A Preliminary Study of Spares Provisioning for the Deep Space Network

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This article gives the results of a preliminary investigation into the problem of developing an efficient Initial Spares Provisioning and Spares Allocation Strategy for the Deep Space Network operational spares. A sparing procedure is given, based on failure and repair rates and specified operational requirements. The procedure was applied to several possible situations and the results are listed. The results of computer simulations of these cases are given.

I. Introduction

Reference 1 establishes the responsibilities and functions necessary to select, procure, allocate, and control spare parts for equipment used by the Deep Space Instrumentation Facility (DSIF). The cognizant operations engineer (COE), the cognizant development engineer (CDE), and a representative of the DSIF Logistics Group determine the initial complement of spare parts at a provisioning conference. This conference is also held for the purpose of identifying the funding sources to be used for procurement, the agent responsible for procurement, the delivery schedule, and the destination. Also described in Ref. 1 is the operational-spare flow of material throughout the Deep Space Network (DSN) Repair-Resupply System. However, the decision as to the number of spares to be provided for a given piece of equipment is at present apparently based solely upon engineering judgment. No systematic analytic technique is provided to determine the number of spares necessary to meet required performance criteria. Moreover, as far as we know, Ref. 2 is the only JPL document that specifies the quantity of spares for DSIF electronic equipment. The rules given in Ref. 2 are strictly rules of thumb and do not take into account such factors as failure rates, repair rates, and shipping time. Consequently, this method of sparing must in many cases result in oversparing or undersparing.

Our primary purpose is to develop efficient procedures for use by the COEs and CDEs to assist them in determining which items should be spared, how much to spare, and an optimal network allocation scheme for these spares, when applicable. Both the sparing and allocation techniques must be subject to initial funding constraints and present maintenance policies. This preliminary report gives the results of our initial investigation of one aspect of the sparing problem. For an individual repairable item we determine the operational availability as a function of failure rate, repair rate, total logistic delay time, and number of spares.

II. General Assumptions

We are concerned with pieces of equipment that are removed, replaced by a spare, and repaired whenever they fail to function. To simplify the discussion we call any such piece of equipment a module. A module is assumed to be in one of three states at all times:

(1) Operating.

(2) Failed and in the repair pipeline.

(3) Spare.

While operating, it is subject to a constant failure rate λ, and while in the spares pool it is assumed not to fail. Once failed, it waits a time T to return to the operating state (or the spare state if it is not needed immediately). It is assumed that T has a fixed distribution independent of all previous waiting times for this module and other modules. The analytic model in Section III assumes T is exponentially distributed. Departures from this assumption about T, are considered in Section IV where it is shown that only the mean value of T has a significant effect on the operational availability which, in general, is defined as the stationary probability that a system is operating or operational at any given time.

III. Determination of Operational Availability

We consider a system of $N + n$ identical modules, n operating with N spares. Under the assumption that T is exponentially distributed with mean $1/\mu$, the operation of the system can be described by a so-called birth-and-death process. Detailed discussions of this process are given in Refs. 3, 4, and 5. The state of this process at any time is the number j, of failed modules among the $N + n$ modules in the system. Assuming that the system operates as long as there is at least one module unfailed, the possible values of j are $0, 1, \cdots, N + n$. The theory of Markov chains gives the result that over a long period of operation the fractions of time the system spends in the various states are given by the so-called stationary distribution $\{P_j\}, j = 0, 1, \cdots, N + n$, which can be obtained from the following formulas:

$$\left.\begin{aligned}
\lambda_j &= \begin{cases} n\lambda & \text{for } j = 0, 1, \cdots, N \\ (N + n - j)\lambda & \text{for } j = N+1, N+2, \cdots, N+n-1 \end{cases} \\
\mu_j &= j\mu \quad \text{for } j = 0, 1, \cdots, N + n \\
q_0 &= 1 \\
q_1 &= \lambda_0/\mu_1 \\
q_{j+1} &= \mu_{j+1}^{-1}\,[q_j(\lambda_j + \mu_j) - q_{j-1}\lambda_{j-1}], \qquad j = 1, 2, \cdots, N + n - 1 \\
P_j &= \frac{q_j}{\sum_{i=0}^{N+n} q_i}, \quad j = 0, 1, \cdots, N + n
\end{aligned}\right\} \quad (*)$$

The above formulas will be denoted collectively as procedure (*).

