Determining the Mass and Ephemeris of Saturn by Radio Tracking of a Jupiter-Saturn-Pluto 1977 Spacecraft

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Preliminary estimates of the accuracies with which the mass and ephemeris of Saturn may be determined from radio tracking of an outer planets spacecraft are presented. It is shown that the determination of these parameters should employ radio data taken simultaneously from two stations. Indications are that the uncertainties may be reduced by approximately three orders of magnitude in the mass and by a factor of two in the ephemeris.

I. Introduction

The preliminary orbit determination capabilities associated with various types of radio data for the Saturn portion of a Jupiter-Saturn-Pluto 1977 (JSP77) mission have been established.[1] It was found that obtaining accurate estimates of the planet-centered spacecraft state required estimating the mass and ephemeris of Saturn. Thus, improved knowledge of Saturn's mass and ephemeris will be a byproduct of the spacecraft orbit determination procedure.

The following sections will show the mass and ephemeris accuracy results for solutions made with long and short arcs of three-station, single-station, and differenced data which are corrupted by data noise, constant station location errors, and particular types of unmodeled spacecraft accelerations. If the data are processed properly and

the above error sources are the only ones present, the knowledge of the mass may be improved three orders of magnitude from its *a priori* value and some of the ephemeris elements may be improved by a factor of four. The surprising conclusion resulting from this study is that apparently the most reliable solutions for the planetary ephemeris may be made with explicitly differenced range and range-rate data taken simultaneously from widely separated stations. Thus, implementation of a tracking system which obtains simultaneous data may allow significant improvement in the results of outer planet celestial mechanics experiments.

II. Data, Error Sources, and Solution Sets

The detailed information describing the trajectory, data, data weights, error sources, *a priori*, estimated parameters, and various solution sets is given in Sections II to V of the article referred to in Footnote 1. Briefly, the solution sets which are studied here involve the use of range and range-rate data taken either from three stations

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[1]See "Preliminary Evaluation of Radio Data Orbit Determination Capabilities for the Saturn Portion of a Jupiter-Saturn-Pluto 1977 Mission," by V. J. Ondrasik, et al., in this issue.

or from a single station, or explicitly differenced range and range-rate data taken simultaneously from widely separated stations. The solution is assumed to be made with a conventional least squares filter which estimates spacecraft state, constant spacecraft accelerations, Saturn's GM, and Saturn's ephemeris in terms of Brouwer and Clemence Set III elements (DA, DE, DMW, DP, DQ, EDW). A discussion of the advantages offered by this set of elements is given in Ref. 1. The data arc starts at either encounter (E) − 120 days (long arc solutions) or E − 34 days (short arc solutions) and generally terminates at E + 4 days or E + 28 days. The batch filter solutions based upon these data are evaluated under the corrupting influence of (1) data noise, (2) 1 m, 2 m, and 5 m errors in the station distance off the spin axis, longitude, and distance from the equator, and (3) unmodeled stochastic spacecraft accelerations of standard deviation 10^{-12} km/s^2 with a correlation time of six days. The effect of the unmodeled accelerations scales directly with their standard deviations.

III. Accuracy of the Mass Solution

Figure 1 shows the accuracy of the estimate of Saturn's mass (GM) using long arcs of three-station, single-station and differenced data. The effects of random data noise and constant station location errors are accounted for in the conventional consider standard deviations which are shown by open bars; the effects of unmodeled spacecraft accelerations only are shown as solid bars.

The figure shows that improvement of GM from its *a priori* value of 3×10^4 km^3/s^2 (0.1%) may occur very early. If spacecraft accelerations are ignored, it is clear that the three-station and single-station solutions have approximately equal accuracy and both are much superior to the differenced data solutions. The fact that conventional data GM solutions are superior to differenced data solutions is not surprising, because most of the information concerning the planetary mass is supplied through the planetary acceleration and, as discussed in Ref. 2, almost all of this information is deleted by the differencing procedure.

