# Spacecraft Orbit Determination Using Long Tracking Arcs

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*This article summarizes a study of the accuracy of planetary approach orbit determination based on long, 100-day arcs. Long arc orbit determination is an attractive means for improving the accuracy of radio metric based navigation in situations when conventional, 30-day "short arc" orbit determination strategies are particularly sensitive to bias deep space station (DSS) location errors. The accuracy analysis is based on the Viking Mission A and B trajectories; nevertheless, the results that are presented can be interpreted in a general way. Several error sources which are not usually included in short arc orbit determination analysis have been evaluated in the study, namely, randomly time-varying (stochastic) DSS location errors, Earth–Moon barycenter ephemeris errors, and spacecraft maneuver errors. The stochastic DSS location errors are included to model error effects arising from imperfect calibration of measurement system errors, e.g., troposphere, ionosphere, space plasma, timing, and polar motion. The inclusion of stochastic station errors in the orbit estimation strategies presented proved to be significant; these errors are shown to limit determination of the spacecraft orbit when the bias components of the station location error are also estimated. Example results using the two Viking trajectories nevertheless indicate that long arc radio orbit determination performs favorably in the presence of these and other errors, including conservative station location bias errors.*

## I. Introduction

Uncertainties in the deep space station (DSS) locations often produce the principal error source affecting the accuracy of radio metric based interplanetary navigation. DSS location error is used as a somewhat loose term, referring to not only Earth-fixed station coordinate uncertainties, but in addition uncertainties in the Earth's rotational dynamics, i.e., universal time (UT) and polar motion. Location errors may also include "effective" location errors arising from transmission media and instrumentation calibration errors that manifest themselves as apparent location errors in the orbit determination process. The collective radio metric "location error" is sometimes referred to as equivalent station location error.

Techniques for improving interplanetary navigation accuracy usually involve means for reducing the equivalent station location errors themselves, for instance, improved transmission media and Earth rotation calibrations as well as improved determinations of the Earth-fixed DSS coordinates. Subjects related to this effort are found in numerous *Deep Space Network Progress Report* articles.

An alternate approach to improving navigation accuracy is the development of orbit determination techniques that exhibit reduced sensitivity to DSS location errors. A technique of this type, orbit determination based on long arcs of radio metric data, is discussed in this article. The term "long arc" refers to a span of radio metric data that is considerably longer than that necessary to minimally determine a spacecraft orbit. Thus, long arcs usually can be assumed to extend 100 days or more, as opposed to the conventional short arc, extending usually no more than 30 days. The motivation for long arcs rests in the interest in shifting from the short arc (essentially geocentric determination of the spacecraft motion) to the long arc (heliocentric determination of the spacecraft motion). This shift of radio metric information content is provided by the "heliocentric bending" due to solar gravitation that can be observed with the longer data arc. This property is somewhat in analogy to the accurate planet-centered determinations of spacecraft motion available during planetary flybys and orbiter phases. Consideration of navigation techniques that are less sensitive to DSS location errors does not suggest that improvements in DSS location accuracies (using the "location" in its general sense) are not necessarily warranted. Any improvements in navigation accuracy should be pursued whenever attractive.

Radio orbit determination based on long data arcs, although usually considered less sensitive to station location errors, is not as yet a widely accepted orbit determination technique despite some favorable operational experiences during the Mariner Mars 1969 and 1971 Missions (see Ref. 1, for example). The principal inhibition in processing long data arcs is a lack of sufficient understanding regarding the effects of major error sources on the estimation parameters. Another disadvantage cited is the assumed requirement that the radio data arc cover an interval of time that is as free as possible from major orbit perturbations, such as midcourse maneuvers. The possibility of significantly reducing the sensitivity of orbit estimates to station location errors is an attractive one, however, and hence the continued interest in long data arc processing techniques for navigation applications. In the following, orbit determination accuracies using long data spans are examined for two Viking trajectories. The data arc is arbitrarily defined to span from 100 days prior to the spacecraft's encounter with the target planet Mars.

