# Filtering Dual-Frequency Radio Metric Data

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*This article introduces a technique for reducing the effect of ionospheric and space plasma charged particles on radio metric measurements. Development of the method is motivated by the difficulty in obtaining complete, two-way range calibrations when dual-frequency measurements are available for only the radio downlink. Using least-squares theory, estimation techniques are derived that allow the downlink calibration to, in effect, be "fed back" to correct unobserved uplink errors. Plausible numerical examples are presented, indicating that such techniques are applicable to precision range measurements for the initial applications of two-station tracking.*

## I. Introduction

Several articles (Refs. 1, 2, and 3) have discussed the use of dual-frequency radio measurements in calibrating the effects of ionospheric and space plasma charged particles. Currently envisioned dual-frequency systems involve a single-frequency uplink/dual-frequency downlink configuration. As stated in Refs. 1, 2, and 3, downlink-only dual-frequency measurements cannot provide complete doppler and range calibrations since, because of the time-varying character of the ionospheric and space plasma electron content, the downlink charged-particle effects cannot be directly related to the uplink effects. There does exist, however, a calculable, statistical relationship between the uplink and the observable downlink effects; and this relationship can be expected to permit a statistical determination to be made of the total uplink/downlink effect. Such a treatment will require the processing of the dual-frequency measurements over an extended period of time (one round-trip time, for instance). Thus, in essence, one filters the dual-frequency measurements to obtain estimates of the charged-particle effect or, more importantly, estimates of the principal quantities of interest, doppler and range.

The following section presents a short discussion of the statistical properties of two-way radio measurements. These results provide a basis for forming statistical estimates of measured quantities, doppler and range, in the presence of errors due to space- and time-varying wave propagation effects. The analysis is concluded with a simplified yet concrete example of estimating the round-trip range to a spacecraft, in the presence of errors due to ionospheric and space plasma charged particles.

## II. Statistical Properties of Two-Way Radio Measurements

The following discussion is restricted to range measurement, i.e., round-trip delay measurement, to simplify the argument. The methods may, however, be applied to round-trip range rate, i.e., doppler, measurements.

The delay measurement, based on a single-frequency uplink, and downlink, can be expressed as a function of reception time:

$$z(t) = \varepsilon_U(t) + \varepsilon_D(t) + 2R + n(t)$$

where $R$ is the one-way range to a spacecraft, and $\varepsilon_U$ and $\varepsilon_D$ are the respective uplink and downlink error contributions due to charged-particle refractive index variations along the ray path. The quantity $n$ represents other error sources, such as instrumentation uncertainties. To simplify the analysis, the spacecraft range shall be assumed constant. This assumption is not restrictive, since in applications, the range change can be tracked with doppler measurements. The range change can be unambiguously determined by comparison of the doppler and range measurements [as in differenced range vs. integrated doppler (DRVID)].

In a dual-frequency downlink configuration, there are two measurements, $z_1(t)$ and $z_2(t)$, where $z_1$ can be expressed as above and

$$z_2(t) = \varepsilon_U(t) + \alpha\varepsilon_D(t) + 2R + n_2(t)$$

where $\alpha$ is a proportionality factor expressing the difference between the downlink charged-particle effects for the two frequencies. For instance, assuming an S-band uplink with X- and S-band downlinks, $z_1$ can be represented as the S-up and -down measurement, and $z_2$ the S-up and X-down measurement. In this case, $\alpha$ is approximately 1/16 (see Ref. 3). The downlink charged-particle effect can be isolated as follows:

$$\frac{z_1(t) - z_2(t)}{1-\alpha} = \varepsilon_D(t) + \frac{n_1(t) - n_2(t)}{1-\alpha}$$

A complete calibration of the range measurement is not available, however, since the uplink and downlink effects cannot be directly related. Nevertheless, it is shown in the following that a statistical relationship between uplink and downlink can be determined, and that spacecraft range measurements can be improved through statistical processing methods, i.e., filtering.

A general statement of the problem is: Estimate $R$ from $z_1(t)$ and $z_2(t)$ for $t_1 \leqq t \leqq t_1 + T$. Conventional least-squares estimation techniques should suffice; therefore, second-order moments of $z_1$ and $z_2$ are required to compute the estimates. As indicated in Ref. 3, the $z_1$ and $z_2$ propagation errors, $\varepsilon_U$ and $\varepsilon_D$, can be expressed as