However, it is to be expected that using three-station or single-station data will yield solutions that may be sensitive to unmodeled spacecraft accelerations. This is indeed the case as shown in Fig. 1. The spacecraft accelerations will seriously degrade the solution for GM made with three-station or single-station conventional data. For example, spacecraft accelerations of 10^{-12} km/s^2 typically degrade the GM solutions based upon three-station and single-station data by two and one orders of magnitude,

respectively, over the accuracies associated with data noise and constant station location errors. Presumably, for the reasons given in Ref. 2, the single-station data solution is less sensitive to the unmodeled accelerations than the three-station data solution because of the difference in the weight given to the range data. When post-encounter data are used, the sensitivity of the GM solution to unmodeled accelerations cannot be reduced below the single-station sensitivity by using differenced data. Thus it appears that the best long arc GM solution would be based upon conventional data with loosely weighted (300 m) range.

Error analyses were also performed using short data arcs. The resulting accuracy estimates were similar to the long arc results and are shown in Fig. 2.

If the spacecraft is subject to unmodeled accelerations less than 10^{-12} km/s^2 and if the estimate uncertainties are not particularly sensitive to the statistics describing these accelerations, it appears that the GM of Saturn can be estimated with an accuracy of 10^2 km^3/s^2 or 0.0003%.

IV. Ephemeris Accuracies

Figure 3 shows how data noise, constant station location errors, and a particular type of unmodeled acceleration may degrade the solution for the elements of Saturn's ephemeris based upon the long arcs of three-station, single-station, and differenced data. Once again, the effects of data noise and constant station location errors are combined and are shown as the open bars in Fig. 3; the effects of unmodeled accelerations only are shown as solid bars.

This figure also shows that, at most, only four of the Brouwer and Clemence Set III elements can be improved, and if the data are not processed properly some of the elements may be degraded from their *a priori* values.

The surprising feature of Fig. 3 is that the differenced data solution is superior to the conventional three-station or single-station solutions even if there are no unmodeled spacecraft accelerations. The superiority of the differenced data solution is substantially increased if the effects of unmodeled accelerations are taken into account, because the differenced data solution has virtually no sensitivity to spacecraft accelerations while the three-station and single-station solutions do.

The effects of data noise, constant station location errors, and unmodeled spacecraft accelerations on ephemeris solutions based upon short arcs of data are shown in

Fig. 4. Comparison of Figs. 3 and 4 shows that the uncertainties due to the data noise and constant station locations are very similar for the long data arc and short data arc solutions. However, the short data arc solutions based upon three-station or one-station conventional data are much less sensitive to unmodeled accelerations than are the long arc solutions. A brief discussion of why this may occur is given in Ref. 3.

From Figs. 3 and 4 it appears that, if a batch filter is used, and if only the error sources considered above are present, the most reliable estimate of Saturn's ephemeris will be based upon explicitly differenced data. Using the differenced data the uncertainties in the Brouwer and Clemence Set III elements DE, DMW, and DP may be reduced by 50% and those in EDW by a factor of four. This corresponds roughly to reducing the one-sigma uncertainty in Saturn's position at encounter from the *a priori* of 1500 to 850 km.

V. Summary and Discussion

The results of the preceding sections indicate that the solutions for Saturn's GM and ephemeris should be based upon different orbit determination procedures. The best radio data GM solutions are apparently made with conventional data having a loose (300 m) data weight assigned to the range data. If the spacecraft experiences 10^{-12} km/s^2 accelerations of the type considered in this article it should be possible to determine GM to 0.0003%. The solution seems to be limited by the effects of these accelerations unless their magnitudes are less than 10^{-13} km/s^2.

The surprising result of this work was that the best ephemeris solutions may be made with explicitly differenced data whether or not the spacecraft is subject to unmodeled accelerations. With the differenced data it should be possible to reduce the uncertainties in the elements DE, DMW, DP, and EDW from their *a priori* values of 0.3×10^{-6}, 0.6×10^{-6}, 0.3×10^{-6}, and 0.3×10^{-6} to 0.2×10^{-6}, 0.4×10^{-6}, 0.2×10^{-6} and 0.07×10^{-6}, respectively.

Although a specific mission to Saturn was considered in this article, the results are probably representative of those for most of the outer planet missions under consideration. Thus, the implementation of a tracking system which provides simultaneous data should significantly increase the precision of the information obtained from outer planet celestial mechanics experiments.