## II. Error Models for Long Arc Analysis

As pointed out, the accuracy characteristics of long arc orbit determination are not as completely understood as those for the short arc. Hence, a rather complete approach in analyzing long arc accuracies is called for. In particular, special care should be taken to include error sources which may significantly corrupt the characteristically different information inherent in the long data arc. Two error sources of this type have been included in this study. These are Earth–Moon barycenter ephemeris errors and also nonconstant (stochastic) station location errors produced by uncertainties in the polar motion, universal time, and transmission media calibrations. In the subsequent discussion, the expression "Earth–Moon barycenter" will be abbreviated as simply "Earth."

In short arc analyses, the short-term Earth ephemeris errors are found to be similar in character to the target planet ephemeris errors, and hence are usually represented as effective target planet ephemeris errors. In processing long data arcs, however, it is to be expected that long-term spacecraft orbit effects can only be poorly distinguished from long-term Earth orbit effects, and thus Earth ephemeris errors must be explicitly modeled for the long arc analysis. An assumption that is made in modeling the Earth and Mars ephemeris errors is that the heliocentric ephemeris errors of the target planet and the Earth can be expected to be strongly interrelated; this is the case since planetary ephemerides are determined from Earth-based observations. The interrelation is accounted for by a complete ($12 \times 12$) position and velocity covariance describing the correlated Earth–Moon barycenter and target planet position and velocity errors. The ephemeris errors are resolved into an (inertial) plane-of-sky type cartesian coordinate system defined as follows (see Fig. 1):

z-axis: points toward Earth from target planet at spacecraft encounter time $T$

x-axis: points in direction of increasing target planet right ascension at time $T$

y-axis: points in direction of decreasing target planet declination at time $T$

Station location errors are representative of physical DSS station location errors plus "equivalent" station location errors that may be attributed to uncalibrated instrumentation and transmission effects. (Reference 2 discusses the concept of equivalent station location errors.) The station location error model assumed for the long arc analysis has both constant and stochastic components. This is in contrast to the more common approach taken in short arc analysis of assuming the errors to be of a bias nature only; in fact, the station location error has a significant stochastic component (see, for example, Ref. 3). Stochastic station location errors refer to errors that can be adequately represented as randomly time-varying station location parameters (in particular, longitude and distance from the spin axis). In processing short arcs, station location parameters are generally not estimated, but the effect of the station location errors is included in the orbit accuracy determination; in this case, the stochastic error effects are negligible (see Chapter 4 of Ref. 4).

The specific DSS location error model is defined as follows: each station location is assumed to have both bias and stochastic errors in longitude and distance from the spin axis. (Preliminary work showed the tracking station distance from the equatorial plane to be inconsequential; this error source was eliminated in the subsequent study.) It is assumed that each station's constant longitude error $\bar{\epsilon}_\lambda$ is composed of an absolute component $\epsilon_{\bar{\lambda}}$ and a relative component $\epsilon_{\Delta\lambda}$, i.e.,

$$\bar{\epsilon}_\lambda = \epsilon_{\bar{\lambda}} + \epsilon_{\Delta\lambda}$$

for each station, where each $\epsilon_{\Delta\lambda}$ is independent from station to station. The absolute component $\epsilon_{\bar{\lambda}}$ represents an error common to all tracking stations (e.g., universal time and polar motion errors). The total longitude error $\epsilon_\lambda$ is composed of the bias term $\bar{\epsilon}_\lambda$ and the stochastic component $\epsilon_{\delta\lambda}$, i.e.,

$$\epsilon_\lambda = \bar{\epsilon}_\lambda + \epsilon_{\delta\lambda}$$

for each station. The distance from the spin axis $\epsilon_{r_s}$ has a bias component $\epsilon_{\bar{r}_s}$ and a stochastic component $\epsilon_{\delta r_s}$:

$$\epsilon_{r_s} = \epsilon_{\bar{r}_s} + \epsilon_{\delta r_s}$$

The stochastic station location parameters are modeled as first-order stationary Markov processes with correlation $R$ of the form

$$R = \sigma^2 \exp\left[-a|\Delta t|\right]$$

The correlation time $1/a$ is set at two days.

