Complex Mixer Error Analysis

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The complex mixer is composed of two channels. One channel contains the signal mixed with the sine of the reference frequency and the other contains the signal mixed with the cosine. Errors unique to this system are gain and phase shifts of one channel with respect to the other. When the power spectrum of the output of the mixer, considered as a complex quantity, is calculated, these errors produce an unwanted image response to each signal component in the true spectrum. This analysis was carried out to ensure that hardware specifications were sufficient to limit these image responses to tolerable levels. Calculations for various gain and phase errors show that image responses in the power spectrum for prototype hardware will be limited to less than 1% of the true signal components.

I. Introduction

The complex mixer is a receiver module which converts the intermediate-frequency (IF) signal from the receiver down to baseband frequency in two parallel channels. One channel mixes the IF signal with the sine of the reference frequency and the other mixes the IF signal with the cosine. The output of the first channel is considered to be the real component of the complex baseband signal and the output of the second channel the imaginary component. A computer samples the outputs of both channels and calculates the power spectrum of the baseband signal.

Differences in the gain and phase characteristics of physical hardware for each channel introduce certain errors in the calculated power spectrum. These errors take the form of image or ghost responses mirrored about the zero-frequency axis from the true signal components of the spectrum. In addition, the amplitudes of the true signal components are also slightly in error. Since the power spectrum is usually normalized, consistent errors in amplitude are of little importance. However, the complex mixer system specifically distinguishes signals whose frequencies are below the mixing frequency from those above. Therefore, the introduction of image responses in the power spectrum is of major concern. The following analysis determines the relative magnitude of the ghost image as a function of the gain and phase errors in the complex mixer channels. It will be shown that specifications limiting the gain and phase errors in constructed complex mixer hardware are sufficient to limit the ghost image to less than 1% of the true signal component.

II. Spectral Calculations for an Ideal System

Prior to the error analysis, calculations of the power spectrum for an ideal system will be reviewed to serve as a basis for comparison. Such a system is shown in the simplified block diagram of Fig. 1. The unit power input signal to both channels is displaced from the IF frequency ω_0 by an amount f and contains an arbitrary phase ϕ. In the upper channel, the input signal is multiplied by the sine of the IF frequency, and in the lower channel, it is multiplied by the cosine. Each channel has a low-pass filter, which blocks the sum terms while passing the difference terms of the multiplication operation. These filters are also used to attenuate signals whose frequencies are outside the sampling bandwidth and which otherwise might be aliased into the calculated spectrum.

Considering the output of the upper channel to be the real part of $f(t)$, $Rf(t)$, and the output of the lower channel to be the imaginary part of $f(t)$, $If(t)$, then

$$Rf(t) = -\frac{\sqrt{2}}{2}\sin(2\pi ft + \phi) \tag{1}$$

$$If(t) = \frac{\sqrt{2}}{2}\cos(2\pi ft + \phi) \tag{2}$$

where t is elapsed time. Combining Eqs. (1) and (2) gives

$$f(t) = \frac{\sqrt{2}}{2}\exp\left[i\left(2\pi ft + \phi + \frac{\pi}{2}\right)\right] \tag{3}$$

The analog-to-digital converters in both channels simultaneously sample the real and imaginary parts of $f(t)$ at a sampling period of Δ_t s to provide the sampled signal f_k:

$$f_k = f(k\Delta_t) \tag{4}$$

where k is the number of the particular sample of $f(t)$. The computer calculates the power spectrum P_n as

$$P_n = \frac{2}{N^2}\,|F_n|^2 = \frac{2}{N^2}F_nF_n^* \tag{5}$$

$$F_n = \sum_{k=0}^{N-1} f_k \exp\left(\frac{-i2\pi nK}{N}\right) \tag{6}$$

where n is the number of a particular spectrum point, N is the number of complex points in f_k used to compute the spectrum, and F_n^* is the complex conjugate of F_n.