$$\varepsilon_U = \int_0^{R_0} U\left(x, t - \frac{(2R_0 - x)}{v}\right) dx$$

$$\varepsilon_D = \int_0^{R_0} U\left(x, t - \frac{x}{v}\right) dx$$

where $U(x,t)$ is the time-varying refractive index function along the ray path, $0 \leqq x \leqq R_0$, and $v$ is the propagation rate. ($R_0$ is the "nominal" range to the spacecraft.) The quantity $U(x,t)$ can be related to charged-particle density, as indicated in Ref. 3. This function shall be assumed in the following to be a random variable depending on $x$ and $t$. The expected product functions of the uplink and downlink errors shall be expressed as follows:

$$\left.\begin{aligned} r_{UU}(s) &= E\left[\varepsilon_U(t)\,\varepsilon_U(t+s)\right] \\ &= \iint_0^{R_0} F\left(\Delta x, s - \frac{\Delta x}{v}\right) dx_1 dx_2 \\ r_{UD}(s) &= E\left[\varepsilon_U(t)\,\varepsilon_D(t+s)\right] \\ &= \iint_0^{R_0} F\left(\Delta x, s + \frac{2(R_0 - \bar{x})}{v}\right) dx_1 dx_2 \end{aligned}\right\} \qquad (1)$$

with

$$\Delta x = x_1 - x_2 \text{ and } \bar{x} = \frac{x_1 + x_2}{2}$$

where it is assumed that $U(x,t)$ is a mean-zero, temporally and locally spatially stationary random variable with covariance function

$$F(\Delta x, s) = E\left[U(x,t)\,U(x+\Delta x, t+s)\right]$$

This assumption is indeed restrictive and not actually necessary for a general development. It would probably be quite sufficient in practice, however, and makes the following analysis more understandable. (The above assumption coincides with the assumptions for the space plasma analysis techniques described in Ref. 4.) Observe from the above development that

$$E\left[\varepsilon_D(t)\,\varepsilon_D(t+s)\right] = r_{UU}(s)$$

and

$$r_{DU}(s) = E\left[\varepsilon_D(t)\,\varepsilon_U(t+s)\right] = r_{UD}(-s)$$

One may argue that such a model is unrealistic; however, this type of model is often sufficient in filtering problems provided that gross effects have been accounted for (i.e., average ionospheric and plasma charged-particle densities have been removed).

## III. Filtering

The particular problem of estimating the constant range $R$ from the dual-frequency data $z_1$ and $z_2$ is most easily treated in the familiar parameter estimation format, i.e.,

$$\bar{z} = Ay + \varepsilon$$

with

$$\bar{z} = \begin{pmatrix} z_1(t_1) \\ z_2(t_1) \\ z_1(t_2) \\ z_2(t_2) \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \qquad A = \begin{pmatrix} 2 \\ 2 \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \qquad y = R$$

and

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_1(t_1) \\ \varepsilon_2(t_1) \\ \varepsilon_1(t_2) \\ \varepsilon_2(t_2) \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

where discrete measurements of $z_1$ and $z_2$ are obtained at times $t_1, \cdots, t_n$, and

$$\varepsilon_1(t) = \varepsilon_U(t) + \varepsilon_D(t) + n_1(t)$$

$$\varepsilon_2(t) = \varepsilon_U(t) + \alpha\varepsilon_D(t) + n_2(t)$$

The solution for $y$ is given by the Gauss-Markov theorem (Ref. 5) and can be represented as follows:

$$\hat{y} = (A^T \Lambda_{\bar{\varepsilon}}^{-1} A)^{-1} A^T \Lambda_{\bar{\varepsilon}}^{-1} z \tag{2}$$

where

$$\Lambda_{\bar{\varepsilon}} = E\left[\bar{\varepsilon}\bar{\varepsilon}^T\right]$$

the variance of the error in estimating $y$ is given by

$$\sigma_{\hat{y}}^2 = E\left[(\hat{y} - y)^2\right] = (A^T \Lambda_{\bar{\varepsilon}}^{-1} A)^{-1}$$

Note that $A^T \Lambda_{\bar{\varepsilon}}^{-1} A$ is a scalar for this problem and that the real difficulty in estimating $y$ is forming $\Lambda_{\bar{\varepsilon}}^{-1}$, since

$$\Lambda_{\bar{\varepsilon}} = \begin{pmatrix} \gamma_{11}(0) & \gamma_{12}(0) & \gamma_{11}(t_1 - t_2) & \gamma_{12}(t_1 - t_2) & \cdots \\ \gamma_{21}(0) & \gamma_{22}(0) & \gamma_{21}(t_1 - t_2) & \gamma_{22}(t_1 - t_2) & \cdots \\ \cdot & \cdot & \gamma_{11}(0) & \gamma_{12}(0) & \cdots \\ \cdot & \cdot & \gamma_{21}(0) & \gamma_{22}(0) & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{pmatrix} \tag{3}$$