Consideration of spacecraft maneuver errors can usually be avoided when analyzing short arc orbit accuracies by choosing data arcs which are free of maneuver perturbations. In this long arc study, however, midcourse maneuvers are assumed to occur at several points during the 100-day data span; this requires the inclusion of the maneuver error effects in the analysis of the data arc. The maneuver motor burn is of relatively short duration and is effectively modeled as a trajectory position and velocity discontinuity.

## III. Orbit Determination Filter Strategy

In this section, we discuss the attempt to choose an appropriate procedure for processing the long data arc: which parameters affecting the data should be estimated, and which, although not estimated, should be considered in the accuracy analysis as affecting the orbit estimate obtained from the data. To arrive at a proper data filtering model, we first list the parameters and error sources affecting the data and spacecraft dynamics:

(1) Spacecraft trajectory (position and velocity), as determined by gravitational motion.

(2) Nongravitational spacecraft accelerations, caused by solar pressure and attitude control (constant and stochastic components).

(3) Spacecraft midcourse maneuver errors (motor pointing and shutoff).

(4) Target planet ephemeris errors.

(5) Earth ephemeris errors, and station location errors (constant and stochastic component).

A batch sequential filter is available to evaluate the effects of the nonconstant parameters (Ref. 5). The idea is to choose a set of parameters which can be effectively estimated from the long arc data, in the presence of errors modelable in terms of parameters whose values are not accurately known. In this light, the approach taken was to postulate, evaluate, and compare the following two filters:

(1) *Filter 1* (station location errors not explicitly estimated)

(a) *Estimated parameters*
Spacecraft state errors (position and velocity)
Constant nongravitational acceleration errors
Spacecraft midcourse maneuver errors

(b) *"Considered" parameters*
(parameters not estimated but whose error covariance degrades error covariance of estimated parameters)
Target planet ephemeris errors
Earth ephemeris errors
Station location errors (constant and stochastic)

(2) *Filter 2* (constant station location errors estimated)

(a) *Estimated parameters*
Spacecraft state errors

Constant nongravitational acceleration errors
Spacecraft midcourse maneuver errors
Absolute component of constant station longitude error
Constant component of station spin axis error

(b) *"Considered" parameters*
Target planet ephemeris errors
Earth ephemeris errors
Constant relative component and stochastic component of station longitude error
Stochastic component of station spin axis error

We have assumed for each filter a stochastic component of nongravitational acceleration, postulated in order to model such random spacecraft perturbations as attitude control gas leakage and unpredictable fluctuations in the solar constant. These small random forces are also modeled as stationary stochastic processes, acting along each spacecraft axis; since relatively little is known of the dynamics involved, the acceleration processes are assumed to be uncorrelated between batches. These acceleration uncertainties, often referred to as process noise, are included because, although they have a negligible direct effect on the spacecraft orbit, they can, if improperly accounted for, limit the capability to actually estimate the orbit. Earth ephemeris errors essentially affect only the data, while the target ephemeris errors are felt only shortly before spacecraft encounter. These variables are strongly correlated and are not estimated. Midcourse maneuver velocity errors can significantly affect the orbit accuracies, and are estimated in each filter. The 3-axis constant components of nongravitational acceleration are easily modeled, and are also estimated in each filter.

