Spectrum of an Asynchronously Biphase Modulated Square Wave

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An exact closed-form expression for the power spectrum of a square-wave carrier (or subcarrier) which is biphase modulated by a random binary data stream is presented. The resulting expression is valid for any carrier frequency and data bit rate, provided that the two sources are not phase-coherently related. Also presented is an approximate expression which can be used to alleviate some computational difficulties of the exact expression at low spectral frequencies.

I. Introduction

Equations for the spectrum of a phase-modulated sinusoidal carrier can be found in almost any elementary textbook on communications. Yet there appears to be little in the literature on spectra resulting from square-wave carriers. Some work has been done by Titsworth and Welch (Ref. 1) and later by Levitt (Ref. 2). These studies, however, were concerned with the case where the data bit stream and carrier are phase coherently related, with the ratio of the data bit to carrier periods an integer, or at most an elementary rational number.

In this article we shall determine an exact, closed-form expression for a square-wave carrier biphase modulated by a random binary data stream of any fixed bit rate. We shall require only that the data stream and carrier be generated in a noncoherent manner.

II. Spectrum Derivation

Let $s(t)$ denote a square-wave carrier taking on the values of $+1$ and -1 with period T_s. The spectrum $S_s(\omega)$ of this signal is given by

$$S_s(\omega) = \sum_{k=1}^{\infty} \frac{4}{\pi^2 (2k-1)^2} \times \{\delta[\omega - (2k-1)\omega_s] + \delta[\omega + (2k-1)\omega_s]\} \tag{1}$$

where $\omega_s = 2\pi/T_s$ and $\delta(.)$ is the Dirac delta function. Also consider a random binary data stream $b(t)$ which assumes the values of $+1$ or -1 with equal probability and with bit period T_b. It is well known that the spectrum of b is given by

$$S_b(\omega) = \frac{4\sin^2\left(\frac{\omega T_b}{2}\right)}{T_b \omega^2} \tag{2}$$

Now, if s and b are not coherently related, then (Ref. 1) the spectrum $S(\omega)$ of $s(t)$, biphase modulated by $b(t)$, could be found by the convolution of $S_s(\omega)$ with $S_b(\omega)$; that is,

$$S(\omega) = \frac{16}{\pi^2}\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\left\{\frac{\sin^2\left[\frac{(\omega-(2k-1)\omega_s)T_b}{2}\right]}{T_b[\omega-(2k-1)\omega_s]^2} + \frac{\sin^2\left[\frac{(\omega+(2k-1)\omega_s)T_b}{2}\right]}{T_b[\omega+(2k-1)\omega_s]^2}\right\} \tag{3}$$

However, a closed form for Eq. (3) does not appear to be readily available. Therefore, let us proceed temporarily in the time domain. From transform theory we know that

$$S(\omega) = F\{R(\tau)\} \tag{4}$$

where $F\{.\}$ is the Fourier transform,

$$R(\tau) = R_s(\tau)R_b(\tau) \tag{5}$$

and $R_s(\tau)$ and $R_b(\tau)$ are the autocorrelation functions of $s(t)$ and $b(t)$ respectively. It is clear that

$$R_s(\tau) = 1 - \frac{4|\tau|}{T_s}; \qquad -\frac{T_s}{2} \leq \tau \leq \frac{T_s}{2} \tag{6}$$

$$R_b(\tau) = 1 - \frac{|\tau|}{T_b}; \qquad -T_b \leq \tau \leq T_b \tag{7}$$

and that $R_s(\tau + kT_s) = R_s(\tau)$, $k = \cdots, -1, 0, 1, 2, \cdots$. The functions $R_s(\tau)$ and $R_b(\tau)$ are shown in Figs. 1 and 2 respectively.

