Signal Suppression in Bandpass Limiters

D. W. Brown
R.F. Systems Development Section

The results of numerous studies and applications of hard and soft limiters are compared. A unifying approach is taken, with the result that the applications considered differ only in regard to the relative level of the input signal.

I. Introduction

At least three different bandpass limiter types or applications are treated in the literature. Most familiar is the hard limiter (Refs. 1–4). Other analyses consider the general case of soft limiters by assuming an error function (erf) transfer characteristic (Refs. 5 and 6). And finally, a special application of the soft limiter appears in the DSN telemetry subcarrier demodulation equipment (Refs.7–10).

The results obtained differ rather markedly in form and perhaps more subtly in application. Is there a unity to these analyses? What differences in application underlie them? It is the purpose of this article to try to answer these questions insofar as they apply to signal suppression.

Signal suppression α is a commonly used factor in expressing the effective signal gain, as a function of input signal-to-noise ratio (SNR), for a bandpass limiter element in a communications system. More rigorous definitions will follow as required.

II. Limiter Types

The hard limiter, in which the input is usually taken as constant in amplitude, and is always in limiting, regardless of input SNR, is commonly characterized by the approximation (Refs. 1 and 2)

$$\alpha_{hl}^2 \approx \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{\mathrm{SNR}}} \tag{1}$$

An approximate form, which will be shown to be equivalent to Eq. (19) of Ref. 5, for the general soft limiter follows:

$$\alpha_s^2 \approx \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{(1+\nu^2)\,\mathrm{SNR}}} \tag{2}$$

where $\nu^2 = \ell^2/S_i G^2$, ℓ^2 is defined as the output power of the signal (only) saturated bandpass limiter, S_i the input signal power, and G^2 the small signal power gain. Comparing with Eq. (1), we see that Eq. (2) immediately reduces to the hard limiter case for $\nu^2 \to 0$. In discussions that follow, it will be useful to recognize the ratio ℓ^2/G^2 as the effective input limit level.

The special application referred to in the introduction utilizes the bandpass limiter *following* the loop phase detector, and since it operates at an intermediate frequency, the limiter is in turn followed by a coherent detector. The interesting result of this mechanization is that the signal input to the bandpass limiter carries the loop phase error information as amplitude. Because the phase-locked loop in this application is designed for low tracking errors, the models (with the notable exception of Ref. 9) inherently assume that the input signal is much less than the input limit level.

One such application (Ref. 7) uses the form

$$\alpha_{sl}^2 \approx \frac{1}{1 + \frac{4}{\pi} \frac{1}{\nu^2 \mathrm{SNR}}} \tag{3}$$

with definitions as above, except that S_i and SNR now both refer to the full or virtual error signal, while assuming as above that the actual signal is negligibly small. Equation (3) was originally developed empirically by applying an effective input ν^2SNR to the approximate hard limiter form. However, Ref. 7 does apply a correcting function developed by Springett (Ref. 4) to account for the effective change in output SNR vs. input SNR in hard limiters.

Before proceeding to consideration of the exact forms, it may be of interest to note that one could speculate on a general form:

$$\alpha^2 \approx \frac{1}{1 + \frac{4}{\pi} \frac{1}{(\gamma^2 + \nu^2)\, \mathrm{SNR}}} \tag{4}$$

where γ^2 relates actual input signal power to virtual signal. When γ^2 is unity, (4) becomes (2), which, as noted above, includes (1) as a special case. And when $\gamma^2 \to 0$ (negligible input signal), (4) approaches (3). We will return later to these forms as special cases of Lesh's result (Ref. 6).

III. A General Model

Equation (29) of Ref. 6 may be rewritten to express the output signal power as[1]

$$S_o = A^2 G^2 \frac{D}{1 + D} \exp\left[- \frac{A^2}{N_o B (1 + D)} \right]$$

$$\times \left\{ I_0 \left[\frac{A^2}{2 N_o B (1 + D)} \right] + I_1 \left[\frac{A^2}{2 N_o B (1 + D)} \right] \right\}^2$$

where

A_2 = actual input signal power

$N_o B$ = input noise power

$D = \dfrac{2L^2}{\pi G^2 N_o B}$, a measure of noise limiting

$L^2 = \dfrac{\pi^2}{8} \ell^2$ = limiter saturation level

and I_0, I_1 are modified Bessel functions.