Thus, two distinct solution strategies are hypothesized; both explicitly estimate the nongravitational acceleration effects and the spacecraft maneuvers. The performance of the two filters is compared on the basis of the estimates obtained when constant station locations are not included among the estimated parameters (filter 1), and in filter 2, which solves for constant errors in the station's absolute longitudes and distance from the spin axis. Both station location parameters are estimated, since it has been determined that the orbit solution errors become increasingly sensitive to spin axis errors, if the longitudes alone are estimated.

## IV. Numerical Examples

The performance of the long arc data filters described above is evaluated using two example Viking trajectories. Trajectory A is representative of the Viking Mission A trajectories, encountering Mars early in the 1976 arrival period; the orbit determination geometry is favorable throughout the heliocentric cruise and encounter period. Trajectory B is representative of the Viking Mission B trajectories, encountering Mars later in the arrival period; its orbit determination performance is known to be degraded, due to low spacecraft declination geometry.

The data types assumed for the sample trajectories are the standard range and range rate. The data pattern takes every other pass of range rate data from DSSs 14, 42, and 61, from encounter minus 100 days ($E-100$) to encounter minus 3 days ($E-3$). Continuous range rate data are assumed from $E-3$ days to encounter, when the spacecraft dynamics begin to change more rapidly. The range datum consists of a single point from DSS 14, taken at $E-30$ days. Table 1 summarizes the data accuracy levels assumed. Note from Table 1 that the range datum is deweighted in the filters from its nominal expected value (in the vicinity of 50 m). This artifice allows Earth-based spacecraft position determinations to be consistent with target planet-centered position determinations, when the target planet ephemeris errors are included. Tables 2 and 3 summarize the parameter *a priori* error levels. The nongravitational acceleration levels and spacecraft maneuver errors are characteristic of the expected magnitudes. Two maneuvers are assumed to occur, at $E-30$ days and $E-10$ days. With regard to the station location *a priori* errors, the assumed 10-m absolute longitude error is postulated to conservatively reflect longitude knowledge degradation. Spin axis error has a nominal level of 1.5 m. The stochastic station error standard deviations are somewhat arbitrarily assigned to be 2 m for both longitude and spin axis, with correlation times set at 2 days, roughly corresponding to the intervals at which data are received from each station. The ephemeris errors accurately reflect the expected levels and are delineated separately in Table 3.

The numerical results of the study are shown in Figs. 2–6, for trajectories A and B, filters 1 and 2. Figures 2–5 show time histories of accuracy prediction in the usual B-plane; the estimate sensitivity to the individual error sources is also given. For the example trajectories, $\mathbf{B} \cdot \mathbf{R}$ errors generally predominate, and the solution time histories are shown in terms of the behavior of only the $\mathbf{B} \cdot \mathbf{R}$ coordinate.

In the case of solved-for parameters, the sensitivity that is presented is the sensitivity to an actual *a priori* error. A special convention is taken regarding the maneuver errors: although the data are filtered assuming that a 0.1-m/s per axis error existed, actual errors are not included in the overall error predictions. The effects of actual maneuver errors can be ascertained from the $\Delta V$ curves shown in Figs. 2–5, however. These curves permit an evaluation of the effects of nominal maneuver errors and also allow an assessment of the effects of maneuver errors that are larger or smaller than the nominal 0.1-m/s error.

Note from Figs. 2 and 3 that the limiting error source, when station locations are not estimated (filter 1), is the absolute longitude component of the longitude bias. Figures 4 and 5 show that the inclusion of the constant tracking station location parameters as solve-for parameters is preferable, given that large station longitude errors are expected. In this case, the errors limiting the determination of the spacecraft orbits are the randomly time-varying (2-m) station location errors.

Perhaps the most significant results of this study are found in the curves which show the effects of station locations for each trajectory/filter case. These results indicate that (1) long arcs are strongly degraded by (10-m) longitude errors if station parameters are not estimated (filter 1); and (2) if station parameters are estimated (filter 2), the stochastic station effects (at 2 m) become the dominant error source.

