Error Probability of Binary Signals With Subcarrier Interference

J. R. Lesh
Network Operations Office

Expressions are presented for the symbol detection error probability of a binary data stream in the presence of an asynchronously related interfering squarewave. These expressions are useful in computing effective symbol energy-to-noise ratio degradation resulting from subcarrier interference.

I. Introduction

The emphasis toward higher data rates has resulted in a steady decrease in the ratios of subcarrier frequencies to data symbol rates. Systems which once employed ratios larger than 10 subcarrier cycles per data symbol are yielding to systems with as few as 1.5 subcarrier cycles per symbol. However, as this ratio decreases, the possibility of subcarrier interference becomes much greater.

Subcarrier interference occurs in the Subcarrier Demodulator Assembly (SDA) as a result of subcarrier and intermediate frequency (IF) mixing. For single subcarrier systems the received IF signal, which can be expanded into IF carrier and IF data components, is applied to the first mixer in the SDA. The purpose of this mixer is to demodulate (remove) the subcarrier signal from the received IF data component. However, this mixer also acts as a modulator to the IF carrier component, modulating onto the carrier the estimated subcarrier. If the ratio of the subcarrier frequency to symbol rate is sufficiently small, both components will pass through the data channel bandpass filter and will be present at the second SDA (coherent IF) mixer. Since the two components are in phase quadrature, the modulated carrier component will be blocked at this mixer provided the receiver phase error (which is also the IF phase error) is zero. When the receiver phase error is nonzero (such as in extreme doppler environments), the second mixer extracts portions of both signals. The result is a baseband symbol stream corrupted by a squarewave subcarrier component, which is subcarrier interference.

In the case of dual-subcarrier modulation, the problem of subcarrier interference is potentially more serious. This is because dual-subcarrier modulation, either conventional or interplex modulation, produces a received signal expansion containing four terms, two of which are in phase while the other two are in the orthogonal direction. For example, with interplex modulation the channel 1 and channel 2 information terms are orthogonal with the intermodulation term in phase with the channel 1 term and the carrier term in phase with the channel 2 term. In this case subcarrier interference can occur (depending on power allocation and frequencies involved) even without a receiver phase error.

In this article expressions are determined for the error probability of a binary data stream corrupted by an additive squarewave. It is assumed that the squarewave is generated asynchronously relative to the data stream so that no fixed phase relationship exists between the two signals. By comparing the results of these expressions with the standard binary detection error probability, it is possible to determine the effective decrease in symbol energy-to-noise ratio resulting from subcarrier interference.

II. Error Probability

Consider a binary data stream $D(t)$ assuming values of $\pm V$ with a data symbol period T_s which is immersed in additive white gaussian noise. The noise $n(t)$ is assumed to be zero mean and have a one-sided spectral density of N_0 W/Hz. Also present is an interfering squarewave having a period T_{sq}. If we let $S(t)$ represent the unit amplitude squarewave, then the composite signal $y(t)$ is given by

$$y(t) = D(t) + \alpha S(t) + n(t) \tag{1}$$

where α is the amplitude of the interfering squarewave. For maximum likelihood symbol detection, we form the quantity

$$z = \int_0^{T_s} y(t)\, dt \tag{2}$$

and compare the result with a threshold of zero. The corresponding symbol error probability is then given by

$$P_e = Q\left(\frac{\sqrt{2}\,x}{\sqrt{N_0 T_s}}\right) \tag{3}$$

where $Q(u)$ is the error probability integral defined by

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} \exp\left[-\frac{\psi^2}{2}\right] d\psi \tag{4}$$

and x is defined by

$$x = \int_0^{T_s} [D(t) + \alpha S(t)]\, dt \tag{5}$$

In order to determine the expression for x, let us define the integer m to be the number of squarewave periods which can be totally contained in one symbol time. Then we have the relation

$$T_s = (m + \gamma)\, T_{sq} \tag{6}$$

where $0 \leq \gamma < 1$. Now, since the integral over complete squarewave periods gives zero contribution, we have (assuming $D(t) = +V$, $0 \leq t \leq T_s$)