A simple way of defining operational availability for our purpose is to specify a number $k, 0 \leq k < n$, and say the system is "up" (i.e., operational) whenever $j \leq N + k$, that is, at least $n - k$ modules are operating. The operational availability is then defined by the *system* up-time ratio (UTR), which is

$$\sum_{j=0}^{N+k} P_j$$

the fraction of time with at least $n - k$ modules operating.

Another approach is to define operational availability as the fraction of time an *individual* operating location in the system is "up" (i.e., has an operating module available). This fraction is, of course, the same for all n operating locations and is obtained from the formula

$$UTR = 1 - \frac{1}{n}(P_{N+1} + 2P_{N+2} + \cdots + nP_{N+n}) \qquad (1)$$

This approach is applicable to the case where n independent identical modules are operating within a DSS or complex with a common pool of spares.

Whichever approach is used to define operational availability, a convenient way of determining the necessary spares complement is to specify the minimum acceptable value, α, of the *UTR* and determine the smallest N for which the *UTR* is greater than or equal to α. Cost constraints may suggest trading off operational availability levels for different modules in such a way as to maximize the overall operational availability of the DSN. The definition and implementation of this approach will be the subject of later investigations.

To illustrate the application of procedure (*) to the problem of sparing, we first considered the case of n identical independently operating modules, for $n = 1, 3, 5, 10, 25$, and 50. Values of N were determined based on $\alpha = 0.99$. Table 1 lists the results. The first five of the six values of λ used are typical failure rates taken from Refs. 6 and 7. We used two values of $1/\mu$, 336 hours (2 weeks) and 1776 hours (two months plus two weeks). The values of N prescribed in Ref. 2 for the above values of n are, respectively, 1, 3, 4, 4, 5, 5, irrespective of failure rates, delay time, and operational availability desired.

We then considered what Rau in Ref. 3 calls an (m, n) system, $0 < m \leqq n$. In this special case, n identical modules are normally in operation, with N spares, and the system requirement is that a minimum of m of them must be in operation at a given time for adequate performance. The definition of this system also assumes that whenever the system is in state $N + n - m + 1$, it is shut down and reactivated only when a spare is available. Procedure (*) can also be applied to this case with a slight modification. In the formulas comprising procedure (*) the possible values of j should be truncated at $N + n - m + 1$ because of the shutdown rule. Then, with $k = n - m$,

$$UTR = \sum_{j=0}^{N+n-m} P_j$$

or, in terms of the downtime ratio (DTR), ($DTR = 1 - UTR$), $UTR = 1 - P_{N+n-m+1}$.

Table 2 gives some of the results of this analysis for an $(m, 5)$ and $(m, 10)$ system.

Procedure (*) gives one the capability of seeing the effect on the *UTR* of increasing N. Putting N successively equal to $0, 1, 2, \cdots$, corresponding values of *UTR* can be computed. Thus one can observe the increments of *UTR* that result from each additional spare. A decision can then be made as to whether a given increment is worth the cost of the extra spare. This flexibility is illustrated in Figs. 1 and 2. In both, *UTR* is plotted against successive values of N for $\lambda = 150.45 \times 10^{-6}$. In Fig. 1, $1/\mu = 336$ hours and in Fig. 2, $1/\mu = 1776$ hours. In each figure, graphs are given for $n = 1, 3, 5$, and 10. The graphs also illustrate the importance of delay time.

