Improved RF Calibration Techniques—A Practical Technique for Accurate Determination of Microwave Surface Resistivity

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In this article, a surface loss measurement technique using a TE_{011}-mode circular waveguide cavity resonator is described. A novel feature of this technique is the use of a standard cavity, with a test sample (a flat plate) forming one of the end walls of the cavity. The unique properties of this cavity eliminate the need for intimate contact between the test surface and the rest of the cavity; a surface contact problem is thereby avoided.

Precision Q-measurement equipment is required for this technique. Formulas for the surface resistivity of the sample as a function of the measured cavity Q are derived. Preliminary test data are presented and compared with predicted values, where available. The method described in this article appears to have a wide range of applicability.

I. Introduction

An important consideration in the design, analysis, and calibration of low-noise and/or high-power antenna systems is the dissipative loss associated with the metallic surfaces. Such losses contribute to the receiver system noise temperature and, if nonlinearities exist, contribute noise from transmitter-induced microwave fields. In addition to noise effects, conductor losses cause heating during high-power transmission, which can cause deterioration or failure of microwave components. The calibration of the loss associated with various possible metallic surfaces is thus of importance.

In this reporting, a technique is described which, given the availability of suitable instrumentation, provides rapid and accurate data on the surface resistivity (and hence losses) of any surface of reasonable conductivity (a metal). The dependence of resistivity upon material, surface finish, and discontinuities in the surface can be rapidly evaluated. The required sample of material is a small plate of reasonable but not excessively good flatness. The technique basically involves the precise measurement of the bandpass characteristics of a resonant cavity, one wall of which is formed by the sample under test. A brief analysis of the cavity resonator characteristics is given in the following section.

II. Properties of the TE_{011} Cavity Resonator

A cylindrical waveguide is defined as one in which the cross-sectional size and shape do not change with the longitudinal dimension. Waves propagate along such a waveguide with characteristics rigorously derivable from the scalar wave equation. If two transverse walls are placed across the waveguide, a propagating wave is reflected when it impinges on the two end walls, and a standing wave is created, thus forming a cavity resonator. The unloaded quality factor Q_0 of the resonator is proportional to the electromagnetic stored energy in the resonator divided by the power dissipated in the walls of the resonator. The functional dependence of Q_0 upon the cavity geometry and the surface resistivity R_S of the walls may be accurately calculated. The starting point of this calculation is the waveguide propagation characteristic.

An important and useful cylindrical waveguide is the one in which the walls are smooth, high-conductivity metal of circular cross-section. The fields in the guide must represent a solution of Maxwell's equations, subject to appropriate boundary conditions at the waveguide wall. Two such types of solutions exist: transverse electric (TE) and transverse magnetic (TM) waves. Within each category, a finite number of independent solutions exist (for propagating waves) which are known as modes. Each mode has a specific set of equations which express the field variations within the waveguide as a function of geometry.

One mode in circular waveguides of particular interest is the so-called "low-loss" mode, the TE_{01} mode. The double subscript index indicates that there is no azimuthal variation of field (see Fig. 1) and one radial field variation. The fields in this waveguide are given by (Ref. 1)

$$H_z = BJ_0(k_c r)\exp\left[\mp j\left(\frac{2\pi}{\lambda_g}\right)z\right] \tag{1a}$$

$$E_z = 0 \tag{1b}$$

$$E_\phi = \pm jBZ_0\left(\frac{f_o}{f_c}\right)J_0'(k_c r)\exp\left[\mp j\left(\frac{2\pi}{\lambda_g}\right)z\right] \tag{1c}$$

$$H_r = -jB\left(\frac{\lambda_o}{\lambda_g}\right)\left(\frac{f_o}{f_c}\right)J_0'(k_c r)\exp\left[\mp j\left(\frac{2\pi}{\lambda_g}\right)z\right] \tag{1d}$$

$$E_r = 0 \tag{1e}$$

$$H_\phi = 0 \tag{1f}$$

where

$j = \sqrt{-1}$

B = an arbitrary constant

$J_0(k_c r)$ = zero-order cylindrical Bessel function of the first kind

$J_0'(k_c r) = \dfrac{d}{d(k_c r)}[J_0(k_c r)]$

f_o = operational frequency

f_c = cutoff frequency

λ_o = operational wavelength

λ_g = guide wavelength $= \dfrac{\lambda_o}{\sqrt{1-\left(\dfrac{\lambda_o}{\lambda_c}\right)^2}}$