$$x = VT_s + \int_0^{T_{sq}} \alpha S(t)\, dt \tag{7}$$

It is clear that the value of the integral term of Eq. (7) will depend on the phase of the squarewave relative to the symbol period. If we fix this phase for the moment by defining t_0 as the time from the beginning of the symbol period to the first leading edge of the squarewave (see Fig. 1), then the expression for x conditioned on t_0 is given by

$$x|_{t_0} = \left\{\begin{array}{ll} VT_s + \gamma T_{sq}\alpha - 2\alpha t_0; & 0 \leq t_0 < \gamma T_{sq} \\ VT_s - \gamma T_{sq}\alpha; & \gamma T_{sq} \leq t_0 < \dfrac{T_{sq}}{2} \\ VT_s - \alpha(\gamma + 1)\, T_{sq} + 2\alpha t_0; & \dfrac{T_{sq}}{2} \leq t_0 < \left(\gamma + \dfrac{1}{2}\right) T_{sq} \\ VT_s + \gamma T_{sq}\alpha; & \left(\gamma + \dfrac{1}{2}\right) T_{sq} \leq t_0 < T_{sq} \end{array}\right\}; \qquad 0 \leq \gamma < \frac{1}{2} \tag{8}$$

or

$$x|_{t_0} = \begin{Bmatrix} VT_s + \alpha(1-\gamma)T_{sq}; & 0 \leqq t_0 < \left(\gamma - \frac{1}{2}\right)T_{sq} \\ VT_s + \gamma T_{sq}\alpha - 2\alpha t_0; & \left(\gamma - \frac{1}{2}\right)T_{sq} \leqq t_0 < \frac{T_{sq}}{2} \\ VT_s - \alpha(1-\gamma)T_{sq}; & \frac{T_{sq}}{2} \leqq t_0 < \gamma T_{sq} \\ VT_s - \alpha(1+\gamma)T_{sq} + 2\alpha t_0; & \gamma T_{sq} \leqq t_0 < T_{sq} \end{Bmatrix}; \quad \frac{1}{2} \leqq \gamma < 1 \tag{9}$$

The conditional error probability $P_e|_{t_0}$ given the value of t_0 can be determined by substituting the appropriate relation from Eqs. (8) and (9) into Eq. (3). Then, since the data stream and squarewave were assumed to be asynchronous, the time interval t_0 will be uniformly distributed in the interval $(0, T_{sq})$, so that the average error probability is given by

$$P_e = \frac{1}{T_{sq}} \int_0^{T_{sq}} P_e|_{t_0}\, dt_0 \tag{10}$$

Performing this integration for $0 \leqq \gamma < 1/2$ yields

$$P_e = \left(\frac{1}{2} - \gamma\right)\left\{Q\left[\frac{\sqrt{2}(VT_s - \gamma T_{sq}\alpha)}{\sqrt{N_0 T_s}}\right] + Q\left[\frac{\sqrt{2}(VT_s + \gamma T_{sq}\alpha)}{\sqrt{N_0 T_s}}\right]\right\} + \frac{1}{\alpha T_{sq}} \int_{VT_s - \gamma T_{sq}\alpha}^{VT_s + \gamma T_{sq}\alpha} Q\left(\frac{\sqrt{2}u}{\sqrt{N_0 T_s}}\right) du; \quad 0 \leqq \gamma < \frac{1}{2} \tag{11}$$

Now, let us define

$$R = \frac{V^2 T_s}{N_0}$$

as the input symbol energy-to-noise ratio without subcarrier interference and

$$\rho = \frac{\alpha}{V}$$

as the ratio of squarewave to data symbol amplitudes. Then form Eq. (6) and the identity

$$\int Q(\sqrt{2}\,ax)\,dx = xQ(\sqrt{2}\,ax) - \frac{e^{-a^2x^2}}{2a\sqrt{\pi}} \tag{12}$$