IV. Computer Simulations of the Sparing Procedures

Even if the actual repair time were exponentially distributed, it is unlikely that our assumption of exponentiality for the total time between the failure of a module and its return to the spares pool is valid. To compare the theoretical *UTR*s obtained using this dubious assumption with those likely to be achieved in a real-life situation, we decided to simulate the cases under consideration. In each case, we generated random failure times that were exponentially distributed with mean $1/\lambda$ and, to make the comparison as stringent as possible, we held the repair time *constant* and equal to $1/\mu$, which means that whenever a module failed, it was assumed that it was returned to the spares pool in exactly 336 or 1776 hours. In view of the fact that the standard deviation of an exponentially distributed random variable with parameter μ is $1/\mu$, this is indeed a rigorous test of how sensitive the *UTR* is to the distribution of the return time of a failed module.

In the case of a single operating module ($n = 1$), we kept track of those intervals of time that began with the failure of the operating module with no spare available for replacement and ended with the return of the next repaired module. We will refer to these intervals as down time. The *DTR* was then defined as the ratio of the sum of the length of these intervals to the total elapsed operating time of the system (including the down time).

For $n > 1$, we kept track of the sum of the lengths of the intervals of time when exactly j of the n operating modules were down simultaneously with no spares, for $j = 1, 2, \cdots, n$. Denoting the jth sum by D_j and the total elapsed time of operation by t, the average *DTR* for each operating module was defined as

$$DTR = \sum_{j=1}^{n} jD_j/nt$$

Table 3 gives the results of the simulation for sparing a single module. The values of the parameters were chosen so that examples would be given for *DTR*s falling within each of the intervals (0.01, 0.05), (0.001, 0.01), and (0, 0.001), corresponding to *UTR*s > 0.95, 0.99, and 0.999. Table 4 gives the results when replacements for any of n independently operating modules are taken from a common spares pool, for $n = 3, 5, 10, 25$, and 50.

For the (m, n) system case, we kept track of the intervals of time when $N + n - m + 1$ of the operating modules were in a failed state. During these intervals of down time we assumed that the system was shut down and hence no additional failures could occur until a spare was available. The *DTR* was defined as the ratio of the total down time to the sum of the total down time and the up time. Table 5 gives the results for various (m, n) systems.

V. Conclusion

The consistency of the results of all three simulations with respect to the close agreement between the simulated *DTR*s and those arrived at by theoretical considerations is, in our opinion, impressive. It strongly indicates that to determine the number of spares necessary to achieve a given *UTR*, all that is needed are reasonably accurate estimates of the failure rate of the module and the average time it takes to return a repaired module to the spares pool once it has failed. How this time interval is subdivided and the nature of the probability distributions of the subintervals of time are, within limits, unimportant. In other words, the simulation results show that the state probabilities obtained from procedure (*) are not sensitive to the actual distribution of the return time. Thus, procedure (*) can be used as a bases for determining a spares complement in many, if not all, practical situations.

Tables 1 and 2 show that the *length* of the return time is important, especially for high failure rates. The establishment of a systematic method for sparing has the added advantage that one can easily consider trade-offs between the number of spares needed and hence their cost, and the cost of shipping them to and from a repair facility. A reduction in the shipping time may result in a smaller number of spares needed and consequently justify a change in the shipping procedure for a particular module.

From our initial investigation we learned that some modules may spend anywhere from two to six weeks at the repair facility waiting for repair and that the basic repair philosophy is "first come first serve." It is our contention that some sort of priority scheme should be implemented for two reasons. First, an efficient sparing procedure that is based on valid statistical considerations must at some time or other result in a dangerously low level of spares. If this never happens, we have overspared. If it happens too often then either our estimates of the parameters are off or the specified *UTR* is too low. However, assuming that the occurrence of this phenomenon is consistent with our spares philosophy for a particular module, a priority scheme that takes into account this level of spares and extends priority to a failed module whenever this level is critical will have a direct impact on station operational availability. Second, reducing the repair time significantly has the same effect as reducing the shipping time. If a trade-off study, particularly for expensive items, shows that less spares are needed if priority is given during repair, a worthwhile cost savings may result by doing so.

The point is that an efficient method for spares provisioning gives us the capability of making sensible decisions based on valid information. Engineerng judgment, as valuable as it is in many situations, is just not precise enough for this purpose. The drastic consequences that may result from either undersparing or oversparing can be avoided only by applying an acceptable scientific method to the problem of sparing. Moreover, in one sense, a priority scheme at the repair facilities can be considered as a hedge against undersparing due to underestimating a failure rate. In another sense, it can be considered as yet another possible means of saving money. Both are consequences devoutly to be wished.