As is the case with all accuracy analysis studies the results shown in the preceding sections are based upon particular error models. Thus, these results are representative of the accuracies actually obtainable only to the extent to which the error model is representative of the true error sources. This study, for example, has not dealt with effects introduced by the gravitational attraction of the spacecraft by either Saturn's rings or by its satellites.

References

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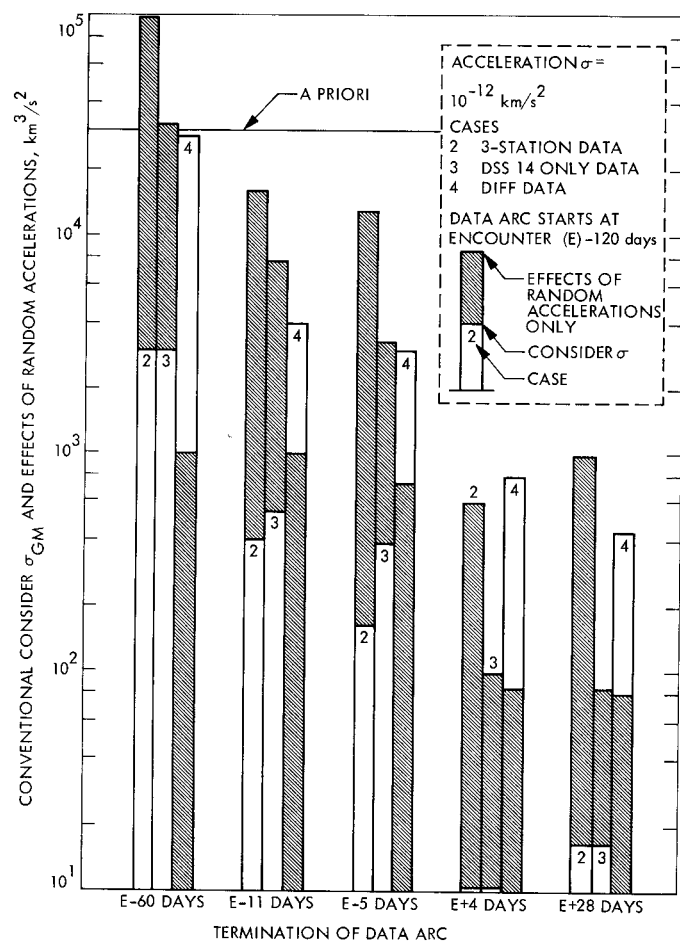
Fig. 1. Effects of data noise, station location errors, and spacecraft accelerations on a GM solution based on a long data arc
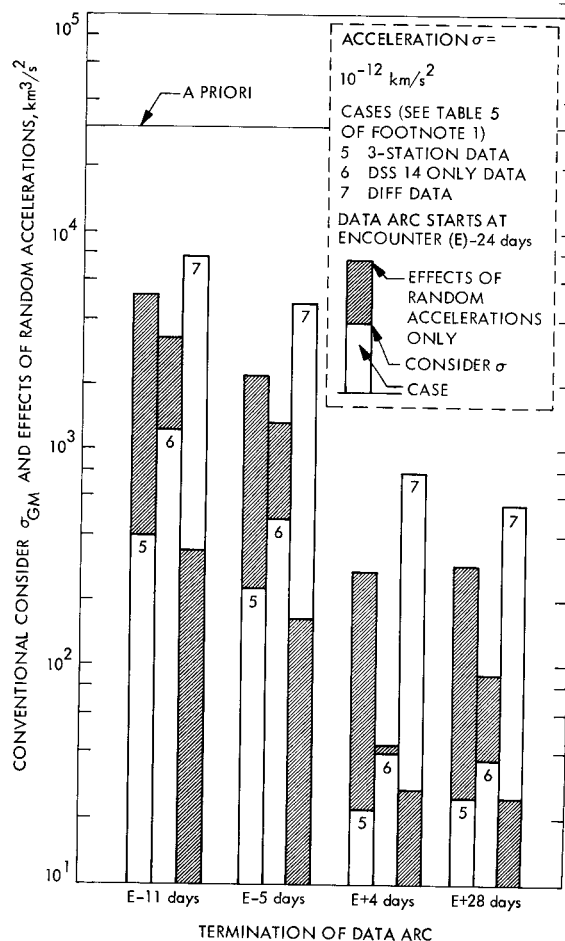