$k_c a$ = first root (not including $x = 0$) of $J_0'(x) = 0$ $= 1.84118$

a = waveguide radius

z_0 = characteristic impedance of the medium (such as air)

If parallel plates are introduced across the waveguide at a separation of one half guide wavelength ($l = \lambda_g/2$), a TE_{011} mode cavity resonator is formed. The internal fields are given by the superposition of $+z$ and $-z$ traveling waves:

$$H_z = -2jBJ_0(k_c r)\sin\left(\frac{\pi}{l}z\right) \tag{2a}$$

$$H_r = -2jB\left(\frac{\lambda_o}{\lambda_g}\right)\left(\frac{f_o}{f_c}\right)J_0'(k_c r)\cos\left(\frac{\pi}{l}z\right) \tag{2b}$$

$$E_\phi = +2Bz_0\left(\frac{f_o}{f_c}\right)J_0'(k_c r)\sin\left(\frac{\pi}{l}z\right) \tag{2c}$$

The total stored energy in the cavity, U_T, is given by the sum of the time-average electric and magnetic stored energies. Since, at resonance (i.e., $l = \lambda_g/2$), the time-average magnetic and electric stored energies are equal,

$$U_T = \frac{1}{2}\epsilon\iiint_v |E_\phi|^2\,dv \tag{3}$$

$$U_T = 2\pi B^2 z_0^2\left(\frac{f_o}{f_c}\right)^2\epsilon\left(\frac{1}{k_c}\right)^2 lI_1 \tag{4}$$

where

$$I_1 = \frac{(k_c a)^2}{2} J_0^2(k_c a) \tag{5}$$

and ϵ = permittivity of medium (such as air)

The total currents $\mathbf{J}$ (r,ϕ,z) flowing in the cavity walls are given (for any reasonable conductor) by the surface currents which would exist in a cavity with walls of infinite conductivity. These currents are given by the boundary condition

$$\mathbf{J}_s = \mathbf{n} \times \mathbf{H} \tag{6}$$

where $\mathbf{n}$ = outward normal (relative to the surface).

By substitution of (2a) and (2b), it is found that

$$\mathbf{J}_{s_{\mathrm{cyl}}} = -2jBJ_0(k_c a) \sin\left(\frac{\pi}{l} z\right) \mathbf{a}_\phi \tag{7}$$

and

$$\mathbf{J}_{s_{\mathrm{end}}} = -2jB\left(\frac{\lambda_o}{\lambda_g}\right)\left(\frac{f_o}{f_c}\right) J_0'(k_c r)\, \mathbf{a}_\phi \tag{8}$$

where

$$J_0'(k_c a) = 0$$

Examination of (7) and (8) reveals the unique and desirable properties of the TE_{011} mode cavity. The current is entirely circumferential and, moreover, is zero at the intersections of the cylindrical and end walls. These two properties make the contact of the test sample end wall and cylindrical wall uncritical and insensitive to problems such as contact resistance, lack of flatness, etc. This valuable property of the cavity is well known (Ref. 2).

The time-average power loss W_L in one of the cavity walls is given by

$$W_{L_{\mathrm{wall}}} = \frac{R_{s_{\mathrm{wall}}}}{2} \iint_s |J_s|^2 \, ds \tag{9}$$

where $R_{s_{\mathrm{wall}}}$ = surface resistivity of the wall in ohms.

Performing the integrations, it is found that

$$W_{L_{\mathrm{cyl}}} = \frac{R_{s_{\mathrm{cyl}}} 4\pi B^2 a l I_1}{(k_c a)^2} \tag{10}$$

$$W_{L_{\mathrm{end}}} = R_{s_{\mathrm{end}}} 4\pi B^2 \left(\frac{\lambda_o}{\lambda_g}\right)^2 \left(\frac{f_o}{f_c}\right)^2 \frac{1}{(k_c)^2} I_1 \tag{11}$$

The total loss W_L is then given by

$$W_L = \frac{4\pi B^2 I_1 a^2}{(k_c a)^2}\left[R_{s_{\mathrm{cyl}}}\left(\frac{l}{a}\right) + \left(\frac{\lambda_c}{\lambda_g}\right)^2 (R_{s_{\mathrm{end}\,1}} + R_{s_{\mathrm{end}\,2}})\right] \tag{12}$$

