Minimum Inertia Design for Gear Trains

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In the design of gear trains, frequently the load inertia is small compared to the inertia of the gearing and thus gear inertia becomes the major resistance to acceleration and also a large source of power consumption. The design program described here emphasizes minimum inertia for the gear train. Some high-acceleration and power-limited radar tracking systems can also benefit from minimum inertia design even though the load inertia is significant. The optimization of gear train ratios for minimum inertia provides smoother tracking and better system response.

The nonlinear differential equations to determine minimum gear train inertia are solved by the Newton–Raphson method. The final design, however, represents a solution to these equations constrained by allowable stresses, stiffness, and other standard American Gear Manufacturers Association (AGMA) specification requirements. Examples demonstrate the effectiveness of the procedure.

I. Introduction

This article describes the mathematical procedure, the implementation, and the results of a computer program to design gear reducers with minimum gear train inertia.

The performance of an initial design, selected by establishing the overall gear ratio requirements and subjectively assigning values to each individual gear mesh, could be improved by optimizing the individual mesh ratios while conforming to the overall ratio. The acceleration of the reflected gear train inertia at the motor shaft reduces the available horsepower to accelerate the load. Therefore, to improve the design of systems such as spacecraft antennas, reconnaissance cameras, high-speed/high-acceleration shipboard radar antennas, or any mechanical system requiring high-acceleration capability, the minimization of drive inertia is the logical approach.

The Minimum Inertia Design for Gear Trains (MIDGET) program has the capability of operating within one of three modes, depending on user option:

(1) Analysis.

(2) Design (with trial ratios).

(3) Design (optimized ratios for minimum inertia).

II. Analysis Mode

The program user inputs gear and shaft data for each mesh, such as the number of teeth in both gear and pinion, diametrical pitch, face width of both gear and pinion, output torque, service factors, efficiency, and material endurance limit. The program makes a complete mathematical analysis of the design, using AGMA formulas for tooth bending stress and overall stiffness of the gearing system. The output of the program consists of complete gear design data at each mesh of gear set (tooth stress, tooth load, wear load, and surface hardness).

A gear schematic is printed at the completion of the individual mesh data. A summary of the gear train design with computed stiffness reflected to the low-speed shaft and the reflected gear train inertia and horsepower required to accelerate the gears and shafts from 0 to 1 deg/s/s completes the analysis. Figure 1 indicates the operations to be performed in this mode.

III. Design Mode A: Trial Ratios

The approach is to develop initial design criteria by utilizing the gear ratios, gear design parameters such as allowable shear and bending stress, service factors, efficiency, output torque, and the overhang at both input and output.

The selection of the number of teeth for both pinion and gear at each mesh is optimized within the program by successive iterations. Since the design technique employed utilizes constraint requirements, it is necessary to proceed iteratively toward the final design. The cyclic steps repeated for each mesh during the iterations are analyses of the current design and determine the need for further parameter change and reanalysis.

Another convenient measure of performance is the stiffness characteristic of the gear train. Stiffness of the drive system determines its capability to respond smoothly to changes in driving forces. Accordingly, the stiffness input constraint would be a prime objective to be fulfilled by the program. Stiffness input requirements of the gearing system are compared to the stiffness calculation of the computer design. The failure of the stiffness analysis to meet the input requirement causes the revising of design parameters and reanalysis. The design variables affecting stiffness would be the gear teeth and the cross-sectional area of the shafts. Successive iterations are made with incremental changes to shaft and gears until the stiffness requirements are achieved.

Since the program originally selected the minimum number of teeth in pinion and gear by use of the input ratio, a recheck is made to insure that gears of the adjacent mesh do not interfere physically with an adjacent shaft. Corrections if any are made and a reanalysis follows. Similarly, as in the Analysis Mode, the output consists of a complete listing of the design data at each mesh and a gear schematic with a summary of data, gear train stiffness, and reflected inertia.

