern can be represented as

$$|G(x)| = \frac{\sin x}{x} \tag{1}$$

where

$x = 8.43\,\theta$

and

θ = antenna pointing angle, in degrees, referenced to a transmitting source

Determination of the mean and variance of $\sin x/x$ can be complicated even though x is a gaussian random variable. But since the region of interest is small and centered at the antenna gain maximum, it is possible to represent $\sin x/x$ as the first two terms of a Taylor series expansion:

$$\begin{aligned}\frac{\sin x}{x} &= \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots}{x}\\ &\approx 1 - \frac{x^2}{6}\\ &\doteq \hat{G}(x)\end{aligned} \tag{2}$$

The approximation is fairly accurate for an angle error as large as $\theta = 0.15$ degrees (3.2% error at $\theta = 0.15$ degrees).

Using the gaussian statistics for the angle error and the approximate equation for antenna gain, it is possible to determine the mean and the variance of the antenna gain as a function of the mean (boresight error) and the variance (angle jitter) of the angle error.

The mean of the antenna gain represented by $\hat{G}(x)$ is

$$\begin{aligned}\eta_G &= E(\hat{G}(x)) = E\left(1 - \frac{x^2}{6}\right)\\ &= 1 - \frac{1}{6}E(x^2)\end{aligned} \tag{3}$$

$E(x^2)$ is the second moment of x:

$$E(x^2) = \sigma_x^2 + \eta_x^2 \tag{4}$$

where

η_x = mean of x

σ_G^2 = variance of x

The variance of $\hat{G}(x)$ is found as follows:

$$\begin{aligned}\sigma_G^2 &= E[G(x)^2] - E[G(x)]^2\\ &= E[G(x)^2] - \eta_G^2\end{aligned} \tag{5}$$

The first term is the second moment of $\hat{G}(x)$:

$$\begin{aligned}E[\hat{G}(x)^2] &= E\left[\left(1 - \frac{x^2}{6}\right)^2\right]\\ &= 1 - \frac{1}{3}E(x^2) + \frac{1}{36}E(x^4)\end{aligned} \tag{6}$$

Combining the mean with the second moment yields

$$\sigma_G^2 = \frac{1}{36}[E(x^4) - E^2(x^2)] \tag{7}$$

The second moment was stated above. The fourth moment is found as follows, using the known moments for gaussian random variables (Ref. 2):

$$\begin{aligned}E(x - \eta_x)^4 &= 3\sigma_x\\ &= E(x^4) - 4\eta_x E(x^3) + 4\eta_x\sigma_x^2 - 5\eta_x^4\end{aligned}$$

Then,

$$E(x^4) = 3\sigma_x^4 - 4\eta_x E(x^3) + 4\eta_x\sigma_x^2 - 5\eta_x^4 \tag{8}$$

Now solving for the third moment in a similar manner,

$$E(x - \eta_x)^3 = 0$$

so that

$$E(x^3) = 3\eta_x\sigma_x^2 - 2\eta_x^3 \tag{9}$$

Substituting this equation into the equation for the fourth moment results in

$$\begin{aligned}E(x^4) = 3\sigma_x^4 &+ 4\eta_x(3\eta_x\sigma^2 - 2\eta_x^2)\\ &- 4\eta_x\sigma_x^2 - 5\eta_x^4\end{aligned} \tag{10}$$

The complete expression for the gain variance can now be determined:

$$\sigma_G^2 = \frac{\sigma_x^4}{18} + \frac{\eta_x^2\,\sigma_x^2}{18} - \frac{2}{9}\eta_x^3 - \frac{1}{9}\eta_x\sigma_x^2 - \frac{\eta_x^4}{6} \tag{11}$$

This equation and Eq. (3) determine the variance and the mean of the antenna gain, respectively, where

$$\eta_x = 8.43\,\eta_\theta$$

$$\eta_\theta = \text{the mean angle error, degrees} \tag{12}$$

and

$$\sigma_x^2 = (8.43)^2\sigma_\theta^2$$

$$\sigma_\theta^2 = \text{the variance of the angle error, degrees} \tag{13}$$

The mean receiver power degradation due to degradation of antenna gain is then

$$d_m = 20\log_{10}\eta_G \tag{14}$$

The additional degradation due to the variance of the antenna gain can be defined as

$$d_v = 10\log(\eta_G^2 + \sigma_G^2) - d_m \tag{15}$$

This equation is not as significant as Eq. (14) but does show the expected variation of the gain.