Figure 6 shows the individual effects of Earth–Moon barycenter ephemeris errors. The effect of Mars ephemeris errors on **B • R** is relatively smaller and somewhat constant at the 100–200-km level (not shown). The implication is clear when comparing these curves to the curves in Figs. 2–5, showing the full Earth/Mars effects: that the detailed relationship between ephemeris errors (principally the Earth), as specified by the full $12 \times 12$ covariance, strongly affects the expected orbit accuracy. In particular, when estimating stations, the combined Mars–Earth effects (shown in Figs. 4 and 5) produce a smaller effect than either their effect when considering stations or the effect of the Mars ephemeris error model. This implies that special care must accompany the specification of the full covariances in subsequent long arc analysis and further effort should be directed toward better understanding the input covariances to assure that they truly reflect the expected planetary errors. Figure 7 gives the time history of the station parameters estimated using filter 2.

## V. Conclusions

The universality of the above results is difficult to determine for different mission applications with different geometries. Nevertheless, several salient points present themselves:

(1) Estimation strategies that include tracking station locations as solution parameters are preferable to those that do not solve for location parameters, given that large station longitude errors are expected. Indeed, for the 100-day data arcs, the orbit determination errors still show considerable sensitivity to station location errors, if station locations are not estimated.

(2) When station locations are estimated, the dominant error source is stochastic station location errors. In the case of trajectory B, the effect of stochastic station location errors actually becomes magnified as station location parameters are solved for.

(3) Coupled Earth ephemeris and planetary (Mars) ephemeris errors do not produce large estimation errors, given that highly correlated errors (obtained from planetary ephemeris solutions) are assumed. Preliminary work has shown that use of uncorrelated ephemeris errors leads to the prediction of catastrophic orbit determination errors, principally through Earth ephemeris errors.

(4) Spacecraft maneuvers can be estimated without difficulty, given that maneuver errors do not greatly exceed assumed magnitudes (e.g., greater than $6\sigma$; $\sigma = 0.1$ m/s).

Also, as an outcome of this analysis, the following two basic recommendations can be made:

(1) Long arc techniques are a feasible and attractive means for alleviating orbit accuracy sensitivity to station locations errors. The techniques of long arc orbit determination differ somewhat from those that have been developed for the short arc. The long arc technology requires further development to assure reliable and economical performance, particularly in the areas of:

   (a) Filter procedure development, to determine the critical parameters of filter design for the long arc methods.

   (b) Data processing economization, particularly in the areas of data compression and data batching techniques, to remove the somewhat unwieldy character of long arc data processing.

   (c) Strategy testing using past mission tracking data, to insure that techniques deemed attractive by general analyses do perform as expected.

(2) The analysis identified stochastic station location errors and barycentric ephemeris errors as major error sources for long arc orbit determination. The understanding of these errors requires further development, in particular:

(a) Simplified, conservative and yet representative models should be developed for describing the barycentric errors. As long as formal and complex error models, such as $12 \times 12$ error covariances, are used for these analyses the reliability of their predicted accuracies shall remain questionable to some degree.

(b) The actual stochastic station location error levels (i.e., rms magnitude) should be assessed rather than just hypothesized, as done in this study. This should be done on an analytical as well as an empirical basis, i.e., estimates could be derived from measurement systems analysis and analysis of past mission tracking data.

## References


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4. Curkendall, D. W., *Problems in Estimation Theory with Applications to Orbit Determination*, Report 7275, UCLA Department of Engineering, Sept. 1972.
5. Nishimura, T., and Nead, M., *ATHENA Filter Sequential Orbit Determination with General Evaluation Capability*, document 900-605, Mar. 8, 1973 (JPL internal document).