we have that the symbol error probability is given by

$$\begin{aligned} P_e = \frac{1}{2}&\left\{Q\left[\sqrt{2R}\left(1 - \frac{\rho\gamma}{m+\gamma}\right)\right] + Q\left[\sqrt{2R}\left(1 + \frac{\rho\gamma}{m+\gamma}\right)\right]\right\} \\ &+ \frac{(m+\gamma)}{\rho}\left\{Q\left[\sqrt{2R}\left(1 + \frac{\rho\gamma}{m+\gamma}\right)\right] - Q\left[\sqrt{2R}\left(1 - \frac{\rho\gamma}{m+\gamma}\right)\right]\right\} \\ &+ \frac{(m+\gamma)}{\rho\sqrt{\pi R}} \exp\left\{-R\left[1 + \frac{\rho^2\gamma^2}{(m+\gamma)^2}\right]\right\} \\ &\times \sinh\left(\frac{2\gamma\rho R}{m+\gamma}\right); \qquad 0 \leqq \gamma < \frac{1}{2} \end{aligned} \tag{13}$$

By the same procedure we obtain for γ larger than 1/2,

$$\begin{aligned} P_e = \frac{1}{2}&\left\{Q\left[\sqrt{2R}\left(1 + \frac{\rho(1-\gamma)}{m+\gamma}\right)\right] + Q\left[\sqrt{2R}\left(1 - \frac{\rho(1-\gamma)}{m+\gamma}\right)\right]\right\} \\ &+ \frac{(m+\gamma)}{\rho}\left\{Q\left[\sqrt{2R}\left(1 + \frac{\rho(1-\gamma)}{m+\gamma}\right)\right] - Q\left[\sqrt{2R}\left(1 - \frac{\rho(1-\gamma)}{m+\gamma}\right)\right]\right\} \\ &+ \frac{m+\gamma}{\rho\sqrt{\pi R}} \exp\left\{-R\left[1 + \frac{\rho^2(1-\gamma)^2}{(m+\gamma)^2}\right]\right\} \\ &\times \sinh\left[\frac{2\rho R(1-\gamma)}{m+\gamma}\right]; \qquad \frac{1}{2} \leqq \gamma < 1 \end{aligned} \tag{14}$$

III. An Example

As an example let us consider a 50-kHz subcarrier biphase modulated by a 33-kbps data stream. The subcarrier in turn phase modulates a carrier with a modulation index of 30 deg. Assume also that the received carrier has been shifted due to doppler such that the receiver static phase error is 30 deg. (It is also assumed that the receiver margin is sufficiently high so that the receiver dynamic phase error is negligible.) Also, let the data symbol energy-to-noise ratio at the SDA second mixer output be 5.0 dB. For this example $m = 1$, $\gamma = 0.515$ and $\rho = 1.0$. From Eq. (14) we find that the symbol error probability is 0.0113, which corresponds to an effective symbol energy-to-noise ratio of 4.1 dB. We conclude that the subcarrier interference alone has caused an effective degradation of 0.9 dB.

IV. Remarks

The above expressions were originally obtained for predicting subcarrier degradation resulting from interplex modulation on the Mariner Venus/Mercury 1973 (MVM'73) mission. When typical mission parameters were used in these expressions, it was found that the effective symbol energy-to-noise ratio degradation was less than 0.2 dB. This was true even in mode 1 for the low-rate channel, where the interfering subcarrier at the symbol detection matched filter has an amplitude 1.13 times that of the symbol amplitude (assuming that the notch filter tuned to the IF carrier is not used ahead of the SDA).

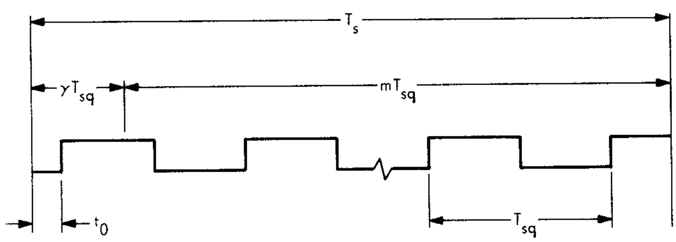

Fig. 1. Time relationship for data symbol and squarewave subcarrier