IV. Design Mode B: Optimized Ratios

In this mode, the user must input the same data as in the previous design mode except that no input ratios are prescribed. The designer writes the equation of reflected inertia for each mesh by reflecting the gear and pinion inertia of the previous mesh to the adjacent shaft by the square of the ratio. Since these ratios are unknown, the result is n equations in n unknowns. Each of the equations is differentiated with respect to inertia and set equal to zero. These resultant equations are input to the DESIGN subroutine, which solves n simultaneous nonlinear equations in n unknowns:

$$f_1(z_1, \cdots, z_n) = 0$$

$$\cdots\cdots$$

$$f_n(z_1, \cdots, z_n) = 0$$

The numerical principle is based on the Newton–Raphson method. With given initial values $(z_1, \cdots, z_n)$, the equations are locally linearized by numerical differentiation. The linearized equations are solved by incremental correction values to the initial values.

The functions $f_1, \cdots, f_n$ are evaluated at the new $(z_1, \cdots, z_n)$, and the norm of the new f-vector is examined (norm is defined here to be the sum of the absolute values of the functions). If the new norm is less than the old, the procedure is iterated. If the new norm is not smaller than the old, the incremental correction values are scaled down. The procedure is iterated until the convergence of $(z_1, \cdots, z_n)$ is such that the relative error has occurred within 10^{-7} (here, relative error = 1 correction value of a_i/value of z_i1).

The output of the DESIGN subroutine is the optimized gear ratios. Similarly, the output of Design Mode B is the same as Design Mode A, described previously. Figures 1 and 2 indicate the operation to be performed in this mode.

V. Sample Problems and Discussion of Results

Tables 1, 2, and 3 compare the results of the Analysis Mode. The program computed the stress levels at each gear mesh and the gear train stiffness. The results can only be evaluated by the good comparison of the stiffness computations, which closely approximate the Philadelphia Gear Corp. computed values. The slight difference results from the method of computing the tooth form factor which is built into MIDGET and the actual value used by Philadelphia Gear Corp. Since the stress levels are all below the boundary condition specified, it appears that the program algorithm is accurate.

The results listed in Tables 4, 5, and 6 compare the existing 64-m antenna gear reducers designed by Philadelphia Gear Corp. and the results of "MIDGET" Design Mode A and show excellent correlation, the main differences being in the face widths, shaft diameters, and stiffness. The difference in shaft diameters is due to the stiffness requirement input to the program. In examining results of previous iterations, it was obvious that the major contributor to stiffness was the cross-sectional area of the shafts. Therefore, the algorithm incorporated shaft section increase rather than face width increase.

The computed stiffness of the "MIDGET" gear train has a stiffness factor of almost twice that of the Philadelphia Gear Corp. designed gear train, while the Philadelphia Gear Corp. design has face widths of nearly twice that of MIDGET. The shafts of Philadelphia Gear Corp. design are less than that of MIDGET, which again confirms the original hypothesis of shaft cross-section area being the major contributor to overall stiffness. The reflected inertia to the high-speed shaft is also twice that of MIDGET. This, however, is directly attributed to the increased face widths of the Philadelphia Gear Corp. design. The increased face width appears to be used to decrease the stress level.

Tables 7, 8, and 9 compare the existing gear train design with that of MIDGET Design Mode B. In this mode, the DESIGN subprogram selected the gear ratios. The results as shown in Table 7 indicate a finer gear tooth size coupled with a narrower face width because of the larger first gear pass. All of the gear sets require smaller gears and reduced mass moment of inertia except the first. The savings in material and machining costs will more than compensate for the increased gear size on the first output gear. The stiffness factor shows an approximately 33% improvement as well as a 50% reduction in reflected inertia.