**Table 1. Data assumptions**

| | |
|---|---|
| Range Rate: | |
| | (1) DSSs 14, 42, and 61 |
| | (2) Every second pass from each station, from $E-100$ days to $E-3$ days |
| | (3) Continuous data from $E-3$ days to encounter |
| | (4) Assumed measurement accuracy is 1 mm/s |
| Range: | |
| | (1) One point at $E-30$ days from DSS 14 |
| | (2) Range point artificially deweighted by assuming a 20-km measurement error |

**Table 2. *A priori* parameter errors**

| | |
|---|---|
| 1. Initial state | $10^5$-km position<br>1-km/s velocity |
| 2. Nongravitational acceleration | $1.2 \times 10^{-12}$-km/s$^2$ per axis bias error |
| 3. Spacecraft maneuvers (assumed to occur at $E-30$ days and $E-10$ days) | 0.1 m/s per axis for each of two maneuvers |
| 4. Equivalent station location errors | (a) *Bias*<br>Absolute longitude<br>$\epsilon_{\bar{\lambda}} = 10$ m<br>Relative longitude<br>$\epsilon_{\Delta\lambda} = 0.95$ m<br>Spin axis $\epsilon_{\bar{r}_s} = 1.5$ m<br>(b) *Stochastic* (correlation time 2 days)<br>Relative longitude<br>$\epsilon_{\delta\lambda} = 2$ m<br>Spin axis $\epsilon_{\delta r_s} = 2$ m |
| 5. Ephemerides | See Table 3 |

**Table 3. Ephemeris errors**

| Trajectory A: Encounter 6/23/76 | | Trajectory B: Encounter 8/21/76 | |
|---|---|---|---|
| Earth–Moon barycenter | Mars | Earth–Moon barycenter | Mars |
| **Heliocentric errors** | | | |
| $\sigma_x = 43$ | $\sigma_x = 92$ | $\sigma_x = 57$ | $\sigma_x = 98$ |
| $\sigma_y = 43$ | $\sigma_y = 77$ | $\sigma_y = 45$ | $\sigma_y = 84$ |
| $\sigma_z = 52$ | $\sigma_z = 51$ | $\sigma_z = 34$ | $\sigma_z = 36$ |
| $\sigma_{\dot{x}} = 1.0 \times 10^{-5}$ | $\sigma_{\dot{x}} = 5.1 \times 10^{-6}$ | $\sigma_{\dot{x}} = 6.9 \times 10^{-6}$ | $\sigma_{\dot{x}} = 4.5 \times 10^{-6}$ |
| $\sigma_{\dot{y}} = 8.2 \times 10^{-6}$ | $\sigma_{\dot{y}} = 7.9 \times 10^{-6}$ | $\sigma_{\dot{y}} = 7.9 \times 10^{-6}$ | $\sigma_{\dot{y}} = 7.1 \times 10^{-6}$ |
| $\sigma_{\dot{z}} = 8.5 \times 10^{-6}$ | $\sigma_{\dot{z}} = 8.5 \times 10^{-6}$ | $\sigma_{\dot{z}} = 1.2 \times 10^{-5}$ | $\sigma_{\dot{z}} = 9.4 \times 10^{-6}$ |
| **Earth–Moon barycentric errors** | | | |
| | $\sigma_x = 133$ | | $\sigma_x = 158$ |
| | $\sigma_y = 100$ | | $\sigma_y = 118$ |
| | $\sigma_z = 10$ | | $\sigma_z = 7$ |
| | $\sigma_{\dot{x}} = 6.6 \times 10^{-6}$ | | $\sigma_{\dot{x}} = 6.3 \times 10^{-6}$ |
| | $\sigma_{\dot{y}} = 1.1 \times 10^{-5}$ | | $\sigma_{\dot{y}} = 1.2 \times 10^{-5}$ |
| | $\sigma_{\dot{z}} = 1.7 \times 10^{-5}$ | | $\sigma_{\dot{z}} = 2.1 \times 10^{-5}$ |

*Assumed:* Full ($12 \times 12$) covariance of Earth–Moon barycenter and Mars plane-of-sky cartesian state errors. The corresponding heliocentric and barycentric position and velocity in km, km/s, are given, using encounter dates for trajectories A and B as reference times.