Now, let us define n as the greatest integer in the quantity

$$\frac{2T_b}{T_s}$$

that is, n is the number of carrier half-periods which can be totally contained in one data bit time. Then, with the aid of Figs. 1 and 2, one can easily show that the spectrum of the modulated carrier is given by

$$S(\omega) = 2\sum_{k=1}^{n}\int_{[(k-1)T_s]/2}^{kT_s/2}(-1)^{k-1}\left(1-\frac{\tau}{T_b}\right)\left[(2k-1)-\frac{4\tau}{T_s}\right]\cos\omega\tau\,d\tau + 2\int_{nT_s/2}^{T_b}(-1)^n\left(1-\frac{\tau}{T_b}\right)\left[(2n+1)-\frac{4\tau}{T_s}\right]\cos\omega\tau\,d\tau \tag{8}$$

After integrating we have, for $\omega \neq 0$,

$$S(\omega) = \frac{32}{T_sT_b\omega^3}\sum_{k=1}^{n}(-1)^k\sin\frac{k\omega T_s}{2} + \frac{8}{T_b\omega^2}\sum_{k=1}^{n}(-1)^{k-1}k\cos\frac{k\omega T_s}{2} + \frac{16}{T_s\omega^2}\sum_{k=1}^{n}(-1)^k\cos\frac{k\omega T_s}{2} + \frac{2}{T_b\omega^2} + \frac{8}{T_s\omega^2} + \frac{2(-1)^n}{\omega^2}\left[\frac{4}{T_s}-\frac{(2n+1)}{T_b}\right]\cos\omega T_b + \frac{16(-1)^{n-1}}{T_sT_b\omega^3}\sin\omega T_b, \qquad \omega \neq 0 \tag{9}$$

Equation (9), even though in closed form, is unattractive computationally since the number of computations depends on the size of n. Fortunately, we can further simplify this expression. First we note that

$$\sum_{k=1}^{n}(-1)^{k-1}k\cos\frac{k\omega T_s}{2} = -\frac{2}{T_s}\frac{d}{d\omega}\left\{\sum_{k=1}^{n}(-1)^k\sin\frac{k\omega T_s}{2}\right\} \tag{10}$$

Then by using the identities (Ref. 3)

$$\sum_{k=1}^{n}(-1)^k\sin kx = \frac{\sin\left[\frac{n(x+\pi)}{2}\right]\sin\left[\frac{(n+1)(x+\pi)}{2}\right]}{\cos\frac{x}{2}} \tag{11}$$

and

$$\sum_{k=1}^{n}(-1)^k\cos kx = \frac{1}{2}\left[\frac{(-1)^n\cos\left(\frac{2n+1}{2}x\right)}{\cos\frac{x}{2}} - 1\right] \tag{12}$$

we have the power spectrum given by

$$S(\omega) = \frac{2(-1)^n}{\omega^2}\left[\frac{4}{T_s} - \frac{(2n+1)}{T_b}\right]\left[\cos\omega T_b + \frac{\cos\frac{(2n+1)\,\omega T_s}{4}}{\cos\frac{\omega T_s}{4}}\right]$$

$$- \frac{2}{T_b\omega^2}\left[\frac{(-1)^n \sin\frac{(2n+1)\,\omega T_s}{4}\sin\frac{\omega T_s}{4}}{\cos^2\left(\frac{\omega T_s}{4}\right)} - \frac{1}{\cos^2\left(\frac{\omega T_s}{4}\right)} - 1\right]$$

$$+ \frac{16}{T_s T_b \omega^3}\left\{(-1)^n\left[\frac{\sin\frac{(2n+1)\,\omega T_s}{4}}{\cos\frac{\omega T_s}{4}} - \sin\omega T_b\right] - \frac{\sin\frac{\omega T_s}{4}}{\cos\frac{\omega T_s}{4}}\right\} \tag{13}$$

where

$$\omega \neq 0, \qquad \omega \neq \frac{2m\pi}{T_s}, \qquad m \text{ is an odd integer.}$$

Equation (13) is the desired expression for the modulated square-wave spectrum. Note, however, that it need not yield the correct result at direct current ($\omega = 0$) and at the carrier frequency harmonics

$$\omega = \frac{2m\pi}{T_s}, \qquad m = \text{odd integer}$$

Consequently, we must determine the expressions for the spectrum at these points. For $\omega = 0$, we can use Eq. (8) without the cosine functions to obtain

$$S(0) = \frac{T_s^2}{24T_b}[1 - (-1)^n]$$

$$+ (-1)^n\left[(2n+1)\,T_b - n(n+1)\,T_s + \frac{n^2T_s^2}{12T_b}(2n+3) - \frac{4T_b^2}{3T_s}\right] \tag{14}$$