From Eqs. (3), (4), (5), and (6), P_n is found to be

$$P_n = \frac{1}{N^2}\,\frac{\sin^2(N\pi f\Delta_t)}{\sin^2\left[\pi\left(f\Delta_t - \frac{n}{N}\right)\right]} \tag{7}$$

It is seen from Eq. (7) that

$$P_n = P_{n+N} \tag{8}$$

and

$$P_n(f) = P_n\left(f + \frac{1}{\Delta_t}\right) \tag{9}$$

Moreover, if the frequency difference between adjacent points of the power spectrum P_n is denoted as Δ_f, then

$$\Delta_f\Delta_t = \frac{1}{N} \tag{10}$$

Use of the Fast Fourier Transform to calculate the power spectrum results in points being calculated for n in the range of zero through $N - 1$. From Eq. (8), this range is equivalent to

$$-\frac{N}{2} \leq n \leq \frac{N}{2} - 1 \tag{11}$$

Frequencies outside this range are attenuated by the low-pass filters to prevent the aliasing indicated by Eq. (9).

III. Error Analysis

Unfortunately, the actual physical hardware in the complex mixer channels introduces small changes in gain and phase from the desired nominal values. Thus, Eqs. (1) and (2) become, respectively,

$$Rf(t) = -\frac{\sqrt{2}}{2}A\sin(2\pi ft + \phi + \epsilon_1) \tag{12}$$

$$If(t) = \frac{\sqrt{2}}{2}B\cos(2\pi ft + \phi + \epsilon_2) \tag{13}$$

where A and B are gains relative to unity, and ϵ is the phase error introduced in the indicated channel. Equation (4) becomes

$$f_k = \frac{\sqrt{2}}{4}\left\{ \exp\left[i\left(\phi + \frac{\pi}{2}\right)\right] [A \exp(i\epsilon_1) + B \exp(i\epsilon_2)] \exp(i2\pi f \Delta_t k) \right.$$

$$\left. + \exp\left[-i\left(\phi + \frac{\pi}{2}\right)\right] [A \exp(-i\epsilon_1) - B \exp(-i\epsilon_2)] \exp(-i2\pi f \Delta_t k) \right\} \tag{14}$$

From Eqs. (5), (6), and (14), the power spectrum is

$$P_n = \frac{\sin^2(\pi N f \Delta_t)}{4N^2}\left[\frac{A^2 + B^2 + 2AB\cos(\epsilon_1 - \epsilon_2)}{\sin^2[\pi(f\Delta_t - n/N)]} + \frac{A^2 + B^2 - 2AB\cos(\epsilon_1 - \epsilon_2)}{\sin^2[\pi(f\Delta_t + n/N)]} \right.$$

$$\left. -2\,\frac{A^2\cos[2(\phi + (N-1)\pi f\Delta_t + \epsilon_1)] - B^2\cos[2(\phi + (N-1)\pi f\Delta_t + \epsilon_2)]}{\sin[\pi(f\Delta_t - n/N)]\sin[\pi(f\Delta_t + n/N)]} \right] \tag{15}$$

The first term in the brackets of Eq. (15) is the desired term, while the second and third terms are error terms. If many spectra are successively taken and averaged, then ϕ may be considered a uniform random variable between zero and 2π, resulting in the third term vanishing. Even for individual spectra, computer simulation of the complex mixer system shows that for 64-point spectra and larger, the worst-case increase in the image produced by the third term is less than 20%. For small ϵ, the cosine of $\epsilon_1 - \epsilon_2$ may be approximated by the first two terms of the cosine power series. Thus, the averaged power spectrum $\overline{P_n}$ is

$$\overline{P_n} = \frac{\sin^2(\pi N f\Delta_t)}{4N^2}\left[\frac{(A+B)^2 - AB\Delta_\epsilon^2}{\sin^2[\pi(f\Delta_t - n/N)]} + \frac{(A-B)^2 + AB\Delta_\epsilon^2}{\sin^2[\pi(f\Delta_t + n/N)]} \right] \tag{16}$$

where Δ_ϵ is the difference in ϵ between the two channels. The denominator of the error term in Eq. (16) shows that the error is in the form of a ghost indication at an n which is the negative of the n for the signal frequency f. The ratio P_r of the power in the error term to the power in the desired term is

$$P_r = \frac{(R-1)^2 + R\Delta_\epsilon^2}{(R+1)^2 - R\Delta_\epsilon^2} \tag{17}$$

where R is the ratio of A to B or the ratio of B to A. Figure 2 is a semi-log plot of Eq. (17) for Δ_ϵ in degrees.