It is our intention to pursue further the development of an optimum spares provisioning procedure for the DSN, incorporating the results already obtained and outlined in this report. Our future investigations will include the problem of spares provisioning with cost constraints and the allocation of spares for modules that are used in more than one DSS. Priority maintenance schemes will be investigated at a later date.

References

1. *Management of DSIF Spare Parts Program,* DSIF Operations Section M & O Document 337S-2B1. Jet Propulsion Laboratory, Pasadena, Calif., July 14, 1972 (JPL internal document).

2. *DSIF General Specification; General Requirements for DSIF Equipment,* Specification 8907A. Jet Propulsion Laboratory, Pasadena, Calif., Oct. 11, 1962 (JPL internal document).

3. Rau, J. G., *Optimization and Probability in Systems Engineering.* Van Nostrand Reinhold Co., New York, 1970.

4. Feller, W., *An Introduction to Probability Theory and Its Applications,* Second Edition, 10th Printing. John Wiley & Sons, Inc., New York, Mar. 1965.

5. Wagner, H. M., *Principles of Operations Research With Applications to Managerial Decisions.* Prentice Hall, New York, 1969.

6. *Reliability Report: Prediction of Data Decoder Assembly Mean Time Between Failure,* DSIF Operations Section M & O Document 337S1-5A3. Jet Propulsion Laboratory, Pasadena, Calif., Feb. 10, 1972 (JPL internal document).

7. *Reliability Report: Prediction of Block Decoder Assembly Mean Time Between Failure,* DSIF Operations Section M & O Document 337S1-5A5. Jet Propulsion Laboratory, Pasadena, Calif., July 25, 1972 (JPL internal document).

Table 1. Number of pooled spares, N, necessary to insure a $UTR \geq 0.99$ for each of n identical modules when they are operating independently[a]

$\lambda/10^{-6}$	$\mu = 1/336$						$\mu = 1/1776$					
	$n = 1$	$n = 3$	$n = 5$	$n = 10$	$n = 25$	$n = 50$	$n = 1$	$n = 3$	$n = 5$	$n = 10$	$n = 25$	$n = 50$
	N	N	N	N	N	N	N	N	N	N	N	N
2.72	0	0	0	0	0	0	0	0	0	0	0	0
47.85	1	1	1	1	1	1	1	1	2	2	3	5
150.44	1	1	1	2	2	3	2	3	4	5	10	16
481.07	2	2	3	4	6	10	3	6	8	13	27	49
791.14	2	3	4	5	9	16	5	8	12	20	42	78
1000.00	2	3	4	6	12	20	5	10	14	24	53	98

[a]Assuming exponentially distributed times to failure and repair; the unit of time is one hour.

Table 2. Number of spares, N, necessary to ensure a $UTR \geq 0.99$ for an $(m, 5)$ and an $(m, 10)$ system[a]

$\lambda/10^{-6}$	$n = 5$						$n = 10$							
	$\mu = 1/336$			$\mu = 1/1776$			$\mu = 1/336$				$\mu = 1/1776$			
	$m = 1$	$m = 3$	$m = 5$	$m = 1$	$m = 3$	$m = 5$	$m = 1$	$m = 4$	$m = 7$	$m = 10$	$m = 1$	$m = 4$	$m = 7$	$m = 10$
	N	N	N	N	N	N	N	N	N	N	N	N	N	N
2.72	0	0	0	0	0	1	0	0	0	0	0	0	0	1
47.85	0	0	1	0	0	2	0	0	0	2	0	0	0	3
150.44	0	0	2	0	2	4	0	0	0	3	0	0	3	7
481.07	0	1	3	1	6	9	0	0	1	5	0	5	11	15
791.14	0	2	4	4	10	13	0	0	3	7	0	10	18	22
1000.00	0	3	5	6	12	16	0	0	4	8	1	14	22	26

[a]Assuming exponentially distributed times to failure and repair; the unit of time is one hour.