If we now further define γ^2 as the fractional input signal, identically equal to $A^2/S_i = A^2/\mathrm{SNR} N_o B$, then we have

$$S_o = A^2 G^2 \frac{D}{1 + D} \exp\left(- \frac{\gamma^2 \mathrm{SNR}}{1 + D} \right)$$

$$\times \left\{ I_0 \left[\frac{\gamma^2 \mathrm{SNR}}{2 (1 + D)} \right] + I_1 \left[\frac{\gamma^2 \mathrm{SNR}}{2 (1 + D)} \right] \right\}^2 \tag{5}$$

where $D = (\pi/4)\, \nu^2 \mathrm{SNR}$.

The notation in use here is somewhat redundant and has evolved from several of the references. Interchangeable forms are applied from time to time and are summarized in the Appendix.

IV. The Hard Limiter

For the ideal hard limiter, $G^2 \to \infty$, $\nu^2 \to 0$, and $D << 1$. And with $\gamma^2 = 1$, Eq. (5) becomes

$$S_o \Big|_{\substack{\text{HARD}\\\text{LIMITER}}} = A^2 G^2 D \exp(-\mathrm{SNR}) \times \left\{ I_0 \left[\frac{\mathrm{SNR}}{2} \right] + I_1 \left[\frac{\mathrm{SNR}}{2} \right] \right\}^2$$

$$\alpha_{hl}^2 = \frac{S_o}{\ell^2} = \frac{\pi}{4} \mathrm{SNR} \exp(-\mathrm{SNR}) \left\{ I_0 \left[\frac{\mathrm{SNR}}{2} \right] + I_1 \left[\frac{\mathrm{SNR}}{2} \right] \right\}^2$$

the exact form for the hard limiter (Ref. 3). Or, if we let

$$\exp(-\mathrm{SNR}) \left\{ I_0 \left[\frac{\mathrm{SNR}}{2} \right] + I_1 \left[\frac{\mathrm{SNR}}{2} \right] \right\}^2 \approx \frac{2/\pi}{2/\pi + \mathrm{SNR}/2} \tag{6}$$

[1]Setting to unity the $\cos\theta$ factor which applies to the coherent detection following the limiter.

Eq. (1), the approximate form, follows immediately. This approximation is asymptotically correct for large and small SNR; it is in error by a few percent when SNR is near unity.

V. The Soft Limiter—Special Application

Before continuing to the general limiter of Eqs. (2) and (4)[2], consider the exact form of Eq. (5) as applied to the $\gamma^2 \to 0$ (i.e., small signal) case:

$$S_o = A^2G^2 \frac{D}{1+D} = \gamma^2 \frac{\ell^2}{\nu^2} \frac{D}{1+D}$$

since the exponential/Bessel product approaches unity for small argument.

And if we define suppression,

$$\alpha_{sl}^2 = \frac{S_o}{\gamma^2 S_i G^2} = \frac{\text{output signal power}}{\text{fractional input signal} \times \text{unlimited virtual output signal}} \tag{7}$$

$$\alpha_{sl}^2 = \frac{D}{1+D} = \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{\nu^2 \text{SNR}}} \qquad \text{(see footnote 3)}$$

Or, if one prefers the definition of suppression to be the ratio of signal output powers with and without noise[4],

$$\alpha_{sl}^2 = \frac{\gamma^2\ell^2 D}{\nu^2(1+D)} \bigg/ \frac{\gamma^2\ell^2}{\nu^2} = \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{\nu^2\text{SNR}}}$$

In any case, it is at first surprising that Eq. (3), derived as it was from the approximate hard limiter form, should turn out to be exact. A little reflection on Eqs. (5) and (6) shows, however, that no approximation is involved in the limit of $\gamma^2 << \nu^2$. In other words, Eq. (3) is correct as it stands (for the small signal case), and it is inappropriate in this application to include Springett's $(\text{SNR})_o/(\text{SNR})_i$ correction as noted above.

For this subcarrier application, Refs. 8 and 10 develop and apply a somewhat different model, with results which can differ from the above, depending upon definitions of input signals. However, the results obtained here are in agreement with Ref. (9), given the small signal assumption.

VI. The Soft Limiter—General Case

Consider now Eq. (5) and apply the approximation (6), but let γ^2 and ν^2 retain generality:

$$S_o \approx \frac{\gamma^2}{\nu^2}\ell^2 \frac{1}{1 + \dfrac{\gamma^2}{\nu^2} + \dfrac{4}{\pi}\dfrac{1}{\nu^2\text{SNR}}} \tag{8}$$

Defining $\alpha^2 = S_o/\ell^2$,

$$\alpha^2 \approx \frac{\gamma^2}{\gamma^2+\nu^2} \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{(\gamma^2+\nu^2)\,\text{SNR}}} \tag{9}$$

Taking into account the necessary normalization of definitions and level $\gamma^2/(\nu^2+\gamma^2)$,we see that the hypothesis of Eq. (4) has been borne out and is applicable to all cases, with small sacrifice of accuracy only when the actual signal-to-noise ratio is near unity *and* the actual input signal is not small compared to the input limit level. One other case in minor error, but seldom if ever occurring in practice, is that of high signal-to-noise ratio *and* actual input signal near the input limit level.