IV. Conclusion

From Fig. 2, it is seen that complex mixer channels that are balanced in gain to within 10% and have less than 10 deg differential phase error will give rise to ghost image responses which are attenuated more than 20 dB. Since actual complex mixer hardware is adjusted for a gain balance within 5% and a phase error within 6 deg at all frequencies of interest, this analysis shows that the problem of ghost images in the computed power spectrum may be considered insignificant.

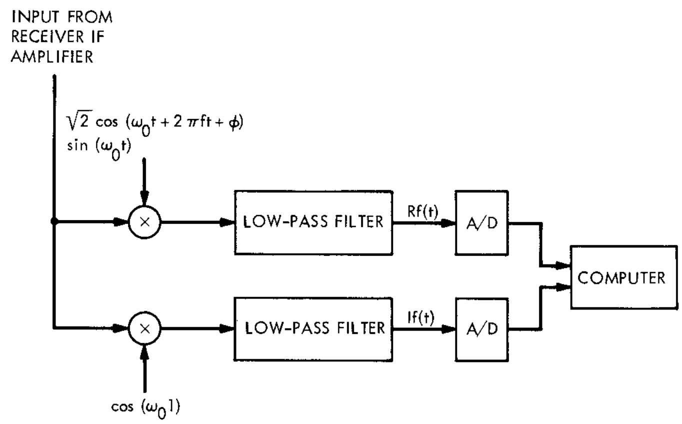

Fig. 1. Ideal complex mixer block diagram

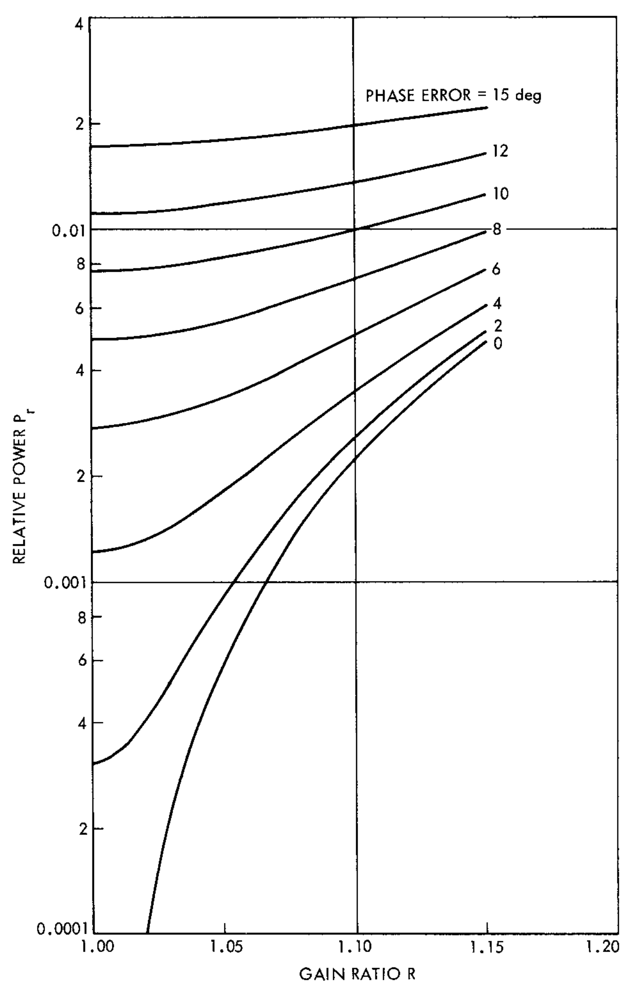

Fig. 2. Ghost image response