Table 3. Results of simulating sparing for a single module[a]

$\lambda/10^{-6}$	$1/\mu$	N	Theoretical DTR	Actual simulated DTR	% difference	Years operated
2.72	336	0	0.00091	0.00099	−8.8	10138
2.72	1776	0	0.0048	0.0048	0	2740
47.85	336	0	0.0158	0.0158	0	556
47.85	336	1	0.00013	0.00015	−15.4	5946
47.85	1776	1	0.0033	0.0033	0	1845
150.44	336	0	0.0481	0.0498	−3.5	1424
150.44	336	1	0.0012	0.0013	−8.3	980
150.44	1776	1	0.0274	0.0267	+2.6	570
150.44	1776	2	0.0024	0.0027	−12.5	960
481.07	336	1	0.0111	0.0109	+1.8	433
481.07	336	2	0.00060	0.00064	−6.7	425
481.07	1776	2	0.0447	0.0484	−8.3	70
481.07	1776	3	0.0095	0.0077	+18.9	440
791.14	336	1	0.0272	0.0261	+4.0	118
791.14	336	2	0.0024	0.0027	−12.5	423
791.14	1776	3	0.0404	0.0403	+0.2	455
1000.00	336	1	0.0405	0.0416	−2.7	59
1000.00	1776	4	0.0252	0.0251	+0.4	216

[a]Return time for repaired modules is constant and equal to $1/\mu$; N denotes number of spares.

Table 4. **Results of simulating pooled sparing for *n* independent modules**[a]

n	$\lambda/10^{-6}$	$1/\mu$	N	Theoretical DTR	Actual simulated DTR	% difference	Years operated
3	2.72	1776	0	0.0048	0.0050	−4.2	6753
	47.85	1776	1	0.0094	0.0104	−10.6	367
	150.45	1776	2	0.0166	0.0148	+10.8	264
	481.07	336	2	0.0046	0.0050	−8.7	122
	481.07	1776	4	0.0406	0.0409	−0.7	123
	791.14	336	2	0.0164	0.0170	−3.7	70
	791.14	1776	7	0.0217	0.0235	−8.3	99
	1000.00	336	2	0.0287	0.0303	−5.6	116
5	2.72	1776	0	0.0048	0.0046	+4.2	4365
	47.85	1776	2	0.0020	0.0024	−20.0	219
	150.45	336	0	0.0481	0.0464	+3.5	165
	150.45	1776	3	0.0107	0.0113	−5.6	152
	481.07	1776	6	0.0342	0.0310	+9.4	100
	1000.00	1776	13	0.0158	0.0150	+5.1	69
10	2.72	1776	0	0.0048	0.0049	−2.1	4167
	47.85	1776	3	0.0013	0.0010	+23.1	239
	150.45	1776	4	0.0183	0.0183	0	78
	481.07	1776	10	0.0384	0.0428	−11.5	61
	791.14	1776	16	0.0396	0.0342	+13.6	62
	1000.00	336	4	0.0370	0.0385	−4.1	59
25	47.85	1776	1	0.0460	0.0460	0	100
	150.45	336	0	0.0481	0.0469	+2.5	65
	150.45	1776	10	0.0053	0.0043	+18.9	91
	481.07	1776	21	0.0490	0.0479	+2.2	16
50	2.72	1776	1	0.00054	0.00057	−5.6	877
	47.85	336	1	0.0050	0.0051	−2.0	48
	47.85	1776	2	0.0432	0.0399	+7.6	52
	150.45	1776	11	0.0467	0.0452	+3.2	48

[a]Return time for repaired modules is constant and equal to $1/\mu$; N denotes the number of pooled spares.