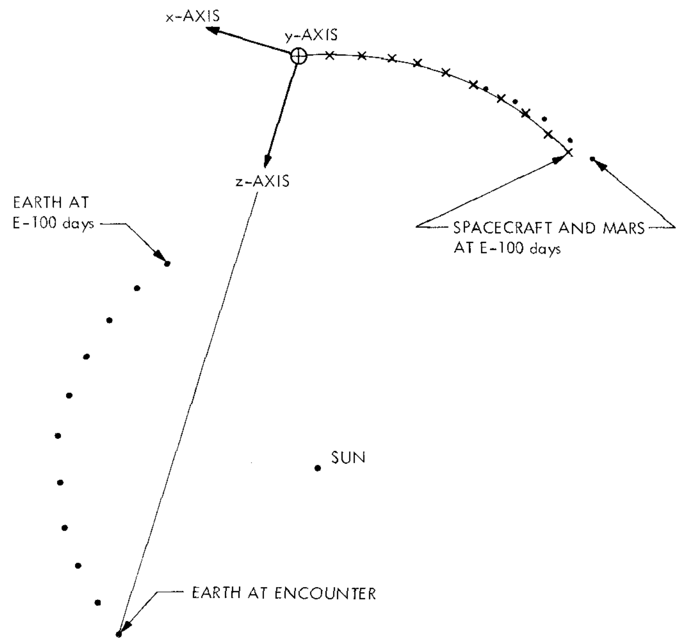


**Fig. 1. Plane-of-sky ephemeris coordinates and trajectory geometry (trajectory A) example**

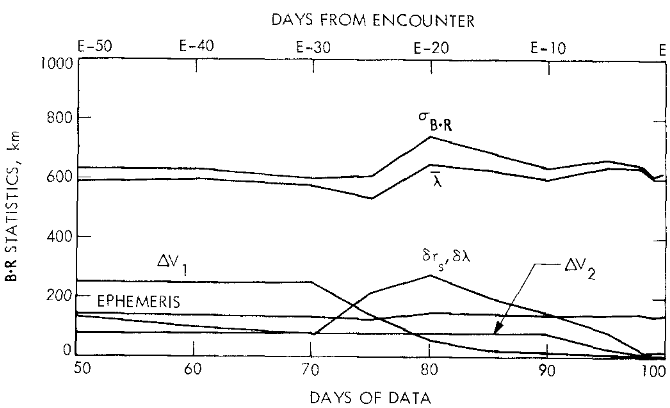


**Fig. 2. Trajectory A, filter 1 error source comparison**

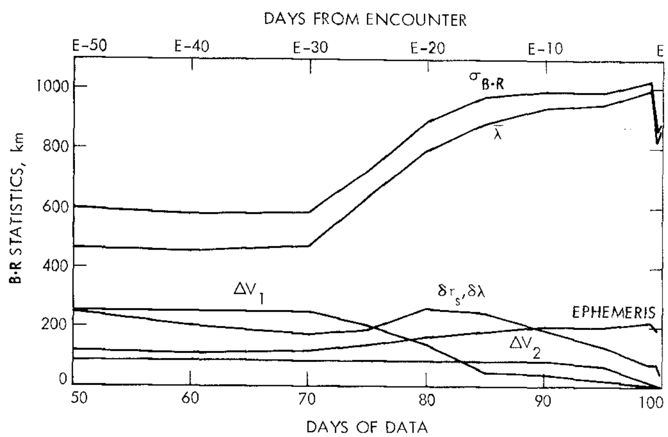


**Fig. 3. Trajectory B, filter 1 error source comparison**

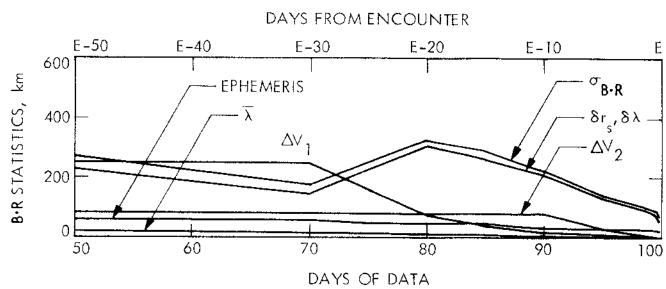