The unloaded cavity Q_0 is given by,

$$Q_0 = \frac{\omega_0 U_T}{W_L} \tag{13}$$

where

$$\omega_0 = 2\pi f_o$$

and from (4), (12), and (13),

$$Q_0 = \frac{\left(\frac{\pi}{2}\right) z_0 \left(\frac{\lambda_c}{\lambda_o}\right)^2 \left(\frac{\lambda_g}{\lambda_o}\right)}{\left[R_{s_{\mathrm{cyl}}}\left(\frac{l}{a}\right) + \left(\frac{\lambda_c}{\lambda_g}\right)^2 (R_{s_{\mathrm{end}\,1}} + R_{s_{\mathrm{end}\,2}})\right]} \tag{14}$$

All of the parameters on the right side of Eq. (14) are known except for the surface resistivities; thus, having measured Q_0, it is possible to solve for the surface resistivities.

III. Experimental Method

Accurate Q measurements for cavity resonators with high unloaded Qs require the use of precision measurement equipment. A Hewlett Packard 8320 stabilized microwave system was available for tests described in this article. A photograph of this system and additional equipment used to measure and record signal frequency, amplitude, and phase is shown in Fig. 2. This system[1] was immediately available for the tests described here. A block diagram of the test setup is shown in Fig. 3. The measurement system is amplitude- and frequency-stabilized. The dynamic range is sufficient to accurately

[1]Designed and assembled by P. Hubbard and R. Berwin for the Communications Elements Research Section.

measure test cavities with up to 50-dB transmission loss. The loaded Q of a cavity is determined from recordings of the amplitude and phase of a test signal as it is swept through the cavity resonant frequency. The unloaded Q is calculated from the measured values which determine loaded Q (Q_l) and the transmission loss.

$$Q_0 = Q_l \left[\frac{L^{1/2}}{L^{1/2} - 1} \right] \tag{15}$$

where L = transmission loss ratio.

An aluminum cavity used to measure surface resistivity with this technique is shown in Fig. 4. The test plate forming one end of the cavity is not shown. The depth and rotation of the coupling loops can be adjusted for a cavity transmission loss that results in sufficient signal level for accurate measurement. The choke joint consists of a 1-mm wide groove which is 7 mm deep. The inner surface of the choke joint is machined back 0.25 mm and does not contact the surface of the test plate, which is clamped against the outer ring of the choke joint. The choke joint eliminates wrong mode resonances and reduces surface contact problems to an undetectable level. The cavity diameter is 5.7 cm, and the depth is 2.73 cm. The cavity dimensions were selected so that a standard aluminum tubing size (material which was on hand) could be used to form a cavity with high surface currents on the cavity ends at a resonant frequency near 8415 MHz.

Figure 5 shows the frequency response of the aluminum cavity with a solid copper test plate.

IV. Experimental Results and Conclusions

For the particular cavity described in the preceding section,

$$Q_0 = \frac{1576.344\,\Omega}{[0.9577\,R_{s_{cy1}} + 0.7329\,(R_{s_{end\,1}} + R_{s_{end\,2}})]} \tag{16}$$

Four test samples were measured with the aluminum cavity described above: (1) aluminum alloy, (2) copper, (3) stainless steel, and (4) perforated aluminum plate. The thickness of the last was such that transmission through the plate was negligible. The test data obtained are shown in Table 1. The agreement between measured and predicted values is felt to be good, considering the preliminary nature of the experiment.[2]

The technique is readily applicable to the measurement of the effect of surface finish and perforated plate geometry. Because the cavity Q_0 is high, very high current densities are obtainable with a relatively modest power input. This fact suggests the possibility of using the technique to investigate noise problems caused by surface irregularities (such as joints) in high-power systems.

[2]Subsequent tests with an OFHC copper version of this cavity and test plates of OFHC copper and 2024 aluminum alloy resulted in measured R_s values of 0.02669 Ω for OFHC copper and 0.04459 Ω for 2024 aluminum alloy.

References

1. Ramo, S., and Whinnery, J. R., *Fields and Waves in Modern Radio*, Second Edition, John Wiley and Sons, New York, 1953, p. 375.
2. Ramo, S., and Whinnery, J. R., *Fields and Waves in Modern Radio*, Second Edition, John Wiley and Sons, New York, 1953, p. 430.