As an application of these equations, assume that the pointing error is equal to zero, i.e.,

$$\eta_\theta = 0$$

Then,

$$\eta_G = 1 - \frac{\sigma_x^2}{6}$$

and

$$\sigma_G^2 = \frac{\sigma_x^4}{18}$$

Values for both d_m and d_v were calculated for various values of angle error variance. The results are presented in Table 1. It can be seen that over the degradation range of interest d_v can be neglected. Note that the table gives the angle error variance for 0.1-dB degradation as 0.031 degrees and 0.305-dB degradation for 0.054 degrees. These are the values which will be maintained during auto tracking and thus define the auto tracking threshold.

III. Angle Error Variance

It was shown in Ref. 1 that the output of the detector multiplier is

$$v_e(t) = \frac{K_r K_\theta}{2}\sqrt{2S}\,\theta\cos\phi_e + \frac{K_r}{2}n_{ce}(t)\cos\phi_e \tag{16}$$

where

θ = antenna pointing error

K_r = gain in the error channel including AGC and the multiplier gain

S = average received power

ϕ_e = receiver tracking loop phase error

K_θ = gain of the effective error pattern

and $n_{ce}(t)$ is gaussian white noise. Equation (16) represents the voltage which drives the antenna servo with the system operating in its linear region. This model is illustrated in Fig. 1.

The phase error ϕ_e from the receiver tracking loop is a function of the noise in the reference channel which can be represented as narrow-band gaussian noise with a two-sided spectral noise density $N_0/2$. The noise $n_{ce}(t)$ is due to the noise in the error channel which has a density $N_{0e}/2$. The densities in the two channels are different since the reference channel uses a low-noise maser front end while the error channels use standard preamplifiers.

The term $n_{ce}(t)\cos\phi_e$ can be shown to be approximately gaussian with the variance equal to the variance of $n_{ce}(t)$ alone if the following condition exists:

$$N_{0e}B_s \gg N_0 B_1$$

where

B_s = noise bandwidth of the error channel

B_1 = noise bandwidth of the receiver carrier tracking

This condition is met due to the different front ends and the narrowness of the carrier tracking loop relative to B_s. Roughly $N_{0e}B_s$ is 20 dB greater than N_0B_1.

With this condition, Eq. (16) can be written as

$$v_e(t) = \frac{K_\theta K_r}{2} \sqrt{2S}\, \theta + \frac{K_r}{2}\, n_{ce}(t)$$

This equation is valid over errors as large as ±½ the antenna beamwidth due to AGC linearizing effect (Ref. 3).

From Eq. (16) the angle error variance can be seen to be

$$\sigma_\theta^2 = \frac{1}{K_\theta^2}\left(\frac{N_{0e}\, B_n}{S}\right)$$

where B_n is the noise bandwidth of the auto track control system.

The gain of the effective error pattern can be written as

$$K_\theta = \frac{K_s}{\theta_B}$$

where

K_s = normalized error—detection slope

θ_B = antenna 3-dB bandwidth

In practice monopulse systems are designed so that K_s ranges between 1.2 and 1.9. The value used when maximum antenna gain is desired is $K_s = 1.57$.

The equation for angle variance was evaluated for a noise temperature of 3000 K (10-dB noise figure) and noise bandwidths of 0.025, 0.05, and 0.1 Hz. These values are ideal parameters so that the resulting curves predict the ideal performance of the auto track system. These results are shown in Fig. 2 which presents angle error variance versus signal level. Also shown in this figure are the values of angle error variance for receiver degradation of 0.3 and 0.1 dB.

Data were taken at DSS 12 using the TD 11 test procedure and software (Ref. 1). The noise bandwidth of the auto track control system had been previously adjusted to be 0.025 Hz and the AGC voltage calibrated to give signal level measurements. The data are presented in Fig. 2.

The data are 0.6 to 1 dBmW different from the theoretical curve. The difference may be due to a number of error contributions, such as accuracy of the AGC calibration, noise bandwidth measurement, and error introduced by the carrier tracking loop. The largest error contribution is probably due to the estimate of noise temperature. It is planned that an error analysis will be performed in order to quantitatively define the errors.

IV. Summary

A method for estimating receiver degradation due to angle error variance has been developed. This result can be used to set requirements for angle error variance tests. The angle error variance equation was determined and the normalized error detection slope K_s was defined. The measured data were found to agree very well with the theoretical curve.