Table 5. Results of simulating sparing for an (m, n) system[a]

n	m	$\lambda/10^{-6}$	$1/\mu$	N	Theoretical DTR	Actual simulated DTR	% differenced	Years operated
10	10	2.72	336	0	0.0090	0.0084	+6.7	2271
5	5	2.72	1776	0	0.0236	0.0256	−8.5	3946
5	4	47.85	1776	0	0.0482	0.0464	+3.7	270
10	9	47.85	1776	1	0.0400	0.0402	−0.5	124
25	21	47.85	1776	2	0.0031	0.0031	0	100
5	4	150.45	336	0	0.0200	0.0221	−10.5	157
5	3	150.45	1776	1	0.0177	0.0179	−1.1	84
10	8	150.45	1776	3	0.0261	0.0241	+7.7	118
25	18	150.45	1776	1	0.0475	0.0468	+1.5	112
5	5	481.07	336	2	0.0396	0.0393	+0.8	49
5	3	481.07	1776	4	0.0397	0.0421	−6.0	55
10	7	481.07	336	1	0.0096	0.0078	+18.8	79
10	3	481.07	1776	0	0.0267	0.0289	−8.2	44
10	7	481.07	1776	11	0.0073	0.0064	+12.3	95
5	1	791.14	1776	1	0.0397	0.0325	+18.1	116
5	3	791.14	336	1	0.0174	0.0173	+0.6	32
5	2	791.14	1776	4	0.0490	0.0461	+5.9	61
5	5	791.14	1776	13	0.0073	0.0068	+6.8	113
5	1	1000.00	336	0	0.0010	0.00087	+13.0	92
5	1	1000.00	1776	2	0.0438	0.0462	−5.5	90
5	2	1000.00	336	1	0.0044	0.0051	−15.9	79
5	2	1000.00	1776	6	0.0423	0.0408	+3.5	59

[a]Return time for repaired modules is constant and equal to $1/\mu$; N denotes the number of spares for the system.

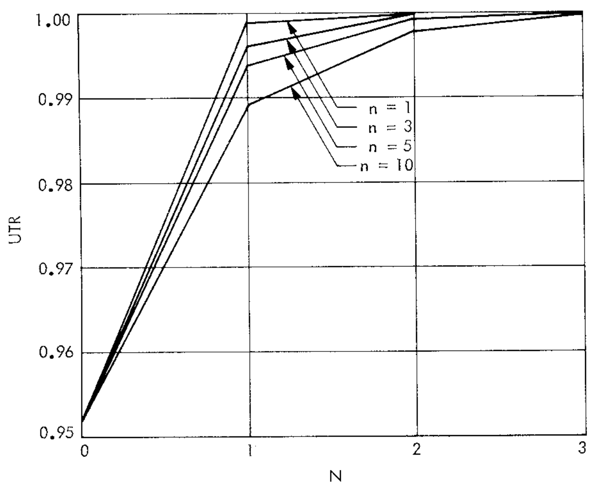

Fig. 1. *UTR* vs *N* for $\lambda = 150.45 \times 10^{-4}$/h, $1/\mu = 336$ hours

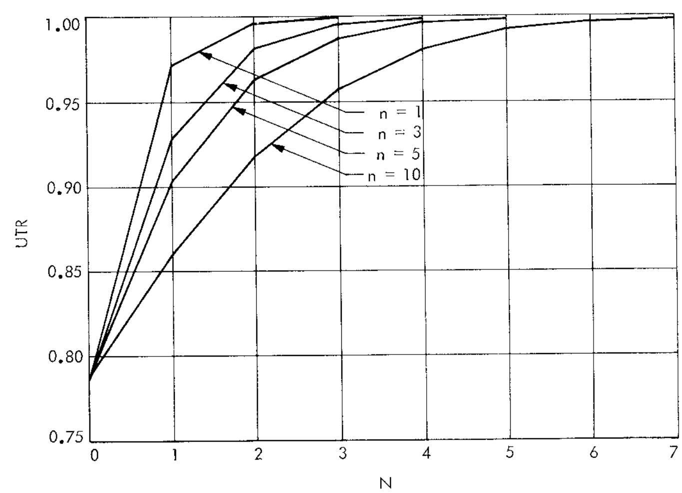

Fig. 2. *UTR* vs *N* for $\lambda = 150.45 \times 10^{-6}$/h, $1/\mu = 1776$ hours