Table 1. Gear reducer design, analysis, with input ratios

Mesh data	Mesh							
	1		2		3		4	
	PG	M	PG	M	PG	M	PG	M
No. teeth								
Gear	77	77	113	113	101	101	134	134
Pinion	20	20	23	23	21	21	20	20
Diameter pitch	2	2	3	3	5	5	8	8
Face width, cm × 2.54	9	9	6.50	6.50	3.25	3.25	2	2
Tooth stress, $N/m^2 \times 0.689 \times 10^4$		13941		9700		15621		19880

PG = Philadelphia Gear design; M = MIDGET computed design.

Table 2. Shaft data, analytic mode

Shaft	Philadelphia gear diameter, cm × 2.54	Computed diameter, cm × 2.54
1	8.5	8.50
2	8.00	8.00
3	5.00	5.00
4	3.00	3.00
5	2.00	2.00
6		

Table 3. Gear reducer stiffness, analytic mode

Philadelphia gear (with housing)		MIDGET computed spring constant	
Computed, × 0.113	Tested, × 0.113	Gears and shafts only, × 0.113	Gears, shafts + Philadelphia Gear housing, × 0.113
3.57×10^8, nm/rad	3.0×10^8, nm/rad	3.46×10^8, nm/rad	

Table 4. Gear reducer design, with input ratios

Mesh data	Mesh							
	1		2		3		4	
	PG	M	PG	M	PG	M	PG	M
No. teeth								
Gear	77	77	113	113	101	101	134	134
Pinion	20	20	23	23	21	21	20	20
Diameter pitch	2	2	3	3	5	5	8	8
Face width, cm × 2.54	9	4.71	6.50	3.14	3.25	2.73	2	1.30
Tooth stress, $N/m^2 \times 0.689 \times 10^4$	13341.5	25480.4	9700	20067	15621.5	18745	19880.8	29550.1

PG = Philadelphia Gear design; M = MIDGET computed design.

Table 5. Shaft data, design mode A

Shaft	Philadelphia gear diameter, cm × 2.54	Computed diameter, cm × 2.54
1	8.5	9.7
2	8.0	6.6
3	5.0	4.1
4	3.0	2.5
5	2.0	1.4
6		

Table 6. Gear reducer stiffness, design mode A

Philadelphia gear (with housing)		MIDGET computed spring constant	
Computed, × 0.113	Tested, × 0.113	Gears and shafts only, × 0.113	Gears, shafts + Philadelphia gear housing, × 0.113
3.57×10^8, nm/rad	3.0×10^8, nm/rad	5.94×10^8, nm/rad	3.55×10^8, nm/rad

Table 7. Gear reducer design, no input ratios

Mesh data	Mesh 1 PG	Mesh 1 M	Mesh 2 PG	Mesh 2 M	Mesh 3 PG	Mesh 3 M	Mesh 4 PG	Mesh 4 M
No. teeth								
Gear	77	367	113	86	101	72	134	65
Pinion	20	18	23	16	21	26	20	32
Diameter pitch	2	4	3	6	5	8	8	10
Face width, cm × 2.54	9	4.6	6.5	3.2	3.25	2.4	2	2
Tooth stress, $N/m^2 \times 0.689 \times 10^4$	13342.5	22209	9700	25671	15621.5	20964	19880.8	21715

PG = Philadelphia Gear design; M = MIDGET computed design.

Table 8. Shaft data, design mode B

Shaft	Philadelphia gear diameter, cm × 2.54	Computed diameter, cm × 2.54
1	8.50	9.74
2	8.00	3.76
3	5.00	2.29
4	3.00	1.71
5	2.00	1.42

Table 9. Gear reducer stiffness, design mode B

Philadelphia gear (with housing)		MIDGET computed spring constant	
Computed, × 0.113	Tested, × 0.113	Gears and shafts only, × 0.113	Gears, shafts + Philadelphia gear housing, × 0.113
3.57×10^8, nm/rad	3.0×10^8, nm/rad	6.49×10^8, nm/rad	3.74×10^8, nm/rad