Further testing will be performed in order to establish the validity of the equation for angle error variance. Once its validity is established, it will be used as detailed specifications for DSIF systems performance.

References

1. Rey, R. D., "Angle Tracking Analysis and Test Development," in *The Deep Space Network Progress Report,* Technical Report 32-1526, Vol. VI, pp. 170–187. Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1971.

2. Papoulis, A., *Probability, Random Variables and Stochastic Processes.* McGraw-Hill Book Co., Inc., New York, 1965.

3. Barton, D. K., *Radar System Analysis.* Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964.

Table 1. Receiver degradation

Angle error variance, deg	Mean receiver degradation, dB	RMS receiver degradation, dB
0.01	0.010	0.000
0.02	0.041	0.0002
0.025	0.065	0.0005
0.03	0.093	0.001
0.031	0.100	0.0012
0.035	0.127	0.0019
0.04	0.166	0.0032
0.045	0.211	0.0053
0.050	0.261	0.0081
0.054	0.305	0.0111
0.055	0.317	0.0120
0.06	0.378	0.0173
0.065	0.446	0.0241
0.07	0.519	0.0329
0.08	0.684	0.0581
0.09	0.875	0.0968
0.1	1.094	0.154
0.11	1.343	0.237
0.12	1.624	0.353
0.13	1.939	0.513
0.14	2.294	0.730
0.15	2.691	1.02

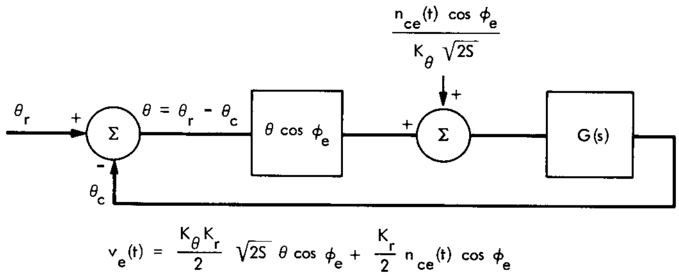

Fig. 1. Angle error model

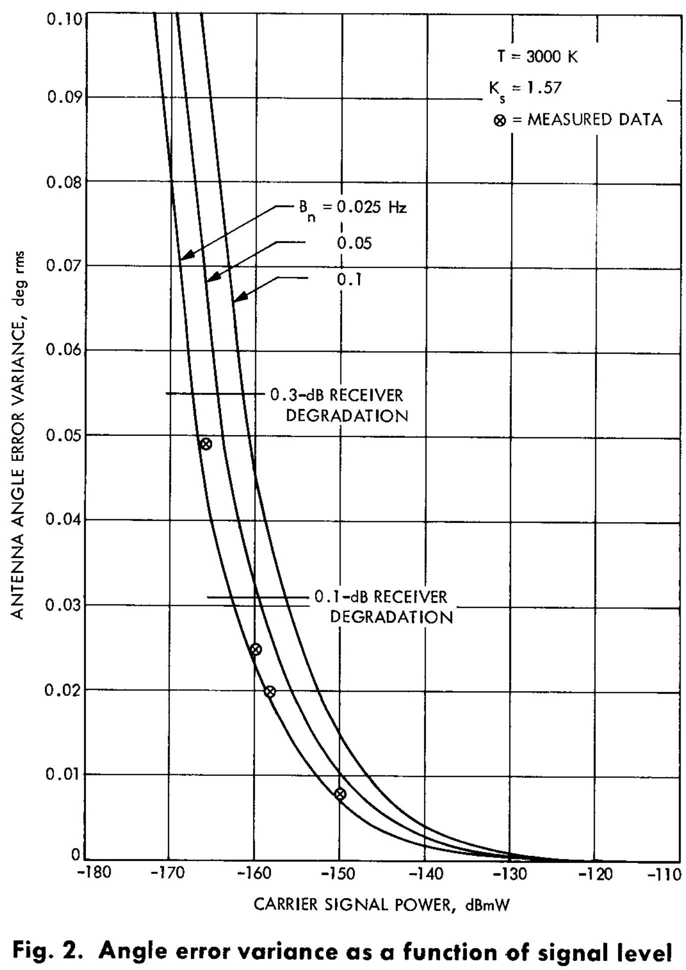

Fig. 2. Angle error variance as a function of signal level