Applying this result to the full signal case $\gamma^2 = 1$,

$$\alpha^2 \approx \frac{1}{1+\nu^2} \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{(1+\nu^2)\,\text{SNR}}} \tag{10}$$

we see the form of Eq. (2) as well as the P_s/L^2 result (approximately) of Fig. 5 of Ref. 5. Applying Tausworthe's definition of α_s as the ratio of signal output powers with and without noise,

$$\alpha_s^2 \approx \frac{1}{1 + \dfrac{4}{\pi}\dfrac{1}{(1+\nu^2)\,\text{SNR}}}$$

[2]Equations (2) and (4) can be seen to be redundant given a $\gamma^2\text{SNR} \to \text{SNR}$ transformation.

[3]In the subcarrier application, it is convenient to write

$$K_d = \frac{\text{limiter output}}{\text{phase error}} = \frac{2}{\pi}\frac{\ell}{\nu}\alpha'\alpha_{sl}\ \text{V/rad}$$

where $\alpha' = (\pi/2)(\gamma/\text{Phase error})$ for this mechanization (Ref. 8).

[4]This definition can be misleading if neither γ^2 or ν^2 is small.

and rearranging,

$$\alpha_s^2 \approx \frac{1}{1 + \dfrac{4}{\pi}\left(\dfrac{1 + \frac{\pi}{4}\nu^2 \mathrm{SNR}}{\mathrm{SNR}}\right)} \Bigg/ \frac{1}{1 + \nu^2}$$

and noting from Eq. (1) that

$$\alpha_{hl}^2(x) \approx \frac{1}{1 + \dfrac{4}{\pi}\left(\dfrac{1}{x}\right)}$$

it follows that

$$\alpha_s^2 \approx \frac{\alpha_{hl}^2\left(\dfrac{\mathrm{SNR}}{1 + \frac{\pi}{4}\nu^2 \mathrm{SNR}}\right)}{\alpha_{hl}^2\left(\dfrac{4}{\pi\nu^2}\right)}$$

It is readily apparent that this is Tausworthe's Eq. (19), where $\rho = \mathrm{SNR}$, since

$$\frac{\pi}{4}\nu^2 = \frac{\pi}{4}\left(\frac{8}{\pi^2}L^2\right)\frac{\gamma^2}{A^2G^2} = \frac{4}{\pi}\frac{L^2}{V^2K^2}$$

given that $V = \sqrt{2}A$, $K = G$, and $L^2 = (\pi^2/8)\,\ell^2$ and, of course, $\gamma^2 = 1$.

VII. Conclusion

It appears that there is a unity to the several treatments of hard and soft limiters, provided careful definitions are made. The significant difference in the various cases lies only in the amplitude of the input signal relative to the effective input limit level, expressed here as the γ^2/ν^2 ratio.

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Appendix

$$\mathrm{SNR} = \frac{S_i}{N_o B} = \text{virtual input signal-to-noise ratio}$$

$$\ell^2 = \frac{8}{\pi^2} L^2 = \text{maximum output signal power}$$

$$\nu^2 = \frac{\ell^2}{S_i G^2} = \frac{4}{\pi} \frac{D}{\mathrm{SNR}} = \frac{\text{input limit level}}{\text{virtual input signal}}$$

$$\nu^2 \mathrm{SNR} = \frac{\ell^2}{G^2 N_o B} = \frac{\text{input limit level}}{\text{input noise level}}$$

$$\gamma^2 = \frac{A^2}{S_i} = \frac{A^2}{\mathrm{SNR}\, N_o B} = \frac{\text{actual input signal}}{\text{virtual input signal}}$$

$$\gamma^2 \mathrm{SNR} = \frac{A^2}{N_o B} = \text{actual input signal-to-noise ratio}$$

$$\frac{\nu^2}{\gamma^2} = \frac{\ell^2}{A^2 G^2} = \frac{\text{input limit level}}{\text{actual input signal}}$$

α = signal suppression factor, general case
$0 < \gamma^2 \leqq 1, \nu^2 \geqq 0$

α_{hl} = signal suppression factor, hard limiter
$\gamma^2 = 1, \nu^2 \rightarrow 0$

α_{sl} = signal suppression factor, special case
$\gamma^2 \rightarrow 0, \nu^2 \geqq 0$

α_s = signal suppression factor, special case
$\gamma^2 = 1, \nu^2 \geqq 0$