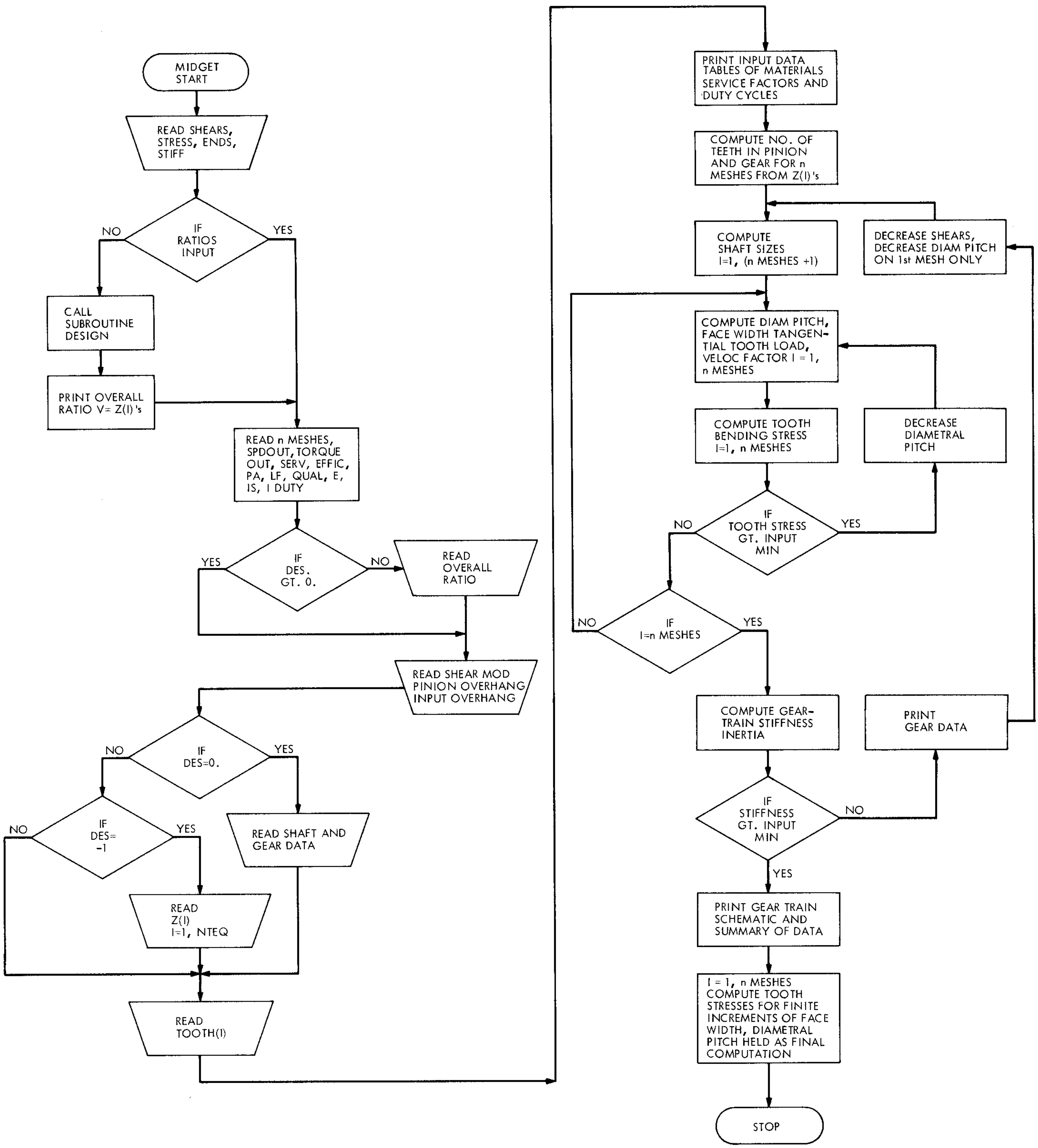

Fig. 1. Minimum inertia design for gear trains