where the above components are given by

$$\gamma_{11}(s) = E\left[\varepsilon_1(t)\,\varepsilon_1(t+s)\right] = r_{UU}(s) + r_{UD}(s) + r_{DU}(s) + r_{DD}(s) + \sigma_{n_1}^2 \delta(s)$$

$$\gamma_{12}(s) = E\left[\varepsilon_1(t)\,\varepsilon_2(t+s)\right] = r_{UU}(s) + \alpha r_{UD}(s) + r_{DU}(s) + \alpha r_{DD}(s) + \sigma_{n_1 n_2} \delta(s)$$

$$\gamma_{22}(s) = E\left[\varepsilon_2(t)\,\varepsilon_2(t+s)\right] = r_{UU}(s) + \alpha\left(r_{UD}(s) + r_{DU}(s)\right) + \alpha^2 r_{DD}(s) + \sigma_{n_2}^2 \delta(s)$$

with $\gamma_{21}(s) = \gamma_{12}(-s)$. Note that the data noise functions $n_1(t)$ and $n_2(t)$ are assumed to be white with the indicated covariance weightings. Numerical procedures that are more efficient than the direct inversion of Eq. (3) can be developed, particularly in this case of stationary noise processes (see Ref. 6). These considerations are not pursued here, since the principal interest in this analysis is a numerical assessment of the filtering techniques for hypothetical error models.

## IV. Error Models

To avoid the complexities of the double integrals in Eq. (1), assume that the charged-particle densities are constant over $N$ specified "cells" along the ray path, and that the densities are statistically independent from cell to cell and individually exponentially correlated. Thus,

$$\varepsilon_U(t) = \sum_{k=1}^{N} U_k\left(t - \frac{2R_0 - x_k}{v}\right)$$

$$\varepsilon_D(t) = \sum_{k=1}^{N} U_k\left(t - \frac{x_k}{v}\right)$$

where $U_k(t)$ is the time-varying charged-particle effect (measured in meters, for instance) that is localized at $x_k$. A visualization of this model is presented in Fig. 1. Functions that correspond to those in Eq. (1) are given by

$$\left.\begin{aligned} r_{UU}(s) &= \sum_{k=1}^{N} F_k(s) \\ r_{UD}(s) &= \sum_{k=1}^{N} F_k\left(s + \frac{2(R_0 - x_k)}{v}\right) \end{aligned}\right\} \qquad (4)$$

with

$$F_k(t) = \sigma_k^2 e^{-|t|/\tau_k}$$

where $\sigma_k$ and $\tau_k$ characterize the statistical properties of the $k$th cell.

## V. Numerical Examples

In the following, three numerical examples are investigated. In each example, the Earth–spacecraft range is assumed to be 2 AU. The charged-particle effect is divided into 10 cells along the ray path. The first cell is located at $x_1 = 0$, and the remaining cells are uniformly distributed between 0 and 2 AU. The three models are illustrated in Fig. 2. In the first model, the particle effect is uniformly distributed along the ray path. Models 2 and 3 include a large ionospheric effect located at $x = 0$, and Model 3 includes a solar corona effect near $x = 1$ AU.

Figure 3 presents the S-up, S-down round-trip autocovariance functions for the three models. Note that the standard deviations for the models are 6.1, 16.2, and 19.7 m, respectively. These values represent the expected round-trip range errors for uncalibrated S-band range measurements. Of particular significance are the peaks for Models 2 and 3 at a lag equal to 2000 s, the signal round-trip time. It is of interest that the formation of estimated signal autocovariance functions, based on actual radio measurements, is a technique for estimating space plasma densities (see Ref. 4).

Figure 4 presents round-trip range error standard deviations, $2 \times \sigma_{\hat{\rho}}$, for the three models as a function of filter length, i.e., the amount of data incorporated into the estimates. It is seen that consistent improvement occurs with dramatic error reductions after one round-trip time. This relates how a filter can remove a large part of the ionospheric error as a result of the special way it influences the radio signals, as indicated in Fig. 3. This property is further illustrated in Fig. 5, recalling that $\varepsilon_U(t)$ and $\varepsilon_D(t)$ are the uplink and downlink effects on the signal received at time $t$. It is seen that large cross correlations exist for Models 2 and 3 for round-trip time lags. Thus, a filter can effectively feed back the downlink errors (measurable with the dual-frequency data) to recover some of the unobserved uplink errors.

## VI. Implications

The principal intent of this analysis is not to develop a general theory but to introduce, with a specific example, a promising approach to the problem of treating radio measurement charged-particle errors. This approach need not be restricted to two-way range measurements with dual-frequency data. Straightforward extensions of it can be applied to dual- or single-frequency range or doppler measurements.