Fig. 2. Effects of data noise, station location errors, and spacecraft accelerations on a GM solution based on a short data arc
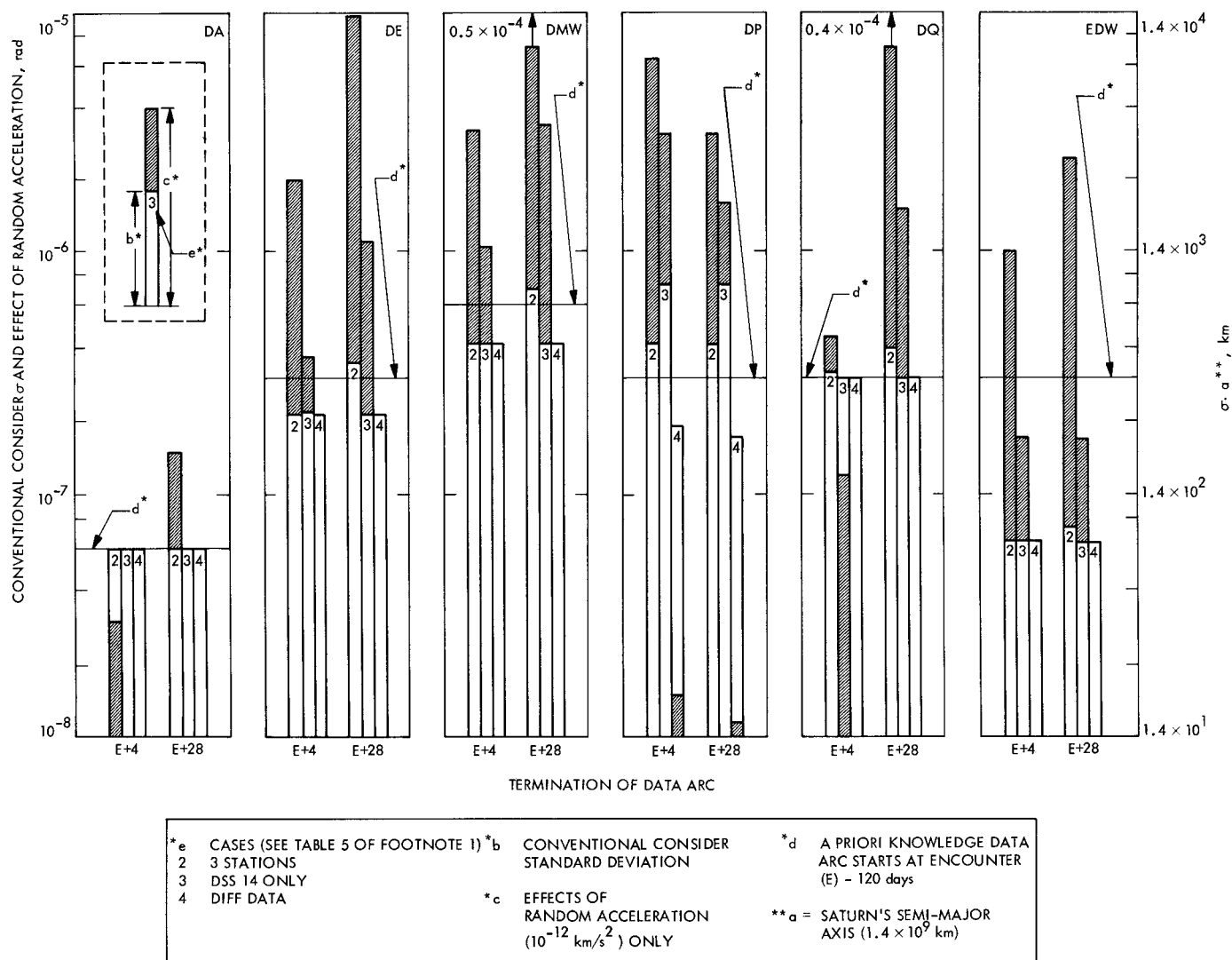
Fig. 3. Effects of data noise, station location errors, and random accelerations on ephemeris solution based on long data arcs

Fig. 4. Effects of data noise, station location errors, and random accelerations on ephemeris solution based on short data arcs