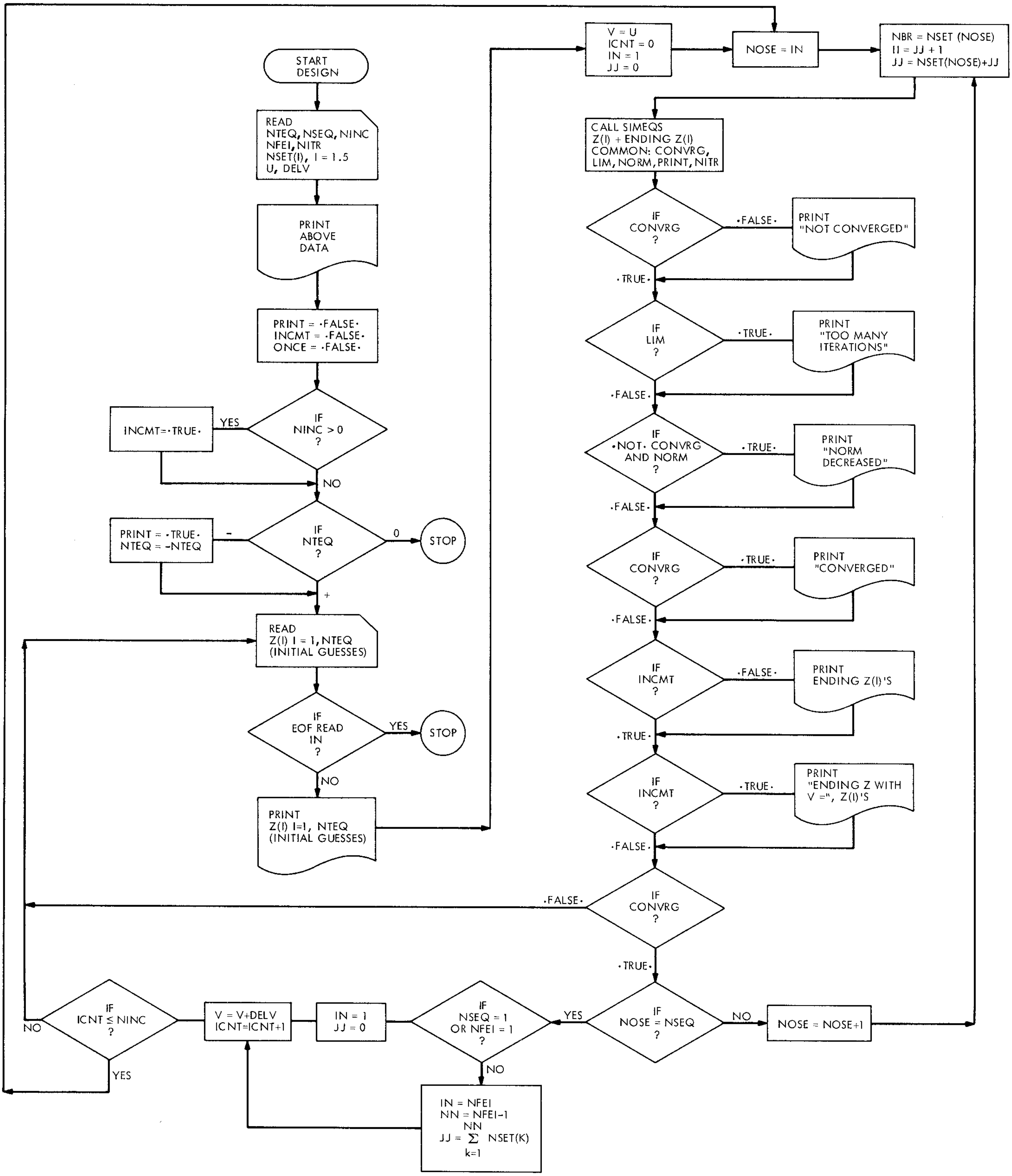

Fig. 2. Flow chart design