Regarding the results of the numerical examples, one observes that the dual-frequency filtering yields large reductions in two-way range measurements provided that one allows for sufficiently long filtering periods (up to the signal round-trip time). The 5-m round-trip range accuracies, based on plausible charged-particle density models, are suitable for the initial two-station tracking applications (see Refs. 7 and 8 concerning near-simultaneous ranging). Note that, for the respective models, the quoted filter performances are optimistic in that statistical model mismatching can be expected to degrade the errors from their optimal values.

# References


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**Fig. 1. Charged-particle density model**

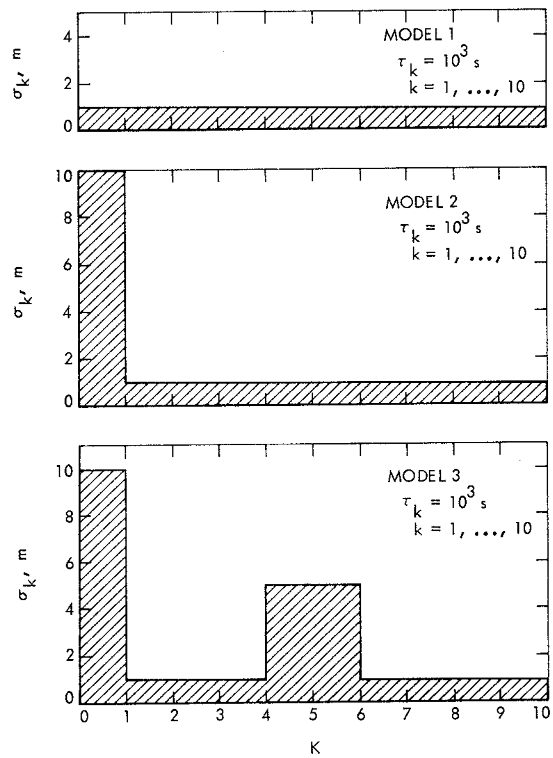


**Fig. 2. Specific charged-particle density models**

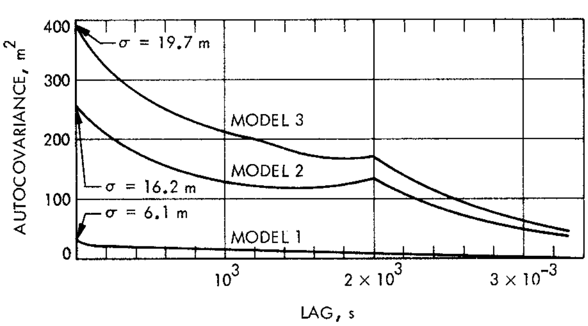


**Fig. 3. Two-way range error autocovariance functions,** $E[\varepsilon_1(t)\,\varepsilon_1(t+s)]$

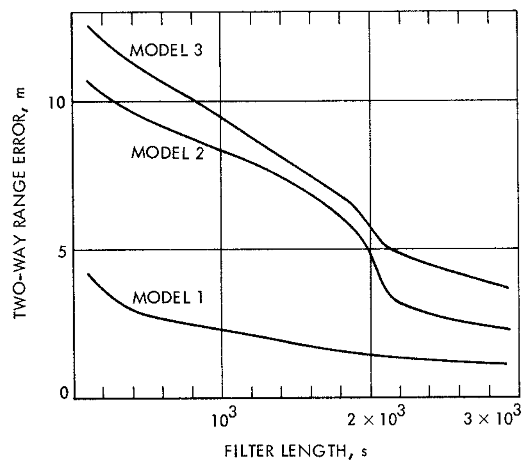


**Fig. 4. Two-way range estimation error,** $2 \times \sigma_{\hat{\rho}}$

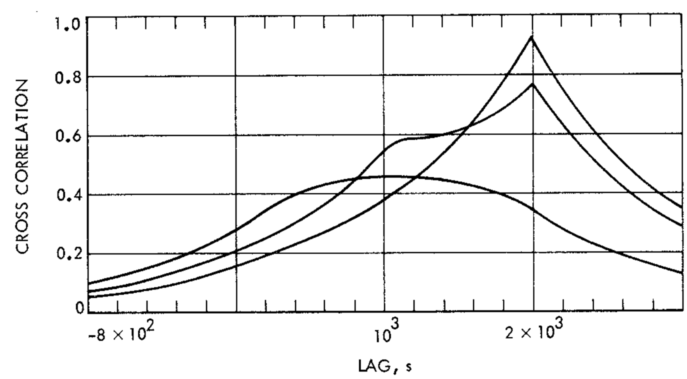


**Fig. 5. Uplink/downlink cross correlation,** $E[\varepsilon_D(t)\,\varepsilon_U(t+s)]/\sigma_{\varepsilon_D}\sigma_{\varepsilon_U}$