When ω corresponds to the carrier harmonics, we can return to Eq. (9) and evaluate the summations directly. The resulting expression is

$$S\left(\frac{2m\pi}{T_s}\right) = \frac{(-1)^n T_s^2}{2\pi^2 m^2}\left[\frac{4}{T_s} - \frac{2n+1}{T_b}\right]\cos\left(\frac{2m\pi T_b}{T_s}\right)$$

$$+ \frac{(2n+1)\,T_s}{\pi^2 m^2} - \frac{2(-1)^n T_s^2 \sin\left(\frac{2m\pi T_b}{T_s}\right)}{\pi^3 m^3 T_b}$$

$$- \frac{(2n^2+2n-1)\,T_s^2}{2\pi^2 m^2 T_b}, \qquad m = \text{odd integer} \tag{15}$$

(Actually, by using limit arguments, one can show that Eq. (13) goes to Eqs. (14) and (15) at the appropriate points and hence is valid for all ω.)

By using Eqs. (13–15), one should be able (at least in theory) to compute the desired spectrum for any value of ω. Practically, however, one finds that a significant error results when Eq. (13) is used at relatively low values of ω. This error results from the computational accuracy needed to compute Eq. (13) at these frequencies. For example, the terms of Eq. (13) may be of the order of 10^{+5} while their sum is of the order of 10^{-5}. To circumvent this problem, we can expand the trigonometric functions of Eq. (13) in their appropriate Taylor series. If we retain only the first two terms of each expansion, Eq. (13) becomes

$$
\begin{aligned}
S(\omega) = {} & \frac{2T_s^2}{3T_b(32-\omega^2T_s^2)^2}\{64[1-(-1)^n]-(-1)^n(6n^4-6n^2-11)\omega^2T_s^2+\omega^2T_s^2\} \\
& + 4(-1)^n\left[\frac{2(2n+3)n^2T_s^2}{3T_b(32-\omega^2T_s^2)}-\frac{T_b^2}{3T_s}-\frac{8n(n+1)T_s}{32-\omega^2T_s^2}\right] \\
& + (-1)^n(2n+1)T_b, \qquad |\omega| \text{ small}
\end{aligned} \tag{16}
$$

Note that Eq. (16) reduces to Eq. (14) when $\omega = 0$.

The frequency below which Eq. (16) should be used rather than Eq. (13) will naturally depend on the problem to be solved. One limit which appears to work quite well is for

$$|\omega| < \frac{0.02}{T_s}$$

However, one may wish to use other limits depending on the carrier frequency, data bit rate, and computational accuracy available.

To see the results of these equations, refer to Figs. 3 and 4. Figure 3 shows the computed spectrum of a 100-Hz square-wave carrier modulated by a 10-bit/s data stream. The data signal spectrum is clearly located around each of the carrier harmonics. Figure 4 shows the spectrum of the same carrier when the data bit rate is increased to 81.3 bits/s. One can easily see the interference resulting from neighboring harmonics.

III. Conclusion

The spectrum of a square-wave carrier asynchronously biphase modulated by binary random data is derived and presented in the form of Eqs. (13–15). These equations are useful in that they are exact expressions and are given in closed form. It was pointed out, however, that Eq. (13) requires extremely high computational accuracies when evaluated at low spectral frequencies. For this reason, an approximate expression (Eq. 16), which is valid at low frequencies, was determined and can be used in place of Eq. (14).

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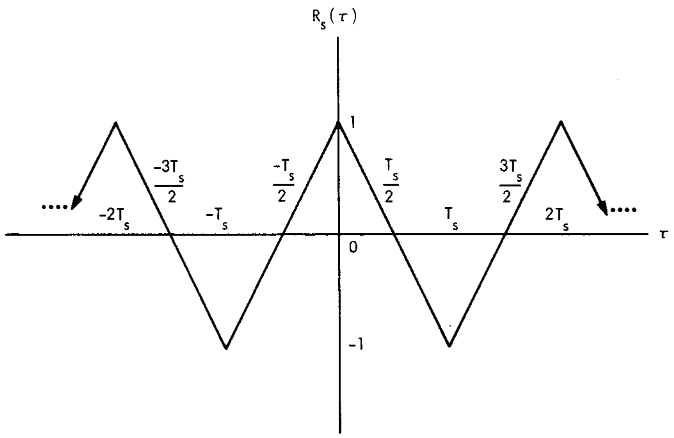

Fig. 1. Autocorrelation function of s (*t*)