Fig. 4. Trajectory A, filter 2 error source comparison

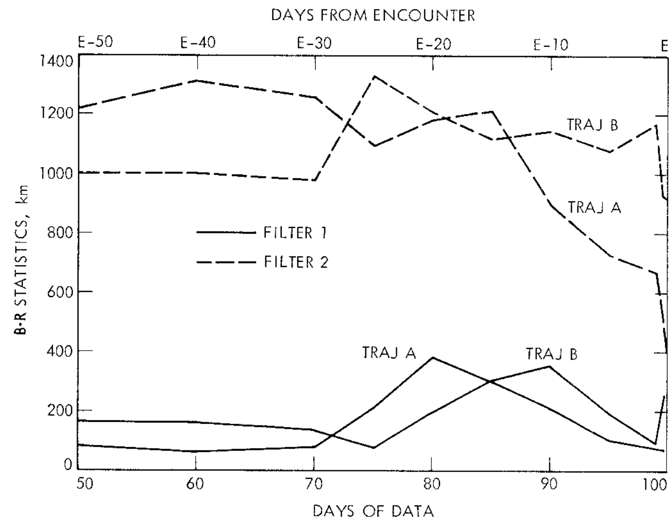


Fig. 6. Earth–Moon barycenter perturbations

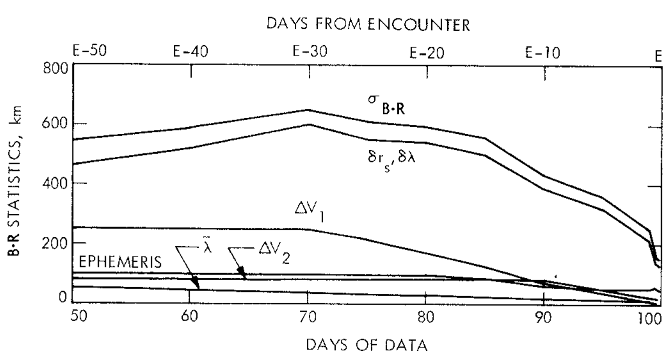


Fig. 5. Trajectory B, filter 2 error source comparison

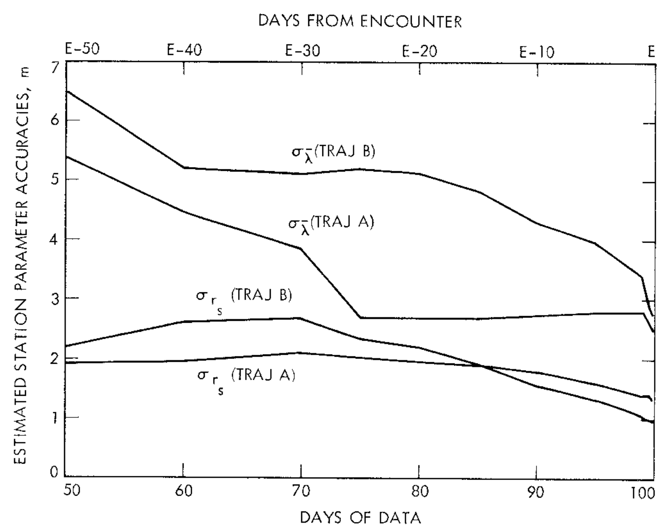


Fig. 7. Time history of station parameter accuracies from filter 2