Table 1. TE_{011} mode cavity test data, aluminum cavity

Sample No.	Sample type	Q_0	R_s, Ω	R_s predicted[a], Ω
1	Aluminum alloy	15,000	0.04336	0.04347 (2024 aluminum alloy)
2	Copper (unknown purity)	16,600	0.02954	0.02519 (OFHC type copper)
3	Stainless steel	2,820	0.6626	–
4	Perforated aluminum plate	10,000	0.11505	–

[a]Supplied by D. A. Bathker, Communications Elements Research Section.

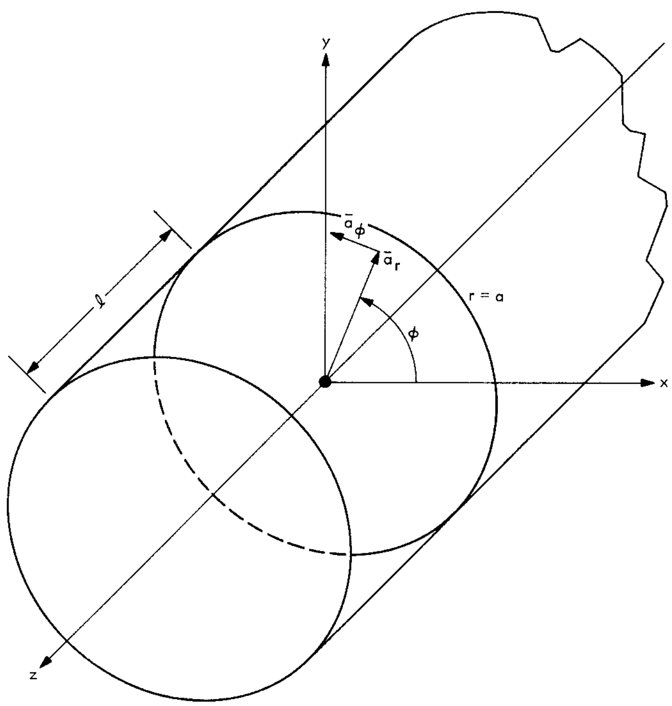

Fig. 1. Circular waveguide geometry

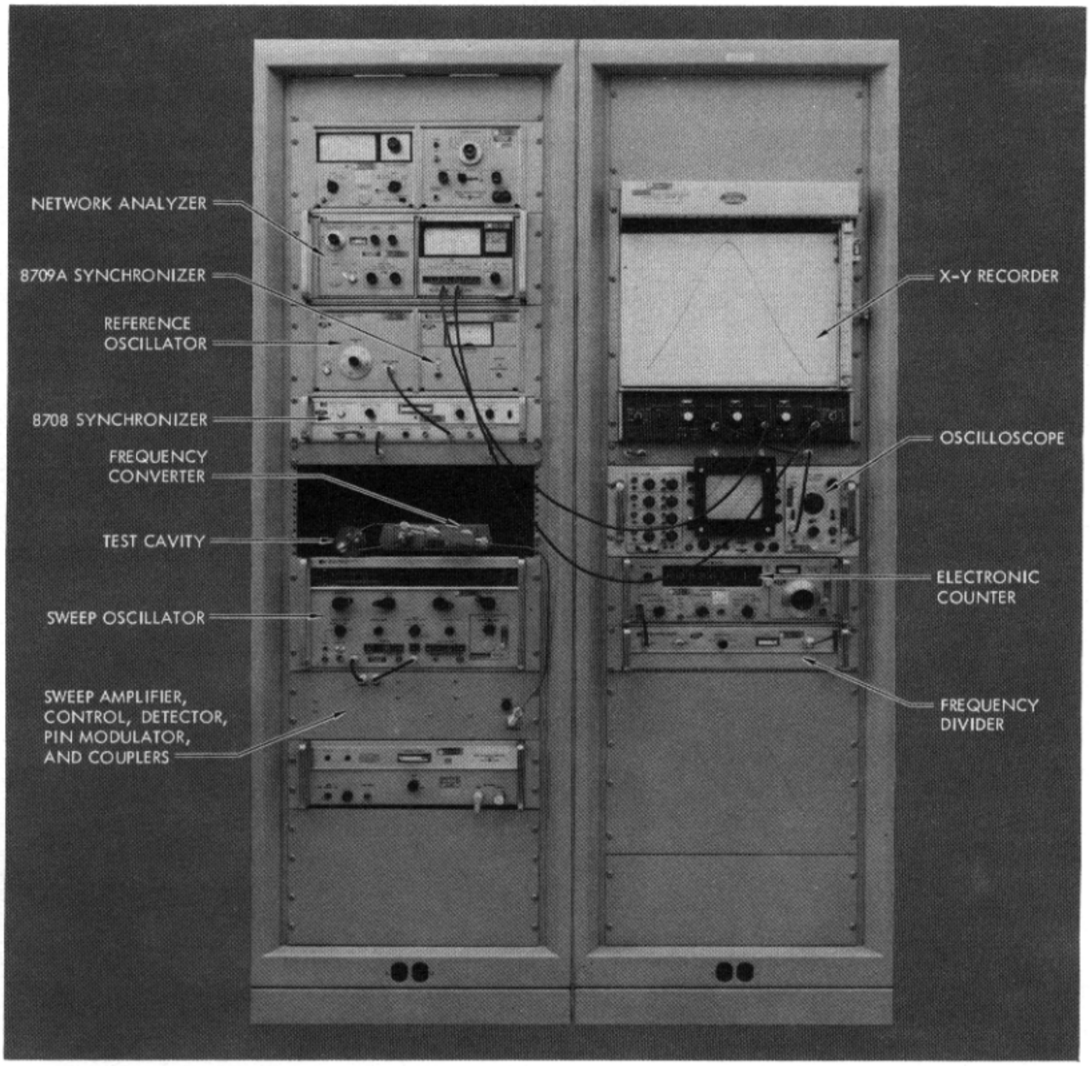

Fig. 2. Microwave equipment used for *Q* measurement

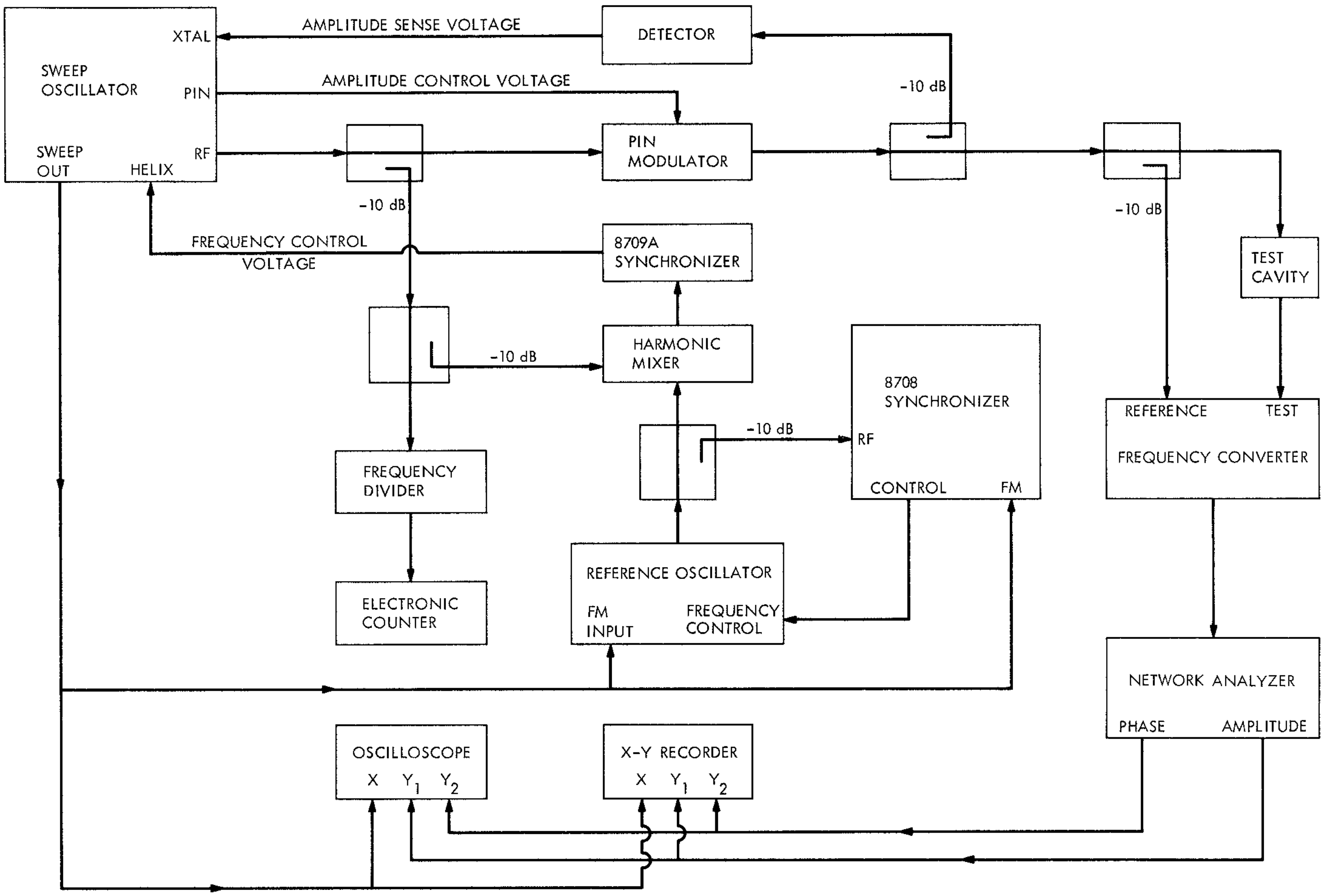

Fig. 3. Q measurement test setup block diagram

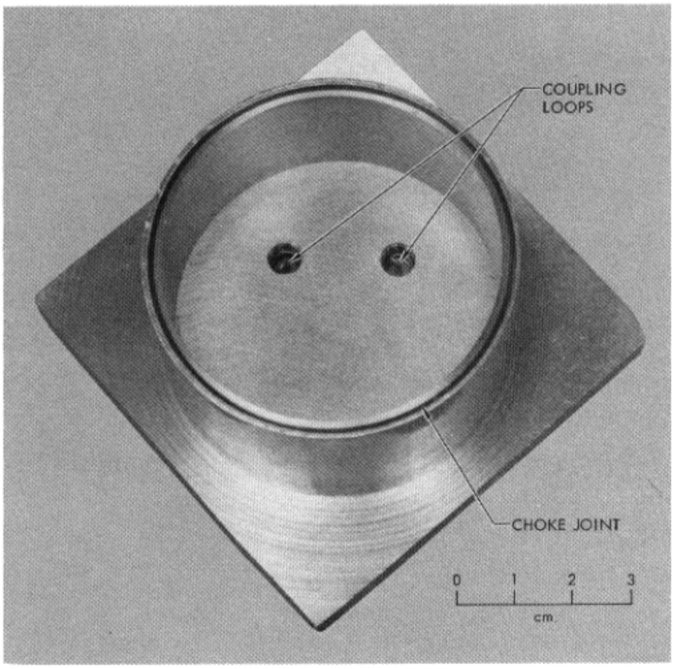

Fig. 4. Aluminum TE_{011} mode cavity resonator

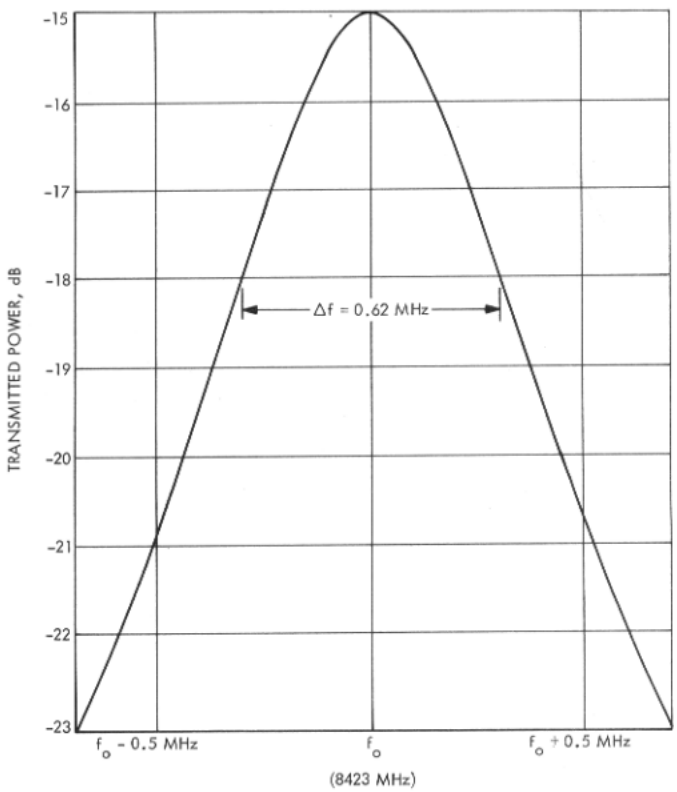

Fig. 5. Frequency response of test cavity resonator