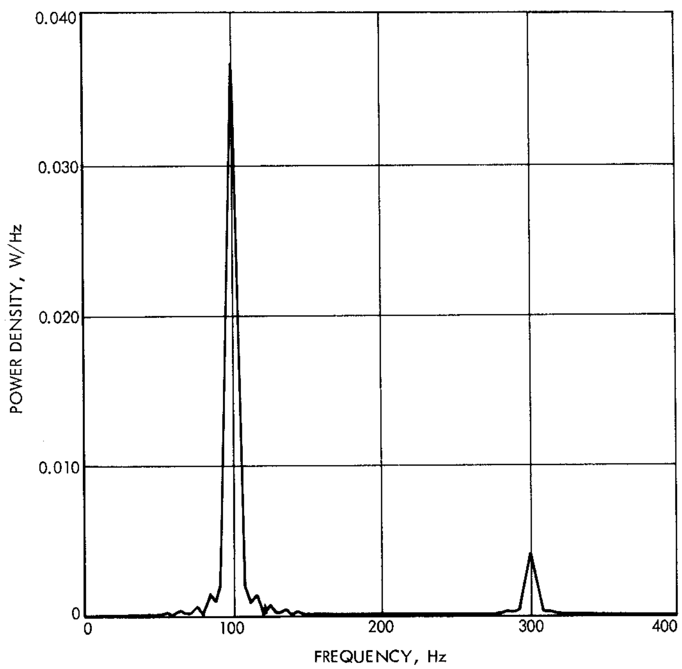

Fig. 3. Computed power spectrum of a 100-Hz carrier modulated by a 10-bit/s data stream

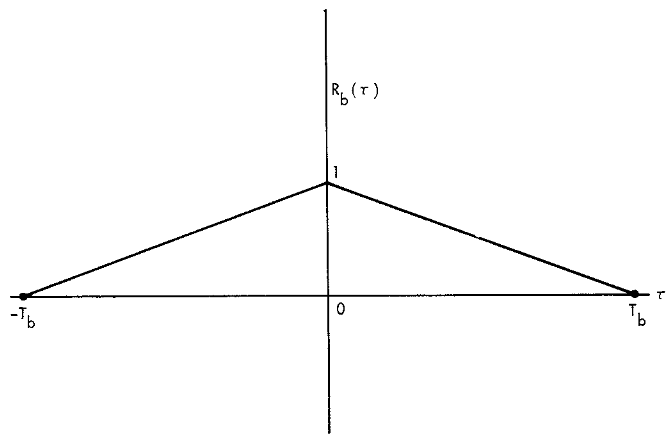

Fig. 2. Autocorrelation function of *b* (*t*)

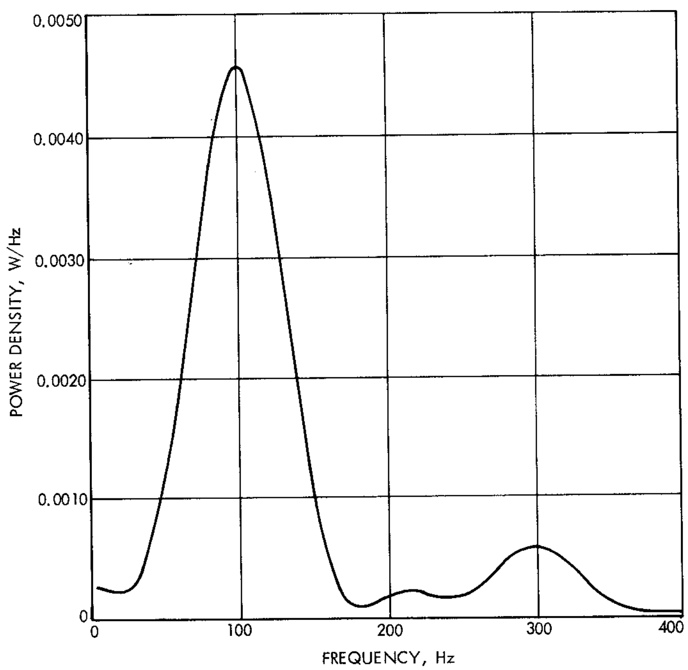

Fig. 4. Computed power spectrum of a 100-Hz carrier modulated by a 81.3